A COUPLED MAGNETO-THERMO-ELASTIC PROBLEM IN A PERFECTLY CONDUCTING ELASTIC HALF-SPACE WITH THERMAL RELAXATION

S.K. ROY-CHOUDHURI
Department of Mathematics
Burdwan University
West Bengal, India

and

GARGI CHATTERJEE (ROY)
R.B.C. College, Naihati
West Bengal, India

(Received January 12, 1988 and in revised form August 14, 1989)

ABSTRACT. In the present paper we consider the magneto-thermo-elastic wave produced by a thermal shock in a perfectly conducting elastic half-space. Here the Lord-Shulman theory of thermoelasticity [1] is used to account for the interaction between the elastic and thermal fields. The solution obtained in analytical form reduces to those of Kaliski and Nowacki [2] when the coupling between the temperature and strain fields and the relaxation time are neglected. The results also agree with those of Massalas and Dalamangas [3] in absence of the thermal relaxation time.

KEY WORDS AND PHRASES. Magneto-thermoelastic wave; Thermal relaxation time.

1980 AMS SUBJECT CLASSIFICATION CODE.

1. INTRODUCTION.

Kaliski and Nowacki [2] investigated the problem of magneto-thermo-elastic disturbances generated by a thermal shock in a perfectly conducting elastic half-space in contact with a vacuum. It was assumed that both in the medium and in the vacuum there acted an initial magnetic field parallel to the plane boundary of the half-space and there was no influence of coupling between temperature and strain fields.

Later, Massalas and Dalamangas [3] considered the same problem where the coupling between the temperature and strain fields was considered. Very recently Chatterjee and Roy Choudhuri [4] extended the problem [3] in generalized thermo-elasticity of Green and Lindsay taking into account the two relaxation times.

In the present paper we extend the problem [3] in generalized thermoelasticity by using the thermal relaxation time of Lord-Shulman theory [1]. The solutions for temperature distribution, deformation and perturbed magnetic field in the vacuum are obtained in analytical form in the first power of the magnetothermo-elastic coupling parameter $\varepsilon$ and relaxation parameter $\tau_0'$. In absence of $\varepsilon$, $\tau_0'$ the solutions agree with those in [2] and in absence of $\tau_0'$, the results agree with those in [3].
Surface stress for different times is calculated and graphically presented. It is believed that this particular problem has not been considered earlier.

2. PROBLEM FORMULATION.

We assume that a magneto-thermo-elastic wave is produced in an elastic half-space \( x_1 > 0 \) due to the thermal shock \( \Theta(t) = \Theta_0 H(t) \) applied on \( x_1 = 0 \) where \( \Theta_0 \) is a constant and \( H(t) \) is the Heaviside function. We also assume that in both the media there is an initial magnetic field acting in the direction of \( x_3 \)-axis. The simplified equations of slowly moving bodies in electrodynamics after linearization are the following:

\[
\begin{align*}
\tilde{\mathbf{E}} &= \frac{4\pi}{c} \phi \\
\tilde{\mathbf{H}} &= -\frac{\mu_0}{c} \frac{\partial \tilde{\mathbf{E}}}{\partial t} \\
\mathbf{\nabla} \cdot \tilde{\mathbf{E}} &= 0, \quad \mathbf{\nabla} \times \tilde{\mathbf{H}} = -\frac{\mu_0}{c} (\dot{\mathbf{u}} \times \tilde{\mathbf{H}})
\end{align*}
\]

where \( \tilde{\mathbf{E}} \) denotes the electric field, \( \tilde{\mathbf{H}} \) is the perturbation of the magnetic field, \( \tilde{\mathbf{H}} \) is the initial constant magnetic field, \( \mathbf{\nabla} \) is the current density vector, \( \mathbf{u} \) denotes the displacement vector, \( \mu_0 \) is the magnetic permeability, \( \sigma \) is the electric conductivity and \( c \) is the velocity of light. The displacement equation of motion in thermo-elasticity including the electromagnetic effect after linearization is,

\[
\nu \mathbf{\nabla}^2 \mathbf{u} + (\lambda + \mu) \mathbf{\nabla} \left( \frac{\mathbf{\nabla} \cdot \mathbf{u}}{4\pi} \right) + \frac{\mu_0}{4\pi} \left( \dot{\mathbf{u}} \times \tilde{\mathbf{H}} \right) + \gamma \tilde{\mathbf{E}} = \rho \frac{\partial \mathbf{u}}{\partial t}.
\]

Also the modified form of Fourier's law of heat conduction taking into account the thermal relaxation time \([1]\) is

\[
\rho c_v \left( \frac{\partial \Theta}{\partial t} + \mathbf{\nabla} \cdot \mathbf{u} \right) + \gamma T_o \left( \lambda + \mu \frac{\partial \mathbf{u}}{\partial t} \right) = K \frac{\partial \Theta}{\partial x_i}, \quad (i=1,2,3)
\]

where \( \lambda, \mu \) are the Lamé' constants, \( \gamma \) is equal to \( (3\lambda + 4\mu) \), \( \alpha_T \) is the co-efficient of linear thermal expansion, \( \Theta \) is equal to \( T - T_0 \); \( T_0, T \) are the reference and absolute temperature of the body respectively; \( K \) is the co-efficient of heat conduction; \( \rho \) is the mass density; \( c_v \) is the specific heat at constant volume; \( \tau_0 \) is the relaxation time. The magneto-thermo-elastic wave propagated in the medium \( x_1 > 0 \) is assumed to depend on \( x_1 \) and time \( t \).

For \( \tilde{\mathbf{H}}_0 = (0, 0, H_3) \) equations (2.1) reduce to

\[
\tilde{\mathbf{E}} = \frac{\mu_0}{c} H_3 (0, 0, 0), \quad \tilde{\mathbf{H}} = -\frac{\mu_0}{c} \mathbf{\nabla} \times \tilde{\mathbf{H}} = \frac{\mu_0}{c} \mathbf{\nabla} \times (0, 0, H_3), \quad \mathbf{\nabla} \cdot \tilde{\mathbf{E}} = 0, \quad \mathbf{\nabla} \times \tilde{\mathbf{H}} = 0.
\]

Equations (2.2) and (2.3) then lead to

\[
(\lambda + 2\mu + \alpha_T) \frac{\partial^2 u_1}{\partial x_1^2} - \gamma \frac{\partial \Theta}{\partial x_1} = \rho \frac{\partial u_1}{\partial t}.
\]
Coupled Magneto-Thermo-Elastic Problem in a Half-Space

\[ \rho C_v \left( \frac{\partial^2 \theta}{\partial t^2} + \frac{1}{\alpha^2} \frac{\partial^2 u_1}{\partial t^2} \right) + \gamma T \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{1}{\alpha^2} \frac{\partial^2 u_1}{\partial t^2} \right) = \frac{E^2}{3x^2} \]

(2.6)

where \( a_0 = \sqrt{\frac{\mu \gamma H^2}{4\pi \rho}} \) is the Alfvén wave velocity. For convenience, we shall use the notations \( u_1 = u, x_1 = x \).

In vacuum the system of equations of electrodynamics are

\[ \left( \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) h_{3} = 0 \]

(2.7)

\[ \left( \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) E_{2} = 0 \]

where \( x' = -x \).

The components \( T_{11} \) and \( T_{11}^0 \) of Maxwell's stress tensor in elastic medium and in vacuum are

\[ T_{11} = \frac{-\mu_0}{4\pi} H_3 H_3 \text{ and } T_{11}^0 = -\frac{1}{4\pi} h_3^0 H_3^3. \]

The normal mechanical and thermal stress is

\[ \sigma_{11} = (\lambda + 2\mu) \frac{\partial u}{\partial x} - \gamma \theta. \]

The boundary conditions to be satisfied are

\[ \sigma_{11} + T_{11} - T_{11}^0 = 0, \quad x = x' = 0 \]

(2.8)

\[ E_2 = E_2^0, \quad x = x' = 0 \]

(2.9)

\[ \theta(\alpha, t) = \theta_0 H(t). \]

(2.10)

3. Solution of the Problem.

To find the solution of the problem we now introduce the following notations and non-dimensional variables

\[ c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_0^2 = a_0^2 + c_1^2, \quad \xi = \frac{c_0 x}{\kappa}, \quad \tau = \frac{c_0 t}{\kappa}, \]
The equations (2.5) - (2.7) and boundary conditions (2.8) - (2.10) become

\[ \frac{\partial^2 U}{\partial \xi^2} - \frac{\partial^2 U}{\partial \tau^2} = 0, \quad \xi > 0 \quad (3.1) \]

\[ \frac{\partial^2 Z}{\partial \xi^2} - \frac{\partial^2 Z}{\partial \tau^2} - \varepsilon \frac{\partial^3 U}{\partial \xi^2 \partial \tau} - \varepsilon \tau_o \frac{\partial^3 U}{\partial \xi^2 \partial \tau^2} = 0, \quad \xi > 0 \quad (3.2) \]

\[ \frac{\partial^2 U}{\partial \xi^2} - \beta^2 \frac{\partial^2 U}{\partial \tau^2} = 0, \quad \xi' > 0 \quad (3.3) \]

\[ \frac{\partial U}{\partial \xi} = z + \beta \frac{H_3}{\gamma T_o} = 0, \quad \xi = \xi' = 0 \quad (3.4) \]

\[ \beta_2 \frac{\partial^2 U}{\partial \tau^2} - \frac{\partial^3 U}{\partial \xi^2 \partial \tau} = 0, \quad \xi = \xi' = 0 \quad (3.5) \]

\[ Z(o, \tau) = \frac{o}{T_o} H(\tau), \quad (3.6) \]

where \( \beta_1 = \frac{H_3}{\gamma T_o \beta}, \quad \beta_2 = \frac{H_3 \gamma T_o}{\rho \beta^2}, \quad \beta = \frac{C_o}{\beta}, \quad \xi' = -\xi \).

Initial conditions in the new variables are

\[ U(\xi, o) = 0, \quad Z(\xi, o) = 0, \quad \frac{\partial z(\xi, 0)}{\partial \xi} = 0. \]

We now introduce a potential function \( \phi \) defined by

\[ U = \frac{\partial \phi}{\partial \xi}. \quad (3.7) \]

Using (3.7) in (3.1) and then integrating we get

\[ z(\xi, \tau) = \left( \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \tau^2} \right) \phi \text{ in } \xi > 0. \quad (3.8) \]
Using (3.7), the equation (3.2) leads to

\[
\frac{\partial^2 z}{\partial z^2} - \frac{\partial z}{\partial \tau} - \tau_0 \frac{\partial^2 z}{\partial \tau^2} - \epsilon \frac{\partial^3 \phi}{\partial z^2 \partial \tau} - \epsilon \tau_0' \frac{\partial^3 \phi}{\partial \tau^2} = 0
\] (3.9)

In the Laplace transform domain the equations (3.8), (3.9) and (3.3) become

\[
\tilde{Z}(\xi,s) = \left(\frac{-\partial^2}{\partial \xi^2} - s^2\right) \phi, \quad \xi > 0
\] (3.10)

\[
\left(\frac{-\partial^2}{\partial \xi^2} - s - \tau_0' \xi^2\right) \tilde{z} = \epsilon s(1+\tau_0's) \frac{\partial^2 \phi}{\partial \xi^2}, \quad \xi > 0
\] (3.11)

\[
\tilde{h}_{3}^{0} = c_3 e^{\beta_3 \xi^*}, \quad \xi > 0.
\] (3.12)

In Laplace transform domain, the boundary conditions (3.4) - (3.6) reduce to

\[
\frac{\partial^2 \phi}{\partial \xi^2} - \tilde{z} + \beta_1 \tilde{h}_3^0 = 0, \quad \xi = 0
\] (3.13)

\[
\beta_2 s \frac{\partial \phi}{\partial \xi} - \frac{\partial \tilde{z}}{\partial \xi} = 0, \quad \xi = \tau_1' = 0
\] (3.14)

\[
\tilde{Z}(0,s) = \frac{\theta_0}{s}.
\] (3.15)

Eliminating \(\tilde{z}\) from (3.10) and (3.11) we get

\[
\frac{\partial^4 \phi}{\partial \xi^4} - \left(1+\epsilon+s+(1+\epsilon) \tau_0's\right)s \frac{\partial^2 \phi}{\partial \xi^2} + s^3(1+\tau_0's) \frac{\partial \phi}{\partial \xi} = 0.
\] (3.16)

The equation (3.16) reduces to (31) in [4] on setting \(\alpha' = \phi' = \tau_0'\).

The general solution of (3.16) vanishing at \(\xi = 0\) is

\[
\phi(\xi,s) = C_1 e^{-\lambda_1 \xi} + C_2 e^{-\lambda_2 \xi}, \quad \xi > 0
\] (3.17)

where \(\lambda_1, \lambda_2\) are given by the roots of the equation

\[
\lambda^4 - s\left(1+\epsilon+s+(1+\epsilon) \tau_0's\right) \lambda^2 + s^3(1+\tau_0's) = 0.
\] (3.18)

Hence

\[
\lambda_{1,2} = \left(\frac{\epsilon s(1+\epsilon)}{2(1+\epsilon+s+(1+\epsilon) \tau_0's)} \pm \left[(1+\epsilon^2 \tau_0'^2 + \tau_0'^2 + 2\epsilon \tau_0' + 2\epsilon \tau_0^2 - 2 \tau_0')s^2 + 2(\epsilon - 1+2\epsilon \tau_0' + \tau_0' + \epsilon \tau_0') s + (1+\epsilon)^2 \right]^{1/2}\right)^{1/2}.
\] (3.19)
The equation (3.19) agrees with that of (34) in [4] for \( \alpha' = \alpha^{*'} = \tau' \). For \( \alpha' = \alpha^{*'} = 0 \), the equations (3.16), (3.19) are in agreement with that of (24) in [3]. Thus the equations (3.1), (3.2), (3.16), (3.19) are more general in the sense that they incorporate the effect of thermal relaxation time of Lord-Shulman theory.

From (3.10) using (3.17) we have

\[
\bar{z}(\xi,s) = C_1(\lambda_1^2 - s^2) e^{-\lambda_1 \xi} + C_2(\lambda_2^2 - s^2) e^{-\lambda_2 \xi}, \quad \xi > 0. \tag{3.20}
\]

From the boundary conditions (3.13) - (3.15) taking into account (3.17) and (3.20) we obtain a linear algebraic system with respect to \( C_1, C_2 \) and \( C_3 \) as

\[
C_1 s^2 + C_2 s^2 + \beta_1 C_3 = 0, \quad \text{at} \quad \xi = \xi' = 0 \tag{3.21}
\]

\[
\beta_2 s \lambda_1 C_1 + \beta_2 s \lambda_2 C_2 - \beta C_3 = 0, \quad \text{at} \quad \xi = \xi' = 0 \tag{3.22}
\]

\[
C_1(\lambda_1^2 - s^2) + C_2(\lambda_2^2 - s^2) = \frac{\theta}{T_0} \tag{3.23}
\]

The constants \( C_i \) \( (i=1,2,3) \) being determined by (3.21) - (3.23), the solutions for \( \bar{\varphi}, \bar{z}, \bar{h}_3 \) are given by

\[
\bar{\varphi}(\xi,s,\xi',T_0) = \frac{\theta}{T_0} \left[ \frac{(s + \beta_1 \beta_2 \lambda_1) e^{-\lambda_1 \xi} - (s + \beta_1 \beta_2 \lambda_2) e^{-\lambda_2 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)} \right] \tag{3.24}
\]

\[
\bar{z}(\xi,s,\xi',T_0) = \frac{\theta}{T_0} \left[ \frac{(\frac{1}{2} s^2 - \xi^2)(s + \beta_1 \beta_2 \lambda_1) e^{-\lambda_1 \xi} - (\frac{1}{2} s^2 - \xi^2)(s + \beta_1 \beta_2 \lambda_2) e^{-\lambda_2 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)} \right] \tag{3.25}
\]

\[
\bar{U}(\xi,s,\xi',T_0) = \frac{\theta}{T_0} \left[ \frac{\lambda_2 (s + \beta_1 \beta_2 \lambda_1) e^{-\lambda_1 \xi} - \lambda_1 (s + \beta_1 \beta_2 \lambda_2) e^{-\lambda_2 \xi}}{s(\lambda_1 - \lambda_2)(\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2)} \right], \quad \xi > 0 \tag{3.26}
\]

\[
\bar{h}_3(\xi',s,\xi',T_0) = \frac{\theta}{T_0} \frac{s \beta_2 e^{-\beta s \xi'}}{\beta_1 \beta_2 s^2 + \beta(\lambda_1 + \lambda_2)s + \beta_1 \beta_2 \lambda_1 \lambda_2}, \quad \xi' > 0. \tag{3.27}
\]

Since \( \varepsilon, \tau' < 1 \) for small thermo-elastic couplings, we expand the functions \( \bar{z}, \bar{U}, \bar{h}_3 \) into Maclaurian's series and retain the first two terms in the series expansion to obtain
COUPLED MAGNETO- THERMO- ELASTIC PROBLEM IN A HALF- SPACE

\[
\tilde{z}(\xi,s,\epsilon, \tau'_0) = \frac{\theta_o}{T_o} \left[ e^{-\xi \sqrt{s}} + \epsilon \left( \begin{array}{c}
\beta e^{-\xi \sqrt{s}} \\
\frac{\beta_1 \beta_2 e^{-\xi \sqrt{s}}}{(\beta + \beta_1 \beta_2)(s-1)^2} + \frac{\beta_1 \beta_2 e^{-\xi \sqrt{s}}}{\sqrt{s} (s-1)^2} + e^{-\xi \sqrt{s}} \\
\end{array} \right) + \frac{\beta_1 \beta_2}{2(\beta + \beta_1 \beta_2) s(s-1)^2} + \frac{\xi \epsilon e^{-\xi \sqrt{s}}}{\sqrt{s} (s-1)^2} - \frac{\xi \epsilon e^{-\xi \sqrt{s}}}{2(\beta + \beta_1 \beta_2) s(s-1)(\sqrt{s}+1)^2} \right]
\]

+ \frac{\beta_1 \beta_2}{2(\beta + \beta_1 \beta_2) s(s-1)^2} + \frac{\xi \epsilon e^{-\xi \sqrt{s}}}{\sqrt{s} (s-1)^2} - \frac{\xi \epsilon e^{-\xi \sqrt{s}}}{2(\beta + \beta_1 \beta_2) s(s-1)(\sqrt{s}+1)^2} \]

\[
\tilde{u}(\xi,s,\epsilon, \tau'_0) = \frac{\theta_o}{T_o} \left[ e^{-\xi \sqrt{s}} - \frac{\beta e^{-\xi \sqrt{s}}}{(\beta + \beta_1 \beta_2)(s-1)^2} + \frac{\beta e^{-\xi \sqrt{s}}}{(\beta + \beta_1 \beta_2)(\sqrt{s}-1)^2} - \frac{\beta e^{-\xi \sqrt{s}}}{2(\beta + \beta_1 \beta_2) s(s-1)(\sqrt{s}+1)^2} \right]
\]

Taking inverse Laplace transform we obtain (Chatterjee (Roy) and Roy Choudhuri [4], Hetnarski [5], Oberhettlner and Badi [6]),
\begin{align}
Z(\xi, \tau, \varepsilon, \tau') &= \frac{\theta_0}{\tau_0} \left[ \text{erfc} \left( \frac{\xi}{2\sqrt{\tau}} \right) + \varepsilon \left( \frac{\theta}{\beta + \beta_1 \beta_2} \right) (\tau - \xi) e^{(\tau - \xi)H(\tau - \xi)} + \frac{\theta_1 \beta_2}{\beta + \beta_1 \beta_2} \sqrt{\frac{\pi}{\xi}} \right] \\
&+ (\tau - \xi - \frac{1}{2}) e^{(\tau - \xi)\text{erf} \sqrt{(\tau - \xi)}} H(\tau - \xi) + \tau f_1(\xi, \tau) - \frac{\xi}{2} f_2(\xi, \tau) - f_1(\xi, \tau) + \text{erfc}(\frac{\xi}{2\sqrt{\tau}}) \\
&+ \frac{\theta_1 \beta_2}{2(\beta + \beta_1 \beta_2)} \left[ f_1(\xi, \tau) - \text{erfc} \left( \frac{\xi}{2\sqrt{\tau}} \right) + \frac{\xi}{2} f_2(\xi, \tau) - \frac{1}{2} f_3(\xi, \tau) \right] \\
&\left[ \frac{1}{4\sqrt{\pi}} (\tau^2 - 2\tau) - \frac{5}{2\tau} e^{-\frac{\xi^2}{4\tau}} \right] \\
(3.31)
\end{align}

\begin{align}
U(\xi, \tau, \varepsilon, \tau') &= \frac{\theta_0}{\tau_0} \left[ f_2(\xi, \tau) - 2\sqrt{\frac{\pi}{\xi}} e^{-\frac{(\xi^2)}{4\tau}} + \varepsilon \text{erfc} \left( \frac{\xi}{2\sqrt{\tau}} \right) - \frac{\theta}{\beta + \beta_1 \beta_2} \left( e^{(\tau - \xi)}H(\tau - \xi) \right. \right. \\
&\left. \left. - \frac{\beta_1 \beta_2}{\beta + \beta_1 \beta_2} \left[ e^{(\tau - \xi)}H(\tau - \xi) + \varepsilon(\frac{\beta_1 \beta_2}{\beta + \beta_1 \beta_2}) \left( \xi f_1(\xi, \tau) - \frac{\xi}{2} f_2(\xi, \tau) - f_1(\xi, \tau) + \text{erfc}(\frac{\xi}{2\sqrt{\tau}}) \right) \right] \right] \\
&\left[ \frac{1}{2\sqrt{\tau}} \right. \left. - \xi \text{erfc} \left( \frac{\xi}{2\sqrt{\tau}} \right) + \frac{\beta_1 \beta_2}{2(\beta + \beta_1 \beta_2)} \left[ \frac{\xi}{2\sqrt{\tau}} \right. \left. + (\tau - \xi - \frac{1}{2}) e^{(\tau - \xi)\text{erfc} \sqrt{(\tau - \xi)}} H(\tau - \xi) \right) \right] \\
&\left. + \frac{\beta_1 \beta_2}{2(\beta + \beta_1 \beta_2)} \left[ (\tau - \xi - \frac{3}{2}) e^{(\tau - \xi)\text{erf} \sqrt{(\tau - \xi)}} - \frac{5}{2} (\tau - \xi - \frac{1}{2}) e^{(\tau - \xi)} \text{erfc} \sqrt{(\tau - \xi)} \right. \right. \\
&\left. + \frac{3}{2} (\tau - \xi) + (\tau - \xi)^2 \text{erf} \sqrt{(\tau - \xi)} + \frac{3}{2} (\tau - \xi) + (\tau - \xi)^2 \text{erf} \sqrt{(\tau - \xi)} \right] \right) \\
&\left. \left. + \frac{\beta_1 \beta_2}{2(\beta + \beta_1 \beta_2)} \right] \right) \\
&\left. \left. + \frac{1}{2} (7\tau - 7\xi - 5) e^{(\tau - \xi)} - \frac{1}{2} (7\tau - 7\xi - \frac{11}{2}) e^{(\tau - \xi)\text{erf} \sqrt{(\tau - \xi)} + \frac{5}{2} (\tau - \xi) + (\tau - \xi)^2 \text{erf} \sqrt{(\tau - \xi)}} \right] \right) \\
\end{align}
COUPLED MAGNETO-THermo-ELASTIC PROBLEM IN A HALF-SPACE

\[ + \frac{5}{2} (\tau - \xi + (\tau - \xi)^2 - 2) H(\tau - \xi) + \frac{\beta^2}{2(\beta + \beta_1 \beta_2)^2} \left[ - \frac{3}{2} (\tau - \xi - \frac{3}{2}) \sqrt{\frac{\xi}{\pi}} - \frac{1}{2} (5\tau - 5\xi - 3) e^{\tau - \xi} \right] \]

\[ + \frac{3}{2} (\tau - \xi - \frac{1}{2}) e^{\tau - \xi} \text{erf} \sqrt{\tau - \xi} - \frac{3}{2} (\tau - \xi + (\tau - \xi)^2) \text{erf} \sqrt{\tau - \xi} + \frac{3}{2} (\tau - \xi) + (\tau - \xi)^2 - 1 \] \ H(\tau - \xi)

\[- \frac{\beta}{2(\beta + \beta_1 \beta_2)} \left[ f_5(\xi, \tau) = 4 \sqrt{\frac{\xi}{\pi}} \right. \left. - \text{erfc} \left( \frac{\xi}{2\sqrt{\tau}} \right) + \frac{3}{2} (\tau - \xi) \text{erf} \sqrt{\tau - \xi} \right] \]

\[ + \frac{\beta \beta_1 \beta_2}{2(\beta + \beta_1 \beta_2)^2} \left[ ((\tau - \xi) - \frac{9}{4}) \sqrt{\frac{\xi}{\pi}} + \frac{1}{2} (7\tau - 7\xi - 5) e^{(\tau - \xi) - \frac{1}{2} (7\tau - 7\xi - \frac{11}{2}) e^{(\tau - \xi) \text{erf} \sqrt{\tau - \xi}} \right] \]

\[ + \left( \frac{5}{2} (\tau - \xi + (\tau - \xi)^2) \text{erf} \sqrt{\tau - \xi} - \frac{5}{2} (\tau - \xi - (\tau - \xi)^2 + 2) H(\tau - \xi) \right) + \frac{1}{2} \beta_2 \left( \tau, \xi \right) = \frac{\beta}{2(\beta + \beta_1 \beta_2)} (\tau - \xi) e^{(\tau - \xi) \text{erf} \sqrt{\tau - \xi} + H(\tau - \xi)) \]

\[ = \frac{\beta}{2(\beta + \beta_1 \beta_2)} \left[ ((\tau - \xi) - \frac{1}{2}) e^{(\tau - \xi) \text{erf} \sqrt{\tau - \xi} + \frac{\xi}{\pi}} H(\tau - \xi)) \right]. \] (3.32)

\[ b_3^{\circ}(\xi', \tau, \epsilon, \tau_0') = \frac{\theta_0}{\tau} \left[ \frac{\beta_2}{\beta + \beta_1 \beta_2} e^{(\tau - \beta \xi')} \text{erfc} \sqrt{\tau - \beta \xi'} H(\tau - \beta \xi') - \epsilon (- \frac{\beta_2}{2(\beta + \beta_1 \beta_2)^2} 2(\tau - \beta \xi') \sqrt{\frac{\beta \xi'}{\pi}} \right] \]

\[ + [1 - 2(\tau - \beta \xi')^2] e^{(\tau - \beta \xi')} \text{erfc} \sqrt{\tau - \beta \xi'} H(\tau - \beta \xi') \] \[ + \tau_0' \left( \frac{\beta_2}{2(\beta + \beta_1 \beta_2)} \right) \frac{1}{\sqrt{\pi(\tau - \beta \xi')}} \]

\[ - 2(\tau - \beta \xi') e^{(\tau - \beta \xi')} \text{erfc} \sqrt{\tau - \beta \xi'} + 2 \sqrt{\frac{\beta \xi'}{\pi}} H(\tau - \beta \xi')). \] (3.33)

where the functions \( f_i(\xi, \tau), i=1,2,3,4,5 \) are given by

\[ f_1(\xi, \tau) = \frac{\xi}{2} \left[ e^{\text{erfc}(\frac{\xi}{2\sqrt{\tau}})} + e^{\text{erfc}(\frac{\xi}{2\sqrt{\tau}})} \right] \]
where $\text{erf}x$ and $\text{erfc}x$ denote the error function and complementary error function respectively.

4. NUMERICAL RESULT.

The surface stress is given by

$$- \frac{T_{11}^{\circ}}{\frac{\beta_3}{\beta(1+\beta_3)}} = e^\tau(1-\text{erf}\sqrt{\tau}) - \frac{e}{2(1+\beta_3)} \left( -2\tau\frac{\sqrt{\tau}}{\pi} + (1-2\tau^2)e^\tau(1-\text{erf}\sqrt{\tau}) \right)$$

$$- \frac{T_{11}^{\prime}\cos(\frac{1}{2\sqrt{\pi}} - \tau e^\tau(1-\text{erf}\sqrt{\tau}))}{\beta(1+\beta_3)}$$

where $\beta_3 = \frac{\beta_1\beta_2}{\beta}$.

If there is no coupling between the electromagnetic field and strain field, $H_3 = 0$, $\beta_2 = 0$, $\beta_3 = 0$ and $\beta$ is finite so that $T_{11}^{\circ} = 0$ on $\xi = a_0$.

In presence of the electromagnetic field and strain field, the surface stress is given by

$$- \frac{T_{11}^{\circ}}{\frac{\beta_3}{\beta(1+\beta_3)}} = \chi(\tau, \xi, \tau')$$
We can assume $\beta_3 << 1$ since $c >> 1$ and $a_o$ and $C_o$ are finite. We take $\beta_3 = .05$. For numerical calculation we take the material of the half-space to be copper for which $c = 0.0168$. If we assume that a representative value of the relaxation time $\tau_o$ is $10^{-11}$ (see [7]), then the non-dimensional thermal wave speed in copper should be approximately equal to 0.66.

Then $\tau_o' = 2.3$ (For thermal properties and sound wave speed in copper, see ref. [8]).

Surface stress $X$ for various values of times $\tau$ are exhibited in the following table and also graphically represented.

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$-10X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.4</td>
</tr>
<tr>
<td>1.5</td>
<td>7.7</td>
</tr>
<tr>
<td>2.0</td>
<td>11.8</td>
</tr>
<tr>
<td>2.5</td>
<td>16.5</td>
</tr>
<tr>
<td>3.5</td>
<td>23.2</td>
</tr>
<tr>
<td>4.0</td>
<td>25.0</td>
</tr>
<tr>
<td>4.5</td>
<td>26.8</td>
</tr>
<tr>
<td>5.0</td>
<td>28.3</td>
</tr>
<tr>
<td>5.5</td>
<td>30.0</td>
</tr>
<tr>
<td>6.0</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Surface stress $-10X$ ---

0:1 division corresponds to 0.1
REFERENCES


Submit your manuscripts at http://www.hindawi.com