ON A CLASS OF p -VALENT CLOSE-TO-CONVEX FUNCTIONS OF ORDER β AND TYPE α

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ABSTRACT. Let $S(A, B, p, \alpha)$ denote the class of functions $g(z) = z^p + \sum_{n=p+1}^{\infty} b_n z^n$ analytic in the unit disc $U = \{z: |z| < 1\}$ and satisfying the condition

$$\frac{zg'(z)}{g(z)} < \frac{p + [pB + (A-B)(p-\alpha)]z}{1+Bz}, \quad z \in U, \quad -1 \leq B < A \leq 1, \quad 0 \leq \alpha < p.$$

Let $C(A, B, p, \beta, \alpha)$ denote the class of functions $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n$ analytic in U , and satisfying the condition

$$\operatorname{Re} \left\{ \frac{zf'(z)}{g(z)} \right\} > \beta, \quad z \in U, \quad g \in S(A, B, p, \alpha).$$

In this paper we determine the coefficient estimates and distortion theorems for the class $C(A, B, p, \beta, \alpha)$.

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1. INTRODUCTION.

Let A_p (p a fixed integer greater than zero) denote the class of functions $f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k$ which are analytic in $U = \{z: |z| < 1\}$. Let Ω denote the class of bounded analytic functions $w(z)$ in U satisfying the conditions $w(0) = 0$ and $|w(z)| \leq |z|$, for $z \in U$. We use P to denote the class of functions $P_1(z) = 1 + \sum_{n=1}^{\infty} d_n z^n$ which are analytic in U and have a positive real part there.

Also we use $P(p, \beta)$ the class of functions that have the form $P(z) = p + \sum_{k=1}^{\infty} c_k z^k$,

which are analytic in U and satisfy the conditions $P(0) = p$ and $\operatorname{Re}\{P(z)\} > \beta$ ($0 \leq \beta < p$) in U . The class $P(p, \beta)$ was introduced by Patil and Thakare [1]. It is well known that a function $P(z)$ is in $P(p, \beta)$ if and only if there exists a function $P_1(z) \in P$ such that

$$P(z) = (p - \beta) P_1(z) + \beta. \quad (1.1)$$

For $-1 \leq B < A \leq 1$ and $0 \leq \alpha < p$, denote by $S(A, B, p, \alpha)$ the class of functions $g(z) \in A_p$ which satisfy

$$\frac{zg'(z)}{g(z)} < \frac{p+[pB+(A-B)(p-\alpha)]z}{1+Bz}, \quad z \in U.$$

By definition of subordination it follows that $g(z) \in S(A, B, p, \alpha)$ has a representation of the form

$$\frac{zg'(z)}{g(z)} = \frac{p+[pB+(A-B)(p-\alpha)]w(z)}{1+Bw(z)}, \quad w \in \Omega. \quad (1.2)$$

Obviously $S(A, B, p, \alpha)$ is a subclass of the class $S_p(\alpha)$, $0 \leq \alpha < p$, of p -valent star-like functions of order α , investigated by Goluzina [2]. The class $S(A, B, p, \alpha)$ introduced by the author [3].

Moreover, let $C(A, B, p, \beta, \alpha)$ denote the class of functions $f(z) \in A_p$ which satisfy

$$\operatorname{Re} \left\{ \frac{zf'(z)}{g(z)} \right\} > \beta, \quad g \in S(A, B, p, \alpha). \quad (1.3)$$

Thus if $f(z) \in C(A, B, p, \beta, \alpha)$, then we may write

$$zf'(z) = g(z) P(z), \quad P(z) \in P(p, \beta). \quad (1.4)$$

We note that $C(1, -1, 1, \beta, \alpha) = C(\beta, \alpha)$, is a subclass of the class of close-to-convex functions of order β and type α introduced by Libera [4].

Let $\phi(z) = \sum_{n=0}^{\infty} \alpha_n z^n$ and $\psi(z) = \sum_{n=0}^{\infty} \beta_n z^n$ be any two functions, then by $\phi(z) * \psi(z)$ we shall mean the Hadamard product or convolution of $\phi(z)$ and $\psi(z)$, that is

$$\phi(z) * \psi(z) = \sum_{n=0}^{\infty} \alpha_n \beta_n z^n.$$

We state below some lemmas that are needed in our investigation.

LEMMA 1 [1]. If $P(z) = p + \sum_{k=1}^{\infty} c_k z^k \in P(p, \beta)$, then

$$|c_n| \leq 2(p - \beta) \text{ for all } n. \quad (1.5)$$

The function $P_0(z)$ defined by

$$P_0(z) = \frac{p+(p-2\beta)\delta_1 z}{1-\delta_1 z}, \quad |\delta_1| = 1 \quad (1.6)$$

shows that the result is sharp for each $n \geq 1$.

LEMMA 2 [1]. If $P(z) \in P(p, \beta)$, then for $|z| = r < 1$

$$\frac{p-|p-2\beta|r}{1+r} \leq |P(z)| \leq \frac{p+|p-2\beta|r}{1-r}. \quad (1.7)$$

Equality occurs for $P_0(z)$ defined by (1.6).

LEMMA 3 [5]. If $\psi(z)$ is regular in U , $\phi(z)$ and $h(z)$ are convex univalent in U such that $\psi(z) < \phi(z)$, then $\psi(z) * h(z) < \phi(z) * h(z)$, $z \in U$.

LEMMA 4 [3]. If $g(z) = z^p + \sum_{n=p+1}^{\infty} b_n z^n \in S(A, B, p, \alpha)$, then for $n \geq p+1$

$$|b_n| \leq \frac{n-(p+1)}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} \quad (1.8)$$

The function $g_o(z)$ defined by

$$g_o(z) = z^p(1+B\delta_2z)^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0, \quad |\delta_2| = 1$$

shows that the result is sharp for each $n \geq p+1$.

2. COEFFICIENT ESTIMATES FOR THE CLASS $C(A, B, p, \beta, \alpha)$.

THEOREM 1. If $f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \in C(A, B, p, \beta, \alpha)$, then for $n \geq p+1$

$$|a_n| \leq \frac{p}{n} \cdot \frac{n-(p+1)}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} + \frac{2(p-\beta)}{n} \left[1 + \sum_{k=0}^{n-(p+2)} \frac{k}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} \right]. \quad (2.1)$$

The result is sharp.

PROOF. Since $f \in C(A, B, p, \beta, \alpha)$, it follows that

$$zf'(z) = g(z)P(z), \quad (2.2)$$

where $g(z) = z^p + \sum_{k=1}^{\infty} b_{p+k} z^{p+k} \in S(A, B, p, \alpha)$ and $P(z) = p + \sum_{k=1}^{\infty} c_k z^k \in P(A, B, p, \beta)$.

Hence

$$[pz^p + \sum_{k=1}^{\infty} (p+k)a_{p+k} z^{p+k}] = [z^p + \sum_{k=1}^{\infty} b_{p+k} z^{p+k}] \times [p + \sum_{k=1}^{\infty} c_k z^k]. \quad (2.3)$$

Equating coefficients of z^n on both sides of (2.3), we obtain

$$na_n = pb_n + c_1 b_{n-1} + c_2 b_{n-2} + \dots + c_{n-p-1} b_{p+1} + c_{n-p}$$

This gives

$$n|a_n| \leq p|b_n| + |c_1||b_{n-1}| + |c_2||b_{n-2}| + \dots + |c_{n-p-1}||b_{p+1}| + |c_{n-p}|. \quad (2.4)$$

Substituting the value from (1.5) and (1.8) in (2.4), we obtain

$$n|a_n| \leq p \cdot \frac{n-(p+1)}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} + 2(p-\beta) \times \\ \left[\frac{n-(p+2)}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} + \frac{n-(p+3)}{\pi} \frac{|(B-A)(p-\alpha)+Bj|}{(j+1)} + \dots + (A-B)(p-\alpha)+1 \right]$$

which, on simplification, takes the form of (2.1). The function $f_o(z)$ defined by

$$f_o(z) = z^{p-1} \cdot \frac{p+(p-2\beta)\delta_1z}{1-\delta_1z} \cdot (1+B\delta_2z)^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0, \\ |\delta_1| = |\delta_2| = 1, \quad (2.5)$$

shows that the bound (2.1) is sharp for each $n \geq p+1$.

Remarks on Theorem 1.

(1) Choosing $p=1$ and $\alpha=\beta=0$ in Theorem 1, we get the result due to Goel and Mehrotra [6].

(2) Choosing $p=1$, $A=1$, $B=-1$ and $\alpha=\beta=0$ in Theorem 1, we get the result due to Reade [7].

3. DISTORTION THEOREMS.

LEMMA 5. If $g \in S(A, B, p, \alpha)$, then for all $|s| \leq 1$, $|t| \leq 1$, ($s \neq t$)

$$\frac{t^p g(sz)}{s^p g(tz)} < \left[\frac{(1+Bsz)}{(1+Btz)} \right]^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0; \quad (3.1)$$

$$\frac{t^p g(sz)}{s^p g(tz)} < \exp [A(p-\alpha)(s-t)z], \quad B=0. \quad (3.2)$$

PROOF. The proof is similar to the one given by Ruscheweyh [8] and Goel and Mehrak [6].

We first consider the case when $B \neq 0$. We have

$$\frac{zg(z)}{g(z)} < \frac{p+[pB+(A-B)(p-\alpha)]z}{1+Bz}, \quad z \in U.$$

This implies that

$$\frac{zg(z)}{g(z)} - p < \frac{p+[pB+(A-B)(p-\alpha)]z}{1+Bz} - p = \frac{(A-B)(p-\alpha)z}{1+Bz}, \quad (3.3)$$

where $\frac{(A-B)(p-\alpha)z}{1+Bz}$ is convex, univalent in U . For $|s| \leq 1$, $|t| \leq 1$, ($s \neq t$),

$$h(z) = \int_0^z \left(\frac{s}{1-su} - \frac{t}{1-tu} \right) du \quad (3.4)$$

is convex, univalent in U . (3.3) and (3.4) satisfy the conditions of lemma 3, and therefore

$$\left[\frac{zg(z)}{g(z)} - p \right]_* h(z) < \frac{(A-B)(p-\alpha)}{(1+Bz)} * h(z). \quad (3.5)$$

Now for every analytic function $q(z)$ with $q(0) = 0$, we have

$$q(z) * h(z) = \int_{tz}^{sz} q(u) \frac{du}{u}. \quad (3.6)$$

By the application of (3.6), (3.5) can be written as

$$\int_{tz}^{sz} \left[\frac{ug'(u)}{g(u)} - p \right] \frac{du}{u} < (A-B)(p-\alpha) \int_{tz}^{sz} \frac{du}{1+Bu}$$

from which (3.1) follows.

Similarly for $B=0$, we obtain (3.2).

LEMMA 6. If $g \in S(A, B, p, \alpha)$, then for $|z| = r < 1$

$$r^p \frac{(A-B)(p-\alpha)}{B} \leq |g(z)| \leq r^p \frac{(A-B)(p-\alpha)}{B}, \quad B \neq 0, \quad (3.7)$$

$$r^p \exp(-A(p-\alpha)r) \leq |g(z)| \leq r^p \exp(A(p-\alpha)r), \quad B = 0. \quad (3.8)$$

$$\left| \arg \frac{g(z)}{z^p} \right| \leq \frac{(A-B)(p-\alpha)}{B} \sin^{-1}(Br), \quad B \neq 0, \quad (3.9)$$

$$\left| \arg \frac{g(z)}{z^p} \right| \leq A(p-\alpha)r, \quad B = 0. \quad (3.10)$$

These bounds are sharp, being attained by the function $g_0(z)$ defined by

$$g_0(z) = z^p (1+B \delta_2 z)^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0,$$

$$= z^p \exp(A(p-\alpha)\delta_2 z), \quad B = 0, \quad |\delta_2| = 1.$$

PROOF. The author [3] proved the results in lemma 6 using a different method. However, we deduce them from lemma 5.

Taking $s=1$, $t=0$ in (3.1) and (3.2), we get

$$\frac{g(z)}{z^p} < (1+Bz)^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0, \quad (3.11)$$

$$\frac{g(z)}{z^p} < \exp(A(p-\alpha)z), \quad B = 0. \quad (3.12)$$

(3.11) implies that $\frac{g(z)}{z^p} = (1+Bw(z))^{\frac{(A-B)(p-\alpha)}{B}}, \quad B \neq 0.$

(i) When $B > 0$

$$\begin{aligned} \left| \frac{g(z)}{z^p} \right| &= \left| (1+Bw(z))^{\frac{(A-B)(p-\alpha)}{B}} \right| \\ &= \left| \exp\left[\frac{(A-B)(p-\alpha)}{B} \log(1+Bw(z))\right] \right| \\ &= \exp \operatorname{Re}\left[\frac{(A-B)(p-\alpha)}{B} \log(1+Bw(z))\right] \\ &= \exp\left(\frac{(A-B)(p-\alpha)}{B} \log|1+Bw(z)|\right) \\ &\leq \frac{(A-B)(p-\alpha)}{B} \\ &\leq (1+Br)^{\frac{(A-B)(p-\alpha)}{B}}. \end{aligned}$$

(ii) When $B < 0$, put $B = -C$, $C > 0$.

$$\begin{aligned} \left| (1+Bw(z))^{\frac{(A-B)(p-\alpha)}{B}} \right| &= \left| ((1-Cw(z))^{-1})^{\frac{(A-B)(p-\alpha)}{C}} \right| \\ &= \left| (1-Cw(z))^{-1} \right|^{\frac{(A-B)(p-\alpha)}{C}} \\ &\leq \left(\frac{1}{1-Cr} \right)^{\frac{(A-B)(p-\alpha)}{C}} \\ &= (1+Br)^{\frac{(A-B)(p-\alpha)}{B}}. \end{aligned}$$

Similarly (3.8) is a direct consequence of (3.12).

For $|z| = r$, from (3.11), we get

$$\left| \arg \frac{g(z)}{z^p} \right| = \frac{(A-B)(p-\alpha)}{B} \left| \arg(1+Bw(z)) \right| \leq \frac{(A-B)(p-\alpha)}{B} \sin^{-1}(Br).$$

Similarly (3.10) is a direct consequence of (3.12).

THEOREM 2. If $f \in C(A, B, p, \beta, \alpha)$, then for $|z| = r < 1$

$$r^{p-1} \frac{p-|p-2\beta|r}{1+r} (1-Br) \frac{(A-B)(p-\alpha)}{B} \leq |\hat{f}(z)| \leq r^{p-1} \frac{p+|p-2\beta|r}{1-r} (1+Br) \frac{(A-B)(p-\alpha)}{B}, \quad B \neq 0, \quad (3.13)$$

$$r^{p-1} \frac{p-|p-2\beta|r}{1+r} \exp(-A(p-\alpha)r) \leq |f'(z)| \leq r^{p-1} \frac{p+|p-2\beta|r}{1-r} \exp(A(p-\alpha)r), \quad B = 0; \quad (3.14)$$

$$\int_0^r r^{p-1} \frac{p-|p-2\beta|r}{1+r} (1-Br) \frac{(A-B)(p-\alpha)}{B} dr \leq |f(z)| \leq \int_0^r r^{p-1} \frac{p+|p-2\beta|r}{1-r} (1+Br) \frac{(A-B)(p-\alpha)}{B} dr, \quad B \neq 0; \quad (3.15)$$

$$\int_0^r r^{p-1} \frac{p-|p-2\beta|r}{1+r} \exp(-A(p-\alpha)r) dr \leq |f(z)| \leq \int_0^r r^{p-1} \frac{p+|p-2\beta|r}{1-r} \exp(A(p-\alpha)r) dr, \quad B = 0 \quad (3.16)$$

All those inequalities are sharp.

PROOF. From (2.2), we have

$$|f'(z)| = \left| \frac{g(z)}{z} \right| |p(z)|. \quad (3.17)$$

Using (1.7), (3.7) and (3.8) in (3.17), we obtain (3.13) and (3.14).

Now

$$|f(z)| = \left| \int_0^r \hat{f}(z) dz \right| \leq \int_0^r |\hat{f}(z)| dr \leq \begin{cases} \int_0^r r^{p-1} \frac{p+|p-2\beta|r}{1-r} (1+Br) \frac{(A-B)(p-\alpha)}{B} dr, & B \neq 0; \\ \int_0^r r^{p-1} \frac{p+|p-2\beta|r}{1-r} \exp(A(p-\alpha)r) dr, & B = 0. \end{cases}$$

Let z_0 , $|z_0| = r$, be chosen in such a way that $|f(z_0)| \leq |f(z)|$, for all z , $|z| = r$. If $L(z_0)$ is the pre-image of the segment $[\overline{0, f(z_0)}]$ in U , then

$$|f(z_0)| = \int_{L(z_0)} |\hat{f}(z)| |dz| \geq \int_{L(z_0)} |\hat{f}(z)| dr \geq \begin{cases} \int_0^r r^{p-1} \frac{p-|p-2\beta|r}{1+r} (1-Br) \frac{(A-B)(p-\alpha)}{B} dr, & B \neq 0; \\ \int_0^r r^{p-1} \frac{p-|p-2\beta|r}{1+r} \exp(-A(p-\alpha)r) dr, & B = 0. \end{cases}$$

Equality signs in (3.13), (3.14), (3.15) and (3.16) are attained by the function $f_0(z)$ defined by (2.5) with $B \neq 0$, and

$$\hat{f}_0(z) = z^{p-1} \frac{p+(p-2\beta)\delta_1 z}{1-\delta_1 z} \exp(A(p-\alpha)\delta_2 z), \quad B=0, \quad |\delta_1| = |\delta_2| = 1.$$

Remarks on Theorem 2.

Choosing $p=1$ and $\alpha=\beta=0$ in Theorem 2, we get the result due to Goel and Mehrotra [6].

4. ARGUMENT OF $\hat{f}(z)$.

LEMMA 7. Let $p(z) \in P(p, \beta)$. Then for $|z| \leq r$,

$$\left| p(z) - \frac{p+(p-2\beta)r^2}{1-r^2} \right| \leq \frac{2(p-\beta)r}{1-r^2}. \quad (4.1)$$

PROOF. It is well known [9] that for $P_1(z) \in P$

$$\left| P_1(z) - \frac{1+r^2}{1-r^2} \right| \leq \frac{2r}{1-r^2}. \quad (4.2)$$

Thus the result follows from (1.1) and (4.2).

LEMMA 8. If $P(z) \in P(p, \beta)$, then

$$|\arg P(z)| \leq \sin^{-1} \frac{2(p-\beta)r}{p+(p-2\beta)r^2}, \quad |z| = r. \quad (4.3)$$

The bound is sharp.

PROOF. The proof follows from Lemma 7. To see that the result is sharp, let

$$P(z) = p \left\{ \frac{1+(1-\frac{2\beta}{p})\delta_1 z}{1-\delta_1 z} \right\},$$

$$\delta_1 = \frac{r}{z} \left\{ \frac{\frac{2\beta}{p}r}{1-(1-\frac{2\beta}{p})r^2} + \frac{i\sqrt{1-(1-\frac{2\beta}{p})^2 r^2} \sqrt{1-r^2}}{1-(1-\frac{2\beta}{p})r^2} \right\}. \quad (4.4)$$

THEOREM 3. If $f \in C(A, B, p, \beta, \alpha)$, then

$$\left| \arg \frac{\hat{f}(z)}{z^{p-1}} \right| \leq \frac{(A-B)(p-\alpha)}{B} \sin^{-1}(Br) + \sin^{-1} \frac{2(p-\beta)r}{p+(p-2\beta)r^2}, \quad B \neq 0, \quad (4.5)$$

$$\left| \arg \frac{\hat{f}(z)}{z^{p-1}} \right| \leq A(p-\alpha)r + \sin^{-1} \frac{2(p-\beta)r}{p+(p-2\beta)r^2}, \quad B=0. \quad (4.6)$$

These inequalities are sharp.

PROOF. From (2.2), we have $\frac{\hat{f}(z)}{z^{p-1}} = \frac{g(z)}{z^p} P(z)$. Thus

$$\arg \frac{\hat{f}(z)}{z^{p-1}} = \arg \frac{g(z)}{z^p} + \arg P(z). \quad (4.7)$$

Using (3.9), (3.10) and (4.3) in (4.7), we obtain (4.5) and (4.6). Equality signs in (4.5) and (4.6) hold for the function $f_1(z)$ and $f_2(z)$ respectively, where

$$f_1'(z) = p z^{p-1} \frac{1 + (1 - \frac{2\beta}{p})\delta_1 z}{1 - \delta_1 z} (1 + B \delta_2 z)^{\frac{(A-B)(p-\alpha)}{B}},$$

$$f_2'(z) = p z^{p-1} \frac{1 + (1 - \frac{2\beta}{p})\delta_1 z}{1 - \delta_1 z} \cdot \exp(A(p-\alpha) \delta_2 z),$$

where

$$\delta_1 = \frac{r}{z} \left\{ \frac{2 \frac{\beta}{p} r}{1 - (1 - \frac{2\beta}{p})r^2} + \frac{i\sqrt{1 - (1 - \frac{2\beta}{p})^2 r^2} \sqrt{1 - r^2}}{1 - (1 - \frac{2\beta}{p})r^2} \right\},$$

$$\delta_2 = \frac{r}{z} [-Br + i\sqrt{1 - B^2 r^2}].$$

Remarks on Theorem 3.

(1) Choosing $p=1$ and $\alpha=\beta=0$ in Theorem 3, we get the result due to Goel and Mehrotra [6].

(2) Choosing $p=1$, $A=1$ and $B=-1$ in Theorem 3, we get the result due to Silverman [10].

(3) Choosing $p=1$, $A=1$, $B=-1$ and $\alpha=\beta=0$ in Theorem 3, we get the result due to Ogawa [11] and Krzyz [12].

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