RESEARCH NOTES

SPLAYING A SEARCH TREE IN PREORDER TAKES LINEAR TIME

R. CHAUDHURI and H. HÖFT

Department of Computer Science
Eastern Michigan University
Ypsilanti, Mich. 48197.

1. Abstract

In this paper we prove that if the nodes of an arbitrary n-node binary search tree T are splayed according to the preorder sequence of T then the total time is O(n). This is a special case of the splay tree traversal conjecture of Sleator and Tarjan [1].

2. Introduction

A binary search tree in which we splay after each access to the node containing the accessed item is called a splay tree. Splaying is a restructuring operation consisting of a sequence of rotations (see [1] for details). Tarjan [2] proved that the nodes of an n-node search tree can be splayed in symmetric order (inorder) in O(n) time.

We define the splay depth (SD) of a node x to be the depth of x at the time we start splaying at x. The set of all the right ancestors of a node z in the subtree rooted at a node x is denoted by $A(z,x)$. Also, left(x) and right(x) denote the left child and the right child of a node x respectively.

3. Main Results

In the following, we simply state our main results.

THEOREM 1

Let T be a binary search tree whose nodes are being splayed according to its own preorder sequence. Let x be a node of T and assume that the splay depth (SD) of x is d. Then

(a) $SD(left(x)) \leqslant d/2 + 3/2$ and

(b) $SD(right(x)) \leqslant 1 + |A(z,x)|$

where z is the preorder predecessor of right(x) and $|A(z,x)|$ denotes the cardinality of the set $A(z,x)$.

Using Theorem 1, we prove that :

THEOREM 2

The total time to splay an n-node binary search tree T according to its own preorder sequence is at most 8n.

The complete paper is being submitted for publication in the Journal of the Association for Computing Machinery.
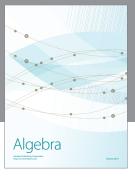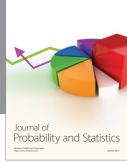
References

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2. R.E.Tarjan: "Sequential access in splay trees takes linear time", Combinatorica, 5(4), 1985, 367-378.
3. R.E.Tarjan: "Data Structures and Network Algorithms", C.B.M.S 44 (1983), S.I.A.M, Philadelphia, Pa.