ABSTRACT. The study of R-type summability methods is continued in this paper by showing that two such methods are identical on the bounded portion of the strong summability field associated with the methods. It is shown that this "bounded consistency" applies for many non-matrix methods as well as for regular matrix methods.

KEY WORDS AND PHRASES. Sequences Spaces, Regular Summability Methods, Zeroclass.

1980 AMS SUBJECT CLASSIFICATION CODES. 40D20, 40F05, 40H05.

1. INTRODUCTION.

In [1] a relation between densities and strong convergence fields was studied for R-type summability methods (RSM). On the space $m$, the space of all bounded real sequences, RSMs are equivalent to the bounded generalized limits (see [2]). In the main Theorem of [3] Freedman actually proved a consistency theorem for the strong convergence fields of generalized limits. This statement will be explained at the end of Section 2. In this paper we extend the results of [1] and [3] to obtain a Bounded Consistency Theorem for strong summability fields of RSMs. We will require a characterization of RSMs in terms of zeroclasses. This paper is a continuation of [2], therefore we will accept notation and definitions of [1], [2] and [3]. In particular a class $X$ of subsets of $I$, the set of positive integers, is called a zeroclass if the following conditions holds:

(a) $A$ is finite $\Rightarrow A \in X$
(b) $A, B \in X \Rightarrow A \cup B \in X$
(c) $A \subseteq B \in X \Rightarrow A \in X$
(d) $I \notin X$.

Further, if $x \in \omega$, $r \in R$ and $A \subseteq I$ with $I - A$ infinite, then by $x \rightarrow r$ (A) we shall mean that for any $\varepsilon > 0$ there exists $N > 0$ such that $|x_n - r| < \varepsilon$ whenever $n \geq N$ and $n \notin A$. If $X$ is a zeroclass, then $\omega_x = \{x \in \omega: x \rightarrow r$ for some $A \in X and real $r\}$ is called the space of all $X$-nearly convergent sequences. The sequence space $\omega_x$ contains $c$, the space of all convergent sequences. By a summability method we will simply mean a real valued linear functional $S$ defined on some spaces $C_x \subseteq \omega$. We shall call $S$ regular if $c \subseteq C_x$ and $S(x) = \lim x$ for each $x \in c$. We let
$C_S^0 = \{ x \in C_S: S(x) = 0 \}$,

$|C_S|^{0} = \{ x \in \omega: |x| \in C_S^0 \}$

and

$|C_S| = \{ x \in \omega: x - r \in |C_S|^{0} \text{ for some real } r \}$.

The set $|C_S|$ is called the strong summability field associated with the method $S$. A method $S$ will be called an RSM when $S$ is regular and $m|C_S|^{0} = |C_S|^{0}$. (i.e., $|C_S|^{0}$ is solid). If $S$ is an RSM, then $|C_S|$, $|C_S|^{0}$ are subspaces of $C_S$ ([1] Proposition 4.9).

If $S: C_S \rightarrow R$ is an RSM, then $X_S = \{ A \subset I: S(\chi_A) = 0 \}$ is a zero class where $\chi_A$ is the characteristic function of $A$. We shall say that $X_S$ is the zero class related to $S$. For any $x \in \omega$ and $\varepsilon > 0$, let

$N_{\varepsilon} (x) = \{ y \in \omega: \sup \{|x_i - y_i|: i = 1, 2, 3, \ldots \} < \varepsilon \}$

Then the class $\{ N_{\varepsilon} (x): \ x \in \omega, \ \varepsilon > 0 \}$ forms a base for the topology $T_\omega$ on $\omega$. For any RSM $S: (C_S, T_\omega) \rightarrow R$, $S$ is continuous ([2]).

2. BOUNDED CONSISTENCY ON STRONG CONVERGENCE FIELDS.

In this section we first develop a theory of strong convergence fields with the help of the zero class concept.

**DEFINITION 2.1.** For any zero class $X$ we denote

$V^0_X = \{ x \in \omega: \text{For any } \alpha > 0, \{ i: i < |x_i| \} \in X \}$

$V_X = \{ x \in \omega: x-r \in V^0_X \text{ for some } r \in R \}$.

**PROPOSITION 2.1.** For any zero class $X$,

$V_X$ is a linear space of sequences. (2.1)

$V^0_X$ is a subspace of $V_X$. (2.2)

**PROOF.** Suppose that $x,y \in V^0_X, r_1, r_2 \in R$ with $x-r_1, y-r_2 \in V^0_X$. For each $i$ and for any $\alpha > 0$,

$|x_i - r_1| \leq \alpha/2$ and $|y_i - r_2| \leq \alpha/2 \implies |x_i + y_i - (r_1 + r_2)| \leq \alpha$.

Thus

$\{ i: \ |x_i + y_i - (r_1 + r_2)| > \alpha \} \subset \{ i: \ |x_i - r_1| > \alpha/2 \} \cup \{ i: \ |y_i - r_2| > \alpha/2 \}$.

By the definition of $V^0_X$ and the properties of zero classes $\{ i: \ |x_i - r_1| > \alpha/2 \} \cup \{ i: \ |y_i - r_2| > \alpha/2 \} \in X$ and $\{ i: \ |x_i + y_i - (r_1 + r_2)| > \alpha \} \in X$. Consequently we get that $x+y \in V_X$. If $k \in R$, then for any $\alpha > 0$,
Therefore for any $\alpha > 0$, \{i: \alpha < |kx_1 - kr_1|\} $\in X$, which implies $kx \in V_x$. Hence $V_x$ is a linear space of sequences.

(2.2) is obvious.

PROPOSITION 2.2. For any zero-class $X$, let $T_x: V_x \to \mathbb{R}$ be the function from $V_x$ to $\mathbb{R}$ defined by $T_x(x) = r$ when $x-r \in V_x^O$. Then $T_x$ is an RSM with domain $V_x$, $|V_x| = V_x$ and $|V_x|^{O} = V_x^{O}$. Further, $X$ is related to $T_x$.

PROOF. In this proof we will write $T_x$ as $T$ for convenience. First, we show $T$ is well defined. If $x \in V_x$, $T(x) = r_1$ and $T(x) = r_2$, so that $x-r_1, x-r_2 \in V_x^O$, then, since $V_x^O$ is a linear space, $(x-r_1) - (x-r_2) = (r_2-r_1)e \in V_x^O$ where $e = (1,1,1,\ldots)$. From the fact that for any $\alpha > 0$

$$\{i: \alpha < |(r_2-r_1)e_i|\} = \begin{cases} \phi, & \text{if } \alpha \geq |r_2-r_1|, \\ I, & \text{if } \alpha < |r_2-r_1| \end{cases}$$

and $\{i: \alpha < |(r_2-r_1)e_i|\} \in X$, it follows that $r_1 = r_2$.

Suppose that $x, y \in V_x$ and $T(x) = r_1$ and $T(y) = r_2$. Then $x-r_1, y-r_2 \in V_x^O$. Since $V_x^O$ is a linear space, $(x+y) - (r_1 + r_2) \in V_x^O$ and $kx - kr_1 \in V_x^O$ for any $k \in \mathbb{R}$. Therefore $T(x+y) = T(x) + T(y)$ and $T(kx) = kT(x)$. Hence $T$ is a linear functional.

Next we have

$$V_x^O = \{x \in \mathbb{R}: \text{for any } \alpha > 0, \{i: \alpha < |x_i|\} \in X\} = \{x \in \mathbb{R}: |x| \in V_x\} = |V_x|^{O},$$

where $V_x = V_x^{O} \bullet e = |V_x|^{O} \bullet e = |V_x|$.

Suppose that $x \in |V_x|^{O}$ and $|y| \leq |x|$ (i.e., $|y_1| \leq |x_1|$ for any i). Thus for any $\alpha > 0$, \{i: \alpha < |y_i|\} $\in X$ and so \{i: \alpha < |y_1|\} $\in X$. Thus we have $y \in |V_x|^{O}$ ([[1] Proposition 1]). Hence $T$ is an RSM.

For any $A \subset I$,

$$\{i: \alpha < \chi_A(i)\} = \begin{cases} A, & \text{if } 0 < \alpha < 1, \\ \phi, & \text{if } 1 \leq \alpha. \end{cases}$$

Hence $\chi_A \in V_x^O = |C_T|^O$ if an only if $A \in X$. Thus $T$ and $X$ are related.
Note that these results can be written in the following notation:

\[ V_x = C_T, \quad |V_x| = |C_T|, \quad |V_x|^o = |C_T|^o. \]

**Proposition 2.3.** For any zero-class \( X \), \( V_x \) is closed with respect to the topological space \((\omega, T_\omega)\).

**Proof.** Suppose that \( x \in V_x \) and choose \( \{x^n\} \subseteq V_x \) such that \( r_n \in R \)

\[ \sup_i |x^n_i - x_i| < 1 \quad (n \geq 1). \]

Suppose that \( T_x(x^n) = r_n \in R \). Since \( x^n \) converges to \( x \) as \( n \to \infty \), we have, for any \( \varepsilon > 0 \), there exists \( N \in I \) such that \( n, m \geq N \Rightarrow |x^n - x|^o < \varepsilon \). We show that \( \lim_{n \to \infty} r_n \) exists. Suppose that \( n, m \geq N \). For each \( i \in I \),

\[ |r_n - r_m| \leq |r_n - x^n_i| + |x^n_i - x^m_i| + |x^m_i - r_m| < |r_n - x^n_i| + \varepsilon + |x^m_i - r_m|. \]

Clearly

\[ I = \{i: |r_n - r_m| - \varepsilon < |r_n - x^n_i| + |x^m_i - r_m| \} \]

\[ \subseteq \{i: (|r_n - r_m| - \varepsilon)/2 < |r_n - x^n_i| \} \cup \{i: i(|r_n - r_m| - \varepsilon)/2 < |x^m_i - r_m|\}. \]

If \( |r_n - r_m| > \varepsilon \), then \( i(|r_n - r_m| - \varepsilon)/2 < |r_n - x^n_i| \) \( \in X \) and

\[ \{i: (|r_n - r_m| - \varepsilon)/2 < |x^m_i - r_m| \} \subseteq X \text{ so that } I \subseteq X, \text{ a contradiction. Hence } |r_n - r_m| \]

\[ \leq \varepsilon \text{ and } \{r_n \} \text{ is a Cauchy sequence of real numbers. Let } \lim_{n \to \infty} r_n = r \in R. \]

Now we show that \( x \in V_x \). For any \( \alpha > 0 \), we choose \( N \in I \) such that \( n > N \Rightarrow |x - x^n| < \alpha/3 \) and \( |r_n - r| < \alpha/3 \). For any \( i \in I \),

\[ |x_i - r| \leq |x_i - x^n_i| + |x^n_i - r_n| + |r_n - r| < 2\alpha/3 + |x^n_i - r_n|. \]

Therefore

\[ \{i: |x_i - r| \}

\[ \subseteq \{i: |x_i - x^n_i| + |x^n_i - r_n| + |r_n - r| < 2\alpha/3 + |x^n_i - r_n| \}. \]

Since \( T(x^n) = r_n \), we have \( \{i: \alpha/3 < |x_i - r_n| \} \subseteq X \). Hence \( x \in V_x \), and so \( V_x \) is closed.

**Proposition 2.4.** For any zero-class \( X \), \( V_x^o \) is a closed subset of \((\omega, T_\omega)\).

**Proof.** \( T_x: (V_x^o, T_\omega) \to R \) is an RSM and so it is continuous. Thus \( T_x^{-1}(0) = V_x^o \)

is closed subset of \((V_x^o, T_\omega)\). Since \( V_x \) is also closed in \((\omega, T_\omega)\), \( V_x^o \) is closed in \((\omega, T_\omega)\).

**Proposition 2.5.** For any zero-class \( X \), \( \overline{V}_x = V_x \) (where \( \overline{V}_x \) denotes the closure of \( V_x \) with respect to the topology \( T_\omega)\).

**Proof.** Suppose that \( x \in \omega_x \), \( r \in R \) and \( A \in X \) with \( x(A) \to r \). Then by the definition of \( x(A) \to r \), we have, for any \( \alpha > 0 \), there exist \( N \in I \) such that \( \{i: |x_i - r| \}

\[ \subseteq A \cap (1, 2, 3, \ldots, N) \text{. Since } A \text{ and } (1, 2, 3, \ldots, N) \subseteq X, \text{ we have } \{i: |x_i - r| \} \subseteq X. \]

Hence
x \in V_x$. Therefore $\omega_x \subset V_x$. Since $V_x$ is closed, we have $\overline{\omega_x} \subset V_x$.

Suppose that $x \in V_x$ and $T(x) = r$. For each $n$, let $\{i: |1/n \cdot |x_i - r|\} = A_n$. Then $A_n \in X$.

Let us define $x^n \in \omega$ by

$$
x^n_i = \begin{cases} 
  r & \text{if } i \in I - A_n \\
  x_i & \text{if } i \in A_n.
\end{cases}
$$

Obviously, $x^n \rightarrow r$ and $A_n \in X$, thus $x^n \in \omega_x$.

Since

$$
|x^n_i - x_i| = \begin{cases} 
  |r - x_i| & \text{if } i \in I - A_n \\
  0 & \text{if } i \in A_n
\end{cases}
$$

we get $|x^n - x|_\infty \leq 1/n$. It follows that $x \in \overline{\omega_x}$. Hence $\overline{\omega_x} = V_x$.

Replacing $\omega_x$ by $\omega_x^o$, $V_x$ by $V_x^o$ and $r$ by $0$ we obtain

**PROPOSITION 2.6.** For any zeroclass $X$, $\overline{\omega_x^o} = V_x^o$ where $\omega_x^o = \{x \in \omega: \exists A \in X, x(A) = 0\}$.

**PROPOSITION 2.7.** (see [1] Proposition 4.10). If the zeroclass $X$ is related to the RSM $S$, then

$$
\omega_x^o \cap m \subset |C_S|^o \subset V_x^o, \quad (2.3)
$$

$$
\omega_x \cap m \subset |C_S| \subset V_x, \quad (2.4)
$$

$S$ and $T_x$ have same value on $|C_S|$. \hfill (2.5)

**PROOF.** (2.3) Let $x \in \omega_x^o \cap m$. Then there exists a set $A \in I$ such that $A \in X$ and $x(A) = 0$. Since $A \in X$, we have $\chi_A \in |C_S|^o$. Since $x \in m$ and $S$ is an RSM, $x \cdot \chi_A \in |C_S|^o$.

Further $x \cdot \chi_{I-A} \in C_o \subset |C_S|^o$ \hfill ([1] Proposition 4.9). Thus $x = x \cdot \chi_A + x \cdot \chi_{I-A} \in |C_S|^o$.

Next, for any $x \in |C_S|^o$ and for any $\alpha > 0$, $\alpha \chi_{\{i: |x_i| \leq \alpha\}} \in |C_S|^o$. Thus

$$
\alpha \chi_{\{i: \alpha < |x_i|\}} \in |C_S|^o \text{ and so } \chi_{\{i: \alpha < |x_i|\}} \in |C_S|^o \text{ equivalently } \{i: \alpha < |x_i|\} \in X.
$$

(2.4) Obviously, $\omega_x \cap m = (\omega_x^o \oplus <e>) \cap m \subset (|C_S|^o \oplus <e>) \cap m = |C_S| \cap m$. also,

$|C_S| = |C_S|^o \oplus <e> \subset V_x^o \ominus <e> = V_x$.

(2.5) Let $x \in |C_S|$. Then there exists $r \in \mathbb{R}$ such that $x - r \in |C_S|^o \subset C_S^o$ so that $S(x-r) = 0$ or $S(x) = r$. By (2.3), $x - r \in V_x^o$. Therefore $T_x(x) = r$. 
PROPOSITION 2.8. If $X_1$ and $X_2$ are zero-classes with $X_1 \subseteq X_2$, then we have

$$V_{X_1} \subseteq V_{X_2},$$

and

$$V_{X_1} \subseteq V_{X_2}.$$  

PROOF. (2.6) Suppose that $x \in V_{X_1}$. Then for any $\alpha > 0$, $\{i : \alpha \leq |x_i|\} \in X_1 \subseteq X_2$. Therefore, for any $\alpha > 0$, $\{i : \alpha \leq |x_i|\} \in X_2$ or $x \in V_{X_2}$. For (2.7) and (2.8) let $x \in V_{X_1}$ and $T_x(x) = r$. Then we have $x - r \in V_{X_1} \subseteq V_{X_2}$. Thus $x - r \in V_{X_2}$ and so $x \in V_{X_2}$ and $T_{X_2}(x) = r = T_{X_2}(x).

PROPOSITION 2.9. (Bounded Consistency Theorem on Strong Convergence Fields). Let $S_1 : C_{S_1} \to R$ be an RSM related with the zero-class $X_1$ and $S_2 : C_{S_2} \to R$ be an RSM related with the zero-class $X_2$. Suppose that $X_1 \subseteq X_2$ and $C_{S_1} \cap m \subseteq C_{S_2}$. Then we have:

$$|C_{S_1}|^o \cap m \subseteq |C_{S_2}|^o \cap m,$$

$$|C_{S_1}| \cap m \subseteq |C_{S_2}| \cap m,$$

$$S_1(|C_{S_1}| \cap m) = S_2(|C_{S_1}| \cap m).$$

PROOF. (2.9) If $x \in |C_{S_1}| \cap m$, then $|x| \in C_{S_1} \cap m$, so $S_1(|x|) = 0$, $|x| \in V_{X_1}$.

(Proposition 2.7) and $T_{X_1}(|x|) = 0$. Since $|x| \in C_{S_1} \cap m \subseteq C_{S_2}$, $S_2(|x|)$ is defined. By the previous proposition $|x| \in V_{X_1} \subseteq V_{X_2} \cap m$ and by Proposition 2.7

$$\omega_{x_2} \cap m \subseteq |C_{S_2}| \cap m \subseteq \omega_{x_2} \cap m.$$ Thus we can find a sequence $\{x^n\}$ in $|C_{S_2}| \cap m$ such that $x^n \to |x|$ in $(\omega, T_\omega)$. Since $S_2$ is an RSM, $S_2$ is continuous. Thus $S_2(x^n) \to S_2(|x|)$.

Since $x^n \in |C_{S_2}| \cap m \subseteq V_{X_2}$, $T_{X_2}(x^n) = S_2(x^n)$. On the other hand $|x| \in V_{X_1} \subseteq V_{X_2}$, thus $0 = T_{X_1}(x) = T_{X_2}(x)$. Hence we have $0 = T_{X_2}(x) = \lim_{n} T_{X_2}(x^n) = \lim_{n} S_2(x^n) = S_2(x)$.

Therefore $x \in |C_{S_2}|^o$.

(2.10) By (2.9),

$$|C_{S_1}| \cap m \subseteq \omega_{x_2} \cap m.$$ Thus we have $|C_{S_2}| \cap m = T_{X_2}(x)$.

(2.11) By Proposition 2.7 (2.5), $S_1 |C_{S_1}| \cap m = T_{X_1}(x)$ and $S_2 |C_{S_2}| \cap m = T_{X_2}(x)$. By (2.10) and the
fact that \( |C_{S_1} \cap m | \geq V_x \), we have the result.

**COROLLARY 1.** Let \( S_1: C_{S_1} \to R \) and \( S_2: C_{S_2} \to R \) be RSMs defined on the same domain \( C_S = C_{S_1} = C_{S_2} \) and with same related zero-class \( X \). Then we have \( |C_{S_1} \cap m | = |C_{S_2} \cap m | \) and \( \lambda (|C_{S_1} \cap m |) \leq \lambda (|C_{S_2} \cap m |) \).

**REMARK.** Let \( F \) be the collection of all RSMs which are related to a fixed zero-class \( X \). Then \( T_x \) is a member of \( F \) and for any RSM \( S: C_S \to R \) in \( F \), \( S \) and \( T_x \) have the same values on the bounded strong convergence field associated with \( S \).

Finally we look at RSMs on \( m \).

**PROPOSITION 2.10.** Let \( S: C_S \to R \) be an RSM and let \( X = X \). Suppose that \( C_S = m \). Then \( |C_S| \geq V_x \cap m \) and \( S(x) = T_x(x) \) for any \( x \in C_S \).

**PROOF.** By Proposition 2.7, we have \( \omega_x \cap m \subseteq |C_S| \subseteq V_x \cap m \). Let \( x \in V_x \cap m \). Since \( V_x \) is the closure of \( \omega_x \) in \( (\omega, T_x) \), we can find a sequence \( \{x_n\} \subseteq |C_S| \) which converges to \( x \) in \( (\omega, T_x) \). Suppose that \( x_n \to x \) in \( |C_S| \). Since \( |C_S| = r_n \) and \( S \) is continuous, we have \( r \to r \) in \( r \). Thus \( |x_n - r| \to |x - r| \) in \( (\omega, T_x) \). Note that \( r \in m \). Thus \( |x - r| \) is also in \( m \), which is the domain of \( S \). Since \( S \) is continuous and \( S(|x_n - r|) = 0 \), \( S(|x - r|) = 0 \), which means \( x \in C_S \).

In the main Theorem of [3], Freedman proved (in the terminology of this paper) the following:

**THEOREM.** If \( Y \) is a zero-class, then \( x \in V_y \cap m \) if and only if for any two RSMs \( S_1, S_2 \) on \( m \) with \( Y \subseteq X_{S_1}, Y \subseteq X_{S_2}, S_1(x) = S_2(x) \).

Suppose that \( S_i: m \to R, X_i (i = 1, 2) \) satisfy the hypothesis of Proposition 2.9, we show that the above Theorem implies that the conclusion of Proposition 2.9 also holds for \( S_1, S_2 \). If \( x \in |C_{S_1} \cap m | \), then \( x \in V_{X_1} \cap m \). It is clear that \( S_1(x) = S_2(x) \) since \( X_1 \subseteq X_2 \).

3. **RSMs WITH A RELATED ULTRAZEROCLASS.**

In [2] we studied RSMs induced from matrices. For a regular matrix \( A \), we define the linear functional \( f_A: C_A \to R \) by \( f_A(x) = \lim_n Ax \) for any \( x \in C_A \). The ordinary Bounded Consistency Theorem (BCT) (see, e.g. [4]) says that for any regular matrices \( A, B \) with \( C_A \cap m \subseteq C_B, f_A(x) = f_B(x) \) for any \( x \in C_A \cap m \).

We can easily see that the BCT for strong convergence fields (Proposition 9) is included in the ordinary BCT for matrices when the RSMs are induced from regular matrices. Therefore we would like to find examples of summabilities such that the bounded consistency in the strong convergence fields of these summabilities is not implied by the matrix BCT.

**DEFINITION 3.1.** An ultrazero-class on \( I \) is a zero-class \( X \) such that there is no zero-class on \( I \) which is strictly finer than \( X \).
PROPOSITION 3.1. Let $X$ be an ultrazeroclass on $I$. Then for any $A \in I^I$, $A \in X$ or $I - A \in X$.

PROOF. Let $F = \{A \in I^I : A \in X\}$. Then $F$ is an ultrafilter. Thus for any $A \in I^I$, $A \in F$ or $I - A \in F$.

PROPOSITION 3.2. $X$ is an ultrazeroclass if and only if $m \in V_x$.

PROOF. Suppose that $X$ is an ultrazeroclass. Then for any $A \in I^I$, $A \in X$ or $I - A \in X$, equivalently, $\chi_A \in V_x$ or $\chi_{(I-A)} \in V_x$, that is $\chi_A \in V_x$ or $1 - \chi_A \in V_x$.

It follows that $\chi_A \in V_x$. Since $V_x$ is linear space, $m_0 \subseteq V_x$. Since $V_x$ is closed in $(\omega,T_m)$, $\overline{m} = m \subseteq V_x$.

Suppose that $X$ is not an ultrazeroclass, then there exists $A \in I^I$ such that $A \notin X$ and $I - A \notin X$. Assume that $\chi_A \in V_x$. Then there exists $r \in R$ such that $\{i : A(i) - r\} \in X$ for any $\alpha > 0$.

If $r = 1$, then $\{i : 1/2 < |A(i) - r|\} = I - A \notin X$.

If $r = 0$, then $\{i : 1/2 < |A(i) - r|\} = A \notin X$.

If $r \notin \{0,1\}$, then $\{i : 0.5 \min(|r|,|1 - r|) < |A(i) - r|\} = I \notin X$.

This is a contradiction. Hence $\chi_A \in m \cdot V_x$.

PROPOSITION 3.3. If $X$ is an ultrazeroclass then there does not exist a regular matrix $A$ such that $f_A$ is an RSM and $|V_x| \cap m = C_A \cap m$.

PROOF. Since $X$ is an ultrazeroclass, $m \subseteq V_x$ and thus $|V_x| \cap m = V_x \cap m = m$. On the other hand, for any regular matrix $A$, $m - C_A \neq \emptyset$.

It follows from the above and Proposition 2.9 that the value of any RSM, $S$, on its bounded strong convergence field is determined by any ultrazeroclass containing the zeroclass related to $S$.

REFERENCES
