POISSON STABILITY IN PRODUCT OF DYNAMICAL SYSTEMS

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(Received July 10, 1986 and in revised form October 14, 1986)

ABSTRACT. This paper deals with Poisson stable and distal dynamical system. It is shown that the product of Poisson stable motions is a Poisson stable motion in dynamical systems.

KEY WORDS AND PHRASES. *Distal dynamical system, compact, Poisson stable, recurrent motions.*

1980 AMS SUBJECT CLASSIFICATION CODE. 34C35.

1. INTRODUCTION.

Product of dynamical systems is a dynamical system and product of semidynamical systems [1] is a semidynamical system. In [1] Bajaj has shown that if $\prod_{\alpha \in I} \Omega_{\alpha}$, $\alpha \in I$ be the product of positive limit sets of the factor systems and Ω_x be the positive limit set of the product system then $\Omega_x \subset \prod_{\alpha \in I} \Omega_{\alpha}$. In general, $\subset$ cannot be replaced by equality. Indeed Ω_x may be empty even if each of Ω_{α} , $\alpha \in I$ is nonempty. He has found the conditions under which $\Omega_1 \times \Omega_2 = \Omega_x$ and Ω_x could be nonempty and compact for a finite product.

We studied Poisson stable and distal dynamical systems in [2]. In this paper we shall mention the conditions under which $\Omega_x = \prod_{\alpha \in I} \Omega_{\alpha}$, $\alpha \in I$ and the product of Poisson stable motions is a Poisson stable motion in dynamical systems.

2. DEFINITIONS AND NOTATIONS.

We shall use the definition and notations of [2] and [3].

3. MAIN RESULTS.

PROPOSITION 3.1. If a motion $\pi(x, t)$ is Poisson stable and distal then its trajectory $\gamma(x)$ is a closed set. [2, theo.2.1].

THEOREM 3.2. If a compact motion is Poisson stable and distal then it is a compact recurrent motion.

PROOF. Let the motion $\pi(x, t)$ be Poisson stable and distal then its trajectory $\gamma(x)$ is closed hence $\gamma(x) = \text{cl } \gamma(x) = \Omega_x$.

As the motion is compact each of the above sets is compact and minimal, and thus by Birkhoff recurrence theorem $\pi(x, t)$ is compact recurrent.

THEOREM 3.3. Let (X, π) be a semidynamical system, let π be Lagrange stable, then π is distal iff for every net t_i in $\mathbb{R}^+$, the phase space $X = \{z \in X : x t_i \rightarrow z \text{ for some } x \in X \text{ and some subnet } t_j \text{ of } t_i\}$. [4, theorem 2.6].

One can show that positive distal dynamical system are distal whenever all positive trajectory closures are compact. Thus the above result for semidynamical systems is applicable in present dynamical systems as well.

THEOREM 3.4. A compact motion $\pi(x, t)$ is Poisson stable and distal iff $\gamma(x) = \text{cl}\gamma(x) = \Omega_x$ is compact and minimal.

PROOF. If a compact motion $\pi(x, t)$ is Poisson stable and distal then $\gamma(x) = \text{cl}\gamma(x) = \Omega_x$ is compact and minimal. Conversely, let $\gamma(x) = \text{cl}\gamma(x) = \Omega_x$ be compact and minimal then it is compact recurrent and therefore compact Poisson stable.

To show the dynamical system (X, π) is distal we shall use theorem 3.3. Here $\gamma(x)$ is compact and minimal, for every x in X the net xt_i in $\gamma(x)$ has a subnet $xt_j \rightarrow z$ in $\gamma(x)$.

Thus $X = \{z \in X : xt_j \rightarrow z \text{ for some net } t_i \text{ in } T \text{ and } t_j \text{ is some subnet of } t_i \text{ in } T\}$.

Which completes the proof.

THEOREM 3.5. The trajectory $\gamma(x)$ of a compact Poisson stable distal motion is complete and totally bounded.

PROOF. Since $\gamma(x)$ is compact in a Hausdorff uniform space X .

Now we shall consider product of such dynamical systems. Let (X_α, π_α) , $\alpha \in I$ be a family of dynamical systems. Let $X = \prod X_\alpha$ be the product space. Let $x \in X$, $x = \{x_\alpha\}$. Define a map π from $X \times T$ into X by $\pi(x, t) = \{x_\alpha t\}$, $\alpha \in I$ then (X, π) is a dynamical system.

The dynamical system (X, π) obtained above is called direct product or product of the family (X_α, π_α) , $\alpha \in I$.

PROPOSITION 3.6. Let (X_α, π_α) , $\alpha \in I$ be a family of dynamical systems and (X, π) the product dynamical system. Let $x \in X$, $x = \{x_\alpha\}$. Then $\Omega_x \subset \prod \Omega_{x_\alpha}$.

THEOREM 3.7. Let (X_α, π_α) , $\alpha \in I$ be a family of {Lagrange stable} {distal} dynamical systems and (X, π) is the product dynamical system. Let $x \in X$ and $x = \{x_\alpha\}$ then (X, π) is {Lagrange stable} {distal}.

THEOREM 3.8. Let (X_α, π_α) , $\alpha \in I$ be Lagrange stable distal dynamical systems. Let $x \in X$, $x = \{x_\alpha\}$ and (X, π) their product dynamical system, then the product $\pi(x, t)$ is Poisson stable motion in the product space (X, π) iff each $\pi_\alpha(x_\alpha, t)$, $\alpha \in I$ is Poisson stable motion.

PROOF. We have $\gamma(x) = \prod \gamma_\alpha(x_\alpha) = \prod \text{cl } \gamma_\alpha(x_\alpha) = \text{cl } \prod \gamma_\alpha(x_\alpha) = \text{cl}\gamma(x)$. Also $\gamma(x) = \prod \gamma_\alpha(x_\alpha) = \prod \Omega_{x_\alpha} = \Omega_x$. Therefore $\gamma(x) = \text{cl}\gamma(x) = \Omega_x$, each being compact minimal and closed.

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