FUZZY Θ -CLOSURE OPERATOR ON FUZZY TOPOLOGICAL SPACES

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ABSTRACT. The paper contains a study of fuzzy Θ -closure operator, Θ -closures of fuzzy sets in a fuzzy topological space are characterized and some of their properties along with their relation with fuzzy δ -closures are investigated. As applications of these concepts, certain functions as well as some spaces satisfying certain fuzzy separation axioms are characterized in terms of fuzzy Θ -closures and δ -closures.

KEY WORDS AND PHRASES. Fuzzy Θ -cluster point, fuzzy Θ -closure, fuzzy δ -closure, q-coincidence, q-neighbourhood.

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1. INTRODUCTION.

It is well-known that the concepts of Θ -closure and δ -closure are useful tools in standard topology in the study of H-closed spaces, Katětov's and H-closed extensions, generalizations of Stone-Weierstrass' theorem etc. For basic results and some applications of Θ -closure and δ -closure operators we refer to Veličko [1], Dickman and Porter [2], Espelie and Joseph [3] and Sivaraj [4]. Due to varied applicabilities of these operators in formulating various important set-topological concepts, it is natural to try for their extensions to fuzzy topological spaces. With this motivation in mind the concept of Θ -closure operator in a fuzzy topological space (due to Chang [5]) was introduced by us in [6] in the light of the notions of quasi-coincidence and q-neighbourhoods of Pu and Liu [7,8]. In the present paper our aim is to continue the same study which ultimately shows that different fuzzy topological concepts can effectively be characterized in terms of fuzzy Θ -closure and δ -closure operators.

In Section 2 of this paper we develop the concept of fuzzy Θ -closure operators and characterize fuzzy Θ -closures of fuzzy sets in a fuzzy topological space in different ways. In literature there can be found several definitions of T_2 -spaces in fuzzy setting. We take the definition of fuzzy T_2 -space as given by Ganguly and Saha [9] and become able to successfully characterize it in our context. Fuzzy regularity has been introduced by many workers from different view points, including one by us in [6]. Since our fuzzy regularity along with the fuzzy T_1 -axiom (of [9]) does not yield the above fuzzy T_2 -axiom, we propose to call it "strong T_2 " in fuzzy setting. Fuzzy semiregularity and almost

regularity were also defined in [6]. We characterize fuzzy regularity and these weaker forms of fuzzy regularity in terms of fuzzy Θ -closure and δ -closure. All these characterizations are incorporated in Section 3 of the paper. Fuzzy weakly continuous functions were first introduced by Azad [10] and were further investigated in [11], whereas the concept of fuzzy Θ -continuous functions was initiated in [6]. Section 3 also includes the characterizations of these functions with the help of the notion of fuzzy Θ -closures.

We now recall some definitions and results of a fuzzy topological space (henceforth fts, for short) (X, T) to be used in this paper excepting very standard ones for which we refer to Zadeh [12], Chang [5] and Pu and Liu [7,8]. The interior and closure of a fuzzy set A in an fts (X, T) will be denoted by $\text{Int } A$ and $\text{Cl } A$ respectively. A fuzzy point [7] with a singleton support x (say) and value α ($0 < \alpha \leq 1$) at x is denoted by x_α . For a fuzzy set A , the support and complement of A are denoted by A_o and A' (or $1 - A$) respectively. For a fuzzy point x_α and a fuzzy set A , we write $x_\alpha \in A$ iff $\alpha \leq A(x)$, and x_α is said to be quasi-coincident (q-coincident, for short) with A , denoted by $x_\alpha q A$, iff $\alpha > A'(x)$. A is said to be a q-neighbourhood (q-nbd, for short) of x_α iff there exists a fuzzy open set B such that $x_\alpha q B \leq A$. For two fuzzy sets A and B , $A \leq B$ iff $A \nsubseteq B'$, and a fuzzy point $x_\alpha \in \text{Cl } A$ iff each q-nbd of x_α is q-coincident with A [7]. For the definitions of fuzzy regularly open, regularly closed, semi-open and semi-closed sets we refer to Azad [10]. Simply by X and Y we shall mean the fuzzy topological spaces (X, T) and (Y, T_1) respectively. The constant fuzzy sets 0_X and 1_X are defined by $0_X(y) = 0$ and $1_X(y) = 1$, for each $y \in X$.

2. FUZZY Θ -CLOSURE AND ITS PROPERTIES.

DEFINITION 2.1. A fuzzy point x_α is said to be a fuzzy Θ -cluster point (δ -cluster point [13]) of a fuzzy set A iff closures of every open q-nbd (resp. iff every regularly open q-nbd) of x_α is q-coincident with A .

The union of all fuzzy Θ -cluster (δ -cluster) points of A is called the fuzzy Θ -closure of A and is denoted by $[A]_\Theta$ (resp. $[A]_\delta$). A fuzzy set A will be called fuzzy Θ -closed (δ -closed) iff $A = [A]_\Theta$ (resp. $A = [A]_\delta$). It is known [6] that for any fuzzy set A in an fts X , $\text{Cl } A \leq [A]_\delta \leq [A]_\Theta$, but the reverse implications are false (see [6] and [13]). However, it is true (see [6]) that for a fuzzy open set A in an fts X , $\text{Cl } A = [A]_\delta = [A]_\Theta$.

THEOREM 2.2. In an fts (X, T) , the following hold:

- (a) Finite union and arbitrary intersection of Θ -closed sets in X is fuzzy Θ -closed.
- (b) For two fuzzy sets A and B in X , if $A \leq B$ then $[A]_\Theta \leq [B]_\Theta$.
- (c) The fuzzy sets 0_X and 1_X are fuzzy Θ -closed.

PROOF. The straightforward proofs are omitted.

REMARK 2.3. The complements of fuzzy Θ -closed sets in an fts (X, T) induce a fuzzy topology T_Θ (say) which is coarser than the fuzzy topology T of the space. Again, for a fuzzy set A in X , $[A]_\Theta$ is evidently fuzzy closed but not necessarily fuzzy Θ -closed as is seen from the next example. Thus, fuzzy Θ -closure operator is not a Kuratowski closure operator. However, it will be shown in the next section that for any fuzzy set A in an fts X , $[A]_\Theta$ is fuzzy Θ -closed if the space X is fuzzy regular (see Corollary 3.6), or iff the space X is fuzzy almost regular (see Theorem 3.10).

EXAMPLE 2.4. Let $X = \{a, b, c\}$ and $T = \{0_X, 1_X, A, B\}$,

where

$$A(a) = 0.5, A(b) = 0.6, A(c) = 0.2$$

and

$$B(a) = 0.4, B(b) = 0.5, B(c) = 0.1.$$

Let U be any fuzzy set given by, $U(a) = U(b) = 0.3$ and $U(c) = 0$. Then, $a_{.6} \in [U]_\Theta, a_{.8} \notin [U]_\Theta$, but $a_{.8} \in [a_{.6}]_\Theta \leq [[U]]_\Theta$. Thus, $[U]_\Theta \neq [[U]]_\Theta$. Hence, $[U]_\Theta$ is not fuzzy Θ -closed.

In the following example, we observe a deviation from the corresponding established result [3] in general topology that $x \in [y]_\Theta$ iff $y \in [x]_\Theta$, if x, y are two points in a topological space.

EXAMPLE 2.5. Let X be an ordinary set with at least two distinct points a, b . Consider the fuzzy topology $T = \{0_X, 1_X A\}$, where $A(a) = \frac{1}{2}$, $A(b) = \frac{2}{5}$ and $A(x) = 0$, for $x \neq a, b (x \in X)$. Let us consider the fuzzy points $a_{\frac{1}{12}}$ and $b_{\frac{4}{5}}$. It can be checked that $a_{\frac{1}{12}} \in [b_{\frac{4}{5}}]_{\Theta}$, but $b_{\frac{4}{5}} \notin [a_{\frac{1}{12}}]_{\Theta}$,

THEOREM 2.6. For any fuzzy set A in an fts (X, T) , $[A]_{\Theta} = \cap \{[U]_{\Theta} : U \in T \text{ and } A \leq U\}$.

PROOF. Obviously, L.H.S. $\leq$ R.H.S. Now, if possible let $x_{\alpha} \in \text{R.H.S.}$ but $x_{\alpha} \notin [A]_{\Theta}$. Then there exists an open q-nbd V of x_{α} such that $\text{Cl } V \not\leq A$ and hence $A \leq 1 - C1V$. Then $x_{\alpha} \in [1 - C1V]_{\Theta}$ and consequently, $\text{Cl } V \leq 1 - C1V$ which is impossible.

According to Pu and Liu [7] a function $S: D \rightarrow J$ is called a fuzzy net in X , where $(D, \geq)$ is a directed set and J denote the collection of all fuzzy points in X . It is denoted by $\{S_n, n \in D\}$ or simply by (S, D) . We now set the following:

DEFINITION 2.7. Let $\{S_n, n \in D\}$ be a fuzzy net and x_{α} a fuzzy point in X .

- (a) x_{α} is called a Θ -cluster point of the fuzzy net iff for every open q-nbd W of x_{α} and for any $n \in D$, there exists $m \geq n$ ($m \in D$) such that $S_m q C1W$.
- (b) The fuzzy net is said to be Θ -converge to x_{α} if for any open q-nbd U of x_{α} , exists $m \in D$ such that $S_n q C1U$, for all $n \geq m$ ($n \in D$). This is denoted by $S \xrightarrow{\Theta} x_{\alpha}$.

THEOREM 2.8. A fuzzy point x_{α} is a Θ -cluster point of a fuzzy net $\{s_n, n \in D\}$ in X iff there is a subnet of $\{S_n, n \in D\}$, which Θ -converges to x_{α} .

PROOF. Let x_{α} be a Θ -cluster point of the given fuzzy net. Let Qx_{α} denote the set of closures of all open q-nbds of x_{α} . Now for any member A of Qx_{α} , there exists an element S_n of the net such that $S_n q A$. Let E denote the set of all ordered pairs (n, A) with the above property, i.e., $n \in D, A \in Qx_{\alpha}$ and $S_n q A$. Then $(E, \gg)$ is a directed set, where $(m, A) \gg (n, B)$, $((m, A), (n, B) \in E)$ iff $m \geq n$ in D and $A \leq B$. Then $T: (E, \gg) \rightarrow (X, T)$ given by $T(m, A) = S_m$ can be checked by a subnet of $\{S_n, n \in D\}$. To show that $T \xrightarrow{\Theta} x_{\alpha}$, let V be any open q-nbd of x_{α} . Then there exists $n \in D$ such that $(n: C1V) \in E$ and then $S_n q C1V$. Now, for any $(m, A) \gg (n, C1V)$, $T(m, A) = S_m q A \leq C1V$. Hence, $T \xrightarrow{\Theta} x_{\alpha}$. Converse is clear.

THEOREM 2.9. Let A be a fuzzy set in X . A fuzzy point $x_{\alpha} \in [A]_{\Theta}$ iff there exists a fuzzy net in A , Θ -converging to x_{α} .

PROOF. Let $x_{\alpha} \in [A]_{\Theta}$. For each open q-nbd U of x_{α} , $C1U q A$. That is, there exist $y^U \in A_{\alpha}$ and real number β_U with $0 < \beta_U \leq A(y^U)$ such that $y_{\beta_U}^U \in A$ and $y_{\beta_U}^U q C1U$. We choose and fix one such $y_{\beta_U}^U$ for each U . Let D denote the set of all open q-nbds of x_{α} . Then $(D, \geq)$ is directed under inclusion relation, i.e., for $B, C \in D, B \geq C$ iff $B \leq C$. Then $\{y_{\beta_U}^U \in A: y_{\beta_U}^U q C1U \text{ and } U \in D\}$ is a fuzzy net in A such that it Θ -converges to x_{α} . Converse is straightforward even if x_{α} is a Θ -cluster point of the fuzzy net in A .

3. CHARACTERIZATIONS OF CERTAIN SEPARATION AXIOMS AND FUNCTIONS IN TERMS OF FUZZY Θ -CLOSURE AND δ -CLOSURE.

DEFINITION 3.1. [9] An fts X is called fuzzy strongly T_2 iff for any two distinct fuzzy points x_{α} and y_{β} in X : whenever $x \neq y, x_{\alpha}$ and y_{β} have fuzzy open nbds U and V respectively such that $U \not\leq V$; and when $x = y, \alpha < \beta$ (say), there exist fuzzy open sets U and V such that $x_{\alpha} \in U, y_{\beta} q V$ and $U \not\leq V$.

LEMMA 3.2. For any two fuzzy open sets A and B in an fts (X, T) , $A \not\leq B \Rightarrow C1A \not\leq B$ and $A \not\leq C1B$.

THEOREM 3.3. An fts (X, T) is fuzzy strongly T_2 iff every fuzzy point of X is fuzzy Θ -closed, and for $x, y \in X$ with $x \neq (C1U)_{\alpha}$.

PROOF. Let X be fuzzy strongly T_2 , and let x_{α} be a fuzzy point in X . In order to show that $[x_{\alpha}]_{\Theta} = x_{\alpha}$, it suffices to establish that for any fuzzy point y_{β} , $y_{\beta} \notin [x_{\alpha}]_{\Theta}$ when either $x \neq y$, or $x = y$ and $\beta > \alpha$. In the first case, there exist fuzzy open nbds U and V of y_1 and x_{α} respectively such that $U \not\leq V$ and then $C1U \not\leq V$ (by Lemma 3.2). Then U is an open q-nbd of y_{β} with $C1U \not\leq x_{\alpha}$ so that $y_{\beta} \notin [x_{\alpha}]_{\Theta}$. In the second case, there exist a fuzzy open nbd U of x_{α} and an open q-nbd V of y_{β} such that $U \not\leq V$. Then $C1V \not\leq U$ so that $C1V \not\leq x_{\alpha}$ and hence $y_{\beta} \notin [x_{\alpha}]_{\Theta}$. Finally, for two distinct points x, y of X , there exist fuzzy open nbds U of x_1 and V of y_1 such that $U \not\leq V$ and hence $C1U \not\leq V$, i.e.,

$y_1 \in V \leq 1 - C1U$. Then $(1 - C1U)(y) = 1 \Rightarrow (C1U)(y) = 0 \Rightarrow y \notin (C1U)_\Theta$. Conversely, let x_α and y_β be two distinct fuzzy points in X .

CASE I. Let $x \neq y$. First suppose that at least one of α and β is less than 1, say $\alpha < 1$. Then there exists $\lambda > 0$ such that $\alpha + \lambda < 1$. Now $x_\lambda \notin [y_\beta]_\Theta$ and hence there exists a fuzzy open nbd U of y_β such that $x_\lambda \notin [U]_\Theta$ (by Theorem 2.6). Then $U \not\supset C1V$, for an open q-nbd V of x_λ . Since $V(x) > 1 - \lambda > \alpha$, V and U are fuzzy open nbds of x_α and y_β respectively such that $U \not\supset V$.

Next, suppose $\alpha = \beta = 1$. By hypothesis, there exists a fuzzy open nbd U of x_1 such that $(C1U)(y) = 0$. Then $(1 - C1U)$ is a fuzzy open nbd of y_1 such that $U \not\supset (1 - C1U)$.

CASE II. Let $x = y$. Suppose $\alpha < \beta$. Then $y_\beta \notin [x_\alpha]_\Theta$ and so $y_\beta \notin [U]_\Theta$, for some fuzzy open nbd U of x_α . Then for an open q-nbd V of y_β , $C1V \not\supset U$ and hence $V \not\supset U$.

DEFINITION 3.4. [6] An fts X is said to be:

- (a) fuzzy regular (semi-regular) iff for each fuzzy point x_α in X and each open q-nbd U of x_α , there exists an open q-nbd V of x_α such that $C1V \leq U$ (resp. $\text{Int } C1V \leq U$);
- (B) fuzzy almost regular iff for each fuzzy point x_α in X and each regularly open q-nbd U of x_α , there exists a regularly open q-nbd V of x_α such that $C1V \leq U$.

THEOREM 3.5. An fts X is:

- (a) fuzzy regular iff for any fuzzy set A in X , $C1A = [A]_\Theta$;
- (b) fuzzy semi-regular iff $[A]_\delta = C1A$, for any fuzzy set A in X .

PROOF. Let X be fuzzy regular. For any fuzzy set A in X it is always true that $C1A \leq [A]_\Theta$. Now, let x_α be a fuzzy point in X such that $x_\alpha \in [A]_\Theta$ and let U be any open q-nbd of x_α . Then by fuzzy regularity of X , there exists an open q-nbd V of x_α such that $C1V \leq U$. Now, $x_\alpha \in [A]_\Theta \Rightarrow C1V \not\supset A \Rightarrow U \not\supset A \Rightarrow x_\alpha \in C1A$. Thus $[A]_\Theta = C1A$.

Conversely, let x_α be a fuzzy point in X and U an open q-nbd of x_α . Then $x_\alpha \notin (1 - U) = C1(1 - U) = [1 - U]_\Theta$. Thus there exists an open q-nbd V of x_α such that $C1V \not\supset (1 - U)$ and then $C1V \leq U$. Hence X is fuzzy regular. (b) Similar to (a) and is omitted.

COROLLARY 3.6. In a fuzzy regular space (X, T) , a fuzzy closed set is fuzzy Θ -closed, and hence for any fuzzy set A in X , $[A]_\Theta$ is fuzzy Θ -closed.

LEMMA 3.7. For any fuzzy semi-open set A in X , $[A]_\delta = C1A$.

PROOF. It suffices to show that $[A]_\delta \leq C1A$. Let $x_\alpha \notin C1A$. Then there exists an open q-nbd V of x_α such that $V \not\supset A$. Then $\text{Int } C1V \leq \text{Int } C1(1 - A) = 1 - C1 \text{Int } A \leq 1 - A$ (since A is fuzzy semi-open). Thus $\text{Int } C1V \not\supset A$ and consequently, $x_\alpha \notin [A]_\delta$.

THEOREM 3.8. An fts X is fuzzy almost regular iff $[A]_\Theta = C1A$, for every fuzzy semi-open set A in X .

PROOF. Let X be fuzzy almost regular and A any fuzzy semi-open set in X . It is enough to show that $[A]_\Theta \leq C1A$. Suppose $x_\alpha \notin C1A$. By Lemma 3.7, there exists an open q-nbd V of x_α such that $\text{Int } C1V \not\supset A$. Since X is fuzzy almost regular, there is a fuzzy regularly open set U such that $x_\alpha U \leq C1U \leq \text{Int } C1V \leq 1 - A$. Then $C1U \not\supset A$ and hence $x_\alpha \notin [A]_\Theta$. Conversely, let U be any fuzzy regularly open q-nbd of a fuzzy point x_α . Then $x_\alpha \notin 1 - U = C1(1 - U) = [1 - U]_\Theta$, since a fuzzy regularly closed set is fuzzy semi-open. Hence, there is an open q-nbd V of x_α such that $C1V \not\supset (1 - U)$. Since $V \leq \text{Int } C1V$, $\text{Int } C1V$ is a regularly open q-nbd of x_α such that $C1 \text{Int } C1V = C1V \leq U$ and X is fuzzy almost regular.

THEOREM 3.9. In an fts X , the following statements are equivalent:

- (a) For any fuzzy open set A in X , $[[A]_\Theta]_\Theta = [A]_\Theta$.
- (b) For any fuzzy set A in X , $[[A]_\Theta]_\Theta = [A]_\Theta$.
- (c) For any fuzzy set A in X , $[A]_\Theta = [A]_\delta$.
- (d) X is fuzzy almost regular.

PROOF. (a) $\Rightarrow$ (d): We first show that for any fuzzy regularly closed set F in X , $F = [F]_\Theta$. In fact, F being fuzzy regularly closed, $F = C1U$, for some fuzzy open set U . Now, $[F]_\Theta = [C1U]_\Theta = [[U]_\Theta]_\Theta$ (since U is fuzzy open) $= [U]_\Theta = C1U = F$. Next, let x_α be a fuzzy point in X and A any fuzzy

regularly open set in X with $x_\alpha qA$. Then $x_\alpha \notin (1-A) = [1-A]_\Theta$, since $(1-A)$ is fuzzy regularly closed. Hence, there exist a fuzzy open set V such that $x_\alpha qV$, but $C1V \not\leq (1-A)$. Let $W = \text{Int } C1V$. Then $x_\alpha qW$, and $C1W = C1V \not\leq (1-A)$. Thus, W is a regularly open q-nbd of x_α such that $C1W \leq A$. Hence, X is fuzzy almost regular.

(d) $\Rightarrow$ (c): For any fuzzy set A , it is clear that $[A]_\delta \leq [A]_\Theta$. Now, let $x_\alpha \in [A]_\Theta$ and U an open q-nbd of x_α . Then $x_\alpha q \text{Int } C1U$. By (d), there exists a regularly open q-nbd V of x_α such that $C1V \leq \text{Int } C1U$. Now, $x_\alpha \in [A]_\Theta \Rightarrow C1V qA \Rightarrow \text{Int } C1U qA \Rightarrow x_\alpha \in [A]_\delta$.

(c) $\Rightarrow$ (b): $[[A]_\Theta]_\Theta = [[A]_\delta]_\Theta = [[A]_\delta]_\delta = [A]_\delta = [A]_\Theta$.

(b) $\Rightarrow$ (a): Obvious.

DEFINITION 3.10. A function $f: X \rightarrow Y$ from an fts (X, T) to another fts (Y, T_1) is called

- (i) fuzzy weakly continuous [10] iff for each fuzzy open set A in Y , $f^{-1}(A) \leq \text{Int } f^{-1}(C1A)$.
- (ii) fuzzy Θ -continuous [6] iff for each fuzzy point x_α in X and each open q-nbd V of x_α , $f(C1V) \leq C1V$, for some open q-nbd U of x_α .

LEMMA 3.11. Let $f: X \rightarrow Y$ be a function. Then for a fuzzy set B in Y , $f(1 - f^{-1}(B)) \leq 1 - B$, where equality holds if f is onto.

PROOF. Let $y \in Y$. If $f^{-1}(y) = \emptyset$, then $[f(1 - f^{-1}(B))](y) = 0 \leq (1 - B)(y)$. If $f^{-1}(y) \neq \emptyset$, then $[f(1 - f^{-1}(B))](y) = \sup_{x \in f^{-1}(y)} [1 - f^{-1}(B)](x) = \sup_{x \in f^{-1}(y)} \{1 - B(f(x))\}$

$$= \sup_{x \in f^{-1}(y)} \{1 - B(y)\} = 1 - B(y) = (1 - B)(y).$$

If f is onto, then for each $y \in Y$, $f^{-1}(y) \neq \emptyset$, and hence we have $f(1 - f^{-1}(B)) = 1 - B$.

THEOREM 3.12. A function $f: X \rightarrow Y$ is:

- (a) fuzzy weakly continuous iff $f(C1U) \leq [f(U)]_\Theta$, for each fuzzy set U in X .
- (b) fuzzy Θ -continuous iff $f([A]_\Theta) \leq [f(A)]_\Theta$, for any fuzzy set A in X .

PROOF. (a) Let f be a fuzzy weakly continuous and U any fuzzy set in X . Suppose $x_\alpha \in C1U$. It is enough to show that $f(x_\alpha) \in [f(U)]_\Theta$. Let A be any open q-nbd of $f(x_\alpha)$. Then $f^{-1}(A) q x_\alpha$. By fuzzy weak continuity of f , $f^{-1}(A) \leq \text{Int } f^{-1}(C1A)$ and hence $\text{Int } f^{-1}(C1A)$ is an open q-nbd of x_α . Since $x_\alpha \in C1U$, we have $\text{Int } f^{-1}(C1A) q U$. Then $f^{-1}(C1A) q U$ and hence $C1A q f(U)$. Thus $f(x_\alpha) \in [f(U)]_\Theta$.

Conversely, for any fuzzy open set U in Y , $f(1 - \text{Int } f^{-1}(C1U))$

$$= f(C1(1 - f^{-1}(C1U))) \leq [f(1 - f^{-1}(C1U))]_\Theta \leq [1 - C1U]_\Theta$$

(by Lemma 3.11)

$$= C1(1 - C1U) = 1 - \text{Int } C1U \leq 1 - U \Rightarrow f(1 - \text{Int } f^{-1}(C1U)) q U \Rightarrow 1 - \text{Int } f^{-1}(C1U) q f^{-1}(U) \\ \Rightarrow f^{-1}(U) \leq \text{Int } f^{-1}(C1U)$$

Hence f is fuzzy weakly continuous.

(b) Let the condition hold. For any fuzzy point x_α in X and any open q-nbd A of $f(x_\alpha)$ in Y , we have by Lemma 3.11, $f(1 - f^{-1}(C1A)) \leq 1 - C1A$. Thus, $C1A q f(1 - f^{-1}(C1A))$ so that $f(x_\alpha) \notin [f(1 - f^{-1}(C1A))]_\Theta$. By hypothesis, $f(x_\alpha) \notin f([1 - f^{-1}(C1A)]_\Theta)$ and hence $x_\alpha \notin [1 - f^{-1}(C1A)]_\Theta$. Then there is an open q-nbd V of x_α such that $C1V \not\leq (1 - f^{-1}(C1A))$ and hence $f(C1V) \leq f f^{-1}(C1A) \leq C1A$. Thus f is fuzzy Θ -continuous.

The converse part was proved in [6].

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