ABSTRACT. The purpose of this paper is to study generic submanifolds with parallel structures, generic product submanifolds and totally umbilical submanifolds of a locally conformal Kaehler manifold. Moreover, we give some examples of generic submanifolds of a locally conformal Kaehler manifold which are not CR-submanifolds.

KEY WORDS AND PHRASES. Locally conformal Kaehler manifold, generic submanifold. CR-submanifold.


1. INTRODUCTION.

Let \(\bar{M}\) be an almost Hermitian manifold with almost Hermitian structure \((J, g)\). The manifold \(\bar{M}\) is called a local conformal Kaehler (briefly, l.c.K.) manifold if for any \(z \in \bar{M}\) there is an open neighborhood \(U\) such that, for some differentiable function \(\sigma: U \rightarrow \mathbb{R}, g' = e^{-\sigma} g\mid_{\bar{U}}\) is a Kaehler metric on \(\bar{U}\). If \(\bar{U} = \bar{M}\) then the manifold is called a globally conformal Kaehler (briefly, g.c.K.) manifold. Let \(\Omega\) be the Kaehler form of an almost Hermitian manifold \(\bar{M}\) defined by \(2(U, V) = g(U, J V)\), for any vector fields \(U, V\) on \(\bar{M}\). Then it is easy to see that \(\bar{M}\) is a l.c.K. manifold if and only if there is a global 1-form \(\omega\) (the Lee form of \(\bar{M}\)) such that

\[
d\omega = 0, \quad d\omega = 0, \quad (1.1)
\]

and \(\bar{M}\) is a g.c.K. manifold if and only if \(\omega\) is exact. For a l.c.K. manifold \(\bar{M}\), the Lee vector field \(B\) is given by

\[
g(B, U) = \omega(U) \quad (1.2)
\]

for any vector field \(U\) on \(\bar{M}\). We denote by \(\nabla\) the Levi-Civita connection of \(g\). We define a torsion-free linear connection \(\nabla\) on \(\bar{M}\) by

\[
\nabla_U V = \nabla_U V - \frac{1}{2} (\omega(U)V + \omega(V)U - g(U, V)B) \quad (1.3)
\]
for any vector fields $U, V$ on $\mathcal{M}$. The linear connection $\nabla$ is called the Weyl connection of $\mathcal{M}$. Then we may easily observe that the Weyl connection $\nabla$ satisfies the condition: $\nabla J = 0$, $\nabla g = 0$ on each neighborhood on which $(J, g^\prime = e^{-\sigma} g |_{q_0})$ is a Kaehler structure.

In general, let $\mathcal{M}$ be a 2n-dimensional almost Hermitian manifold and $M$ be an m-dimensional Riemann manifold isometrically immersed in $\mathcal{M}$. Let $\nabla$ be the Levi-Civita connection on $M$ induced by $\nabla$. Then the Gauss and Weingarten formulas are given respectively by

$$\nabla_U V = \nabla_U V + h(U, V), \quad (1.4)$$

$$\nabla_U N = -A_N U + \nabla_{\left[ N \right]} U \quad (1.5)$$

for any vector fields $U, V$ tangent to $M$ and $N$ normal to $M$, where $h$ is the second fundamental form of $M$ in $\mathcal{M}$ and $\nabla \left[ N \right]$ is the normal connection on the normal bundle $T^\perp (M)$ with respect to the Levi-Civita connection $\nabla$. Then we have $g(A_N U, V) = g(h(U, V), N)$, for any vector fields $U, V$ tangent to $M$. For any vector field $U$ tangent to $M$, we put

$$JU = PU + FU \quad (1.6)$$

where $PU$ and $FU$ are tangential and normal components of $JU$, respectively. Then $P$ is an endomorphism of the tangent bundle $T(M)$ of $M$ and $F$ is a normal bundle valued 1-form on $T(M)$. For any vector field $N$ normal to $M$, we put

$$JN = tN + fN, \quad (1.7)$$

where $tN$ and $fN$ are the tangential and normal components of $JN$, respectively. Then $t$ is an endomorphism of the normal bundle $T^\perp (M)$ of $M$ in $\mathcal{M}$ and $t$ is a tangent bundle valued 1-form on $T^\perp (M)$.

**DEFINITION:** Let $M$ be a submanifold of an almost Hermitian manifold $\mathcal{M}$. The holomorphic subspace $D_x$ of $T_x M$ at $x \in M$ is defined by $D_x = T_x M \cap JT_x M$. $D_x$ is the maximal complex subspace of $T_x \mathcal{M}$ which is contained in $T_x M$. If the dimension of $D$ is constant along $M$, and furthermore, $D$ defines a differentiable distribution on $M$, then $M$ is called a generic submanifold of $\mathcal{M}$.

Let $M$ be a generic submanifold of an almost Hermitian manifold $\mathcal{M}$. We call the distribution $D$ the holomorphic distribution and the orthogonal complementary distribution $D^\perp$ the purely real distribution. They satisfy the following relations:

$$D_x \cap D_x^\perp = \{0\}, \quad D_x^\perp \cap JD_x^\perp = \{0\} \text{ for each } x \in M.$$  

Let $\nu_x$ be the holomorphic normal space of $M$ at $x$, i.e.,

$$\nu_x = T_x^\perp M \cap JT_x^\perp M.\quad (1.8)$$

Then $\nu_x (x \in M)$ defines a differentiable vector subbundle $\nu$ of $T^\perp (M)$ satisfying

$$T^\perp (M) = FD^\perp + \nu \text{ (direct sum), } t(T^\perp (M)) = D^\perp. \quad (1.8)$$

Furthermore, we have

$$D \perp D^\perp, PD = D \text{ and } D^\perp \supset PD^\perp. \quad (1.9)$$
We put \( \dim D = 2p \) and \( \dim D^\perp = q \). If \( p, q \geq 1 \), then the generic submanifold \( M \) is said to be proper. In the sequel, we shall consider only proper generic submanifolds. We put

\[
(\nabla U)^P V = \nabla U(PIV) - P(\nabla U V),
\]

and

\[
(\nabla U)^F V = \nabla U^F (FV) - F \nabla U V
\]

for any vector fields \( U, V \) tangent to \( M \). We say that \( P \) (resp. \( F \)) is parallel if \( (\nabla U)^P V = 0 \) (resp. \( (\nabla U)^F V = 0 \)) for any vector fields \( U, V \) tangent to \( M \). If a generic submanifold \( M \) of an almost Hermitian manifold \( \tilde{M} \) satisfies the condition \( JD^\perp \subset T^\perp (M) \), then \( M \) is called a CR-submanifold of \( \tilde{M} \). Dragomir ([4]) studied CR-submanifolds of l.c.K. manifolds. The present paper is a continuation of the previous work [5].

2. PRELIMINARIES.

Let \( M \) be a generic submanifold of a l.c.K. manifold \( \tilde{M} \). For the Lee vector field \( B \) of \( \tilde{M} \), we put

\[
B = B^T + B^\perp \quad \text{along } M,
\]

where \( B^T \) (resp. \( B^\perp \)) is the tangential (resp. normal) component of \( B \). Furthermore, we put

\[
B^T = B^D + B^{D^\perp} \quad \text{along } M,
\]

where \( B^D \) (resp. \( B^{D^\perp} \)) is the \( D \)-component (resp. \( D^\perp \)-component) of \( B \). Since \( \tilde{\nabla} J = 0 \) with respect to the Weyl connection \( \tilde{\nabla} \), taking account of (1.3) \~ (1.7), (1.11), (1.12), (2.1) and (2.2), we have

\[
(\nabla X)^P Y - \frac{1}{2}\omega(JY)X + \frac{1}{2}\omega(Y)JX - \text{th}(X,Y)
+ \frac{1}{2}g(X,JY)B^T - \frac{1}{2}g(X,Y)P^T - \frac{1}{2}g(X,Y)h^T = 0,
\]

(2.3)

\[
h(X,Y) = F \nabla X Y + \frac{1}{2}g(X,JY)B^T
- \frac{1}{2}g(X,Y)FB^T - \frac{1}{2}g(X,Y)h^T - fh(X,Y) = 0,
\]

(2.4)

\[
(\nabla X)^P Z - A F Z X - \frac{1}{2}\omega(JZ)X + \frac{1}{2}\omega(Z)JX - \text{th}(X,Z) = 0,
\]

(2.5)

\[
(\nabla X)^F Z + h(X,PZ) - fh(X,Z) = 0,
\]

(2.6)

\[
(\nabla Z)^P X - \frac{1}{2}\omega(JX)Z + \frac{1}{2}\omega(X)PZ - \text{th}(X,Z) = 0,
\]

(2.7)

\[
F \nabla Z X - h(JX,Z) + fh(X,Z) = 0,
\]

(2.8)

\[
(\nabla Z)^P W - A F W Z - \frac{1}{2}\omega(JW)Z + \frac{1}{2}\omega(W)PZ + \frac{1}{2}\omega(Z,JW)B^T
- \frac{1}{2}g(Z,W)PB^T - \frac{1}{2}g(Z,W)h^T - \text{th}(Z,W) = 0,
\]

(2.9)

\[
(\nabla Z)^F W + h(Z,PW) + \frac{1}{2}g(Z,JW)B^T + \frac{1}{2}\omega(W)FZ
- \frac{1}{2}g(Z,W)FB^T - \frac{1}{2}g(Z,W)h^T - fh(Z,W) = 0,
\]

(2.10)

for any \( X, Y \in D \) and \( Z, W \in D^\perp \).

We recall the conditions for the distributions \( D \) and \( D^\perp \) to be integrable.
The distribution $D^\perp$ is integrable if and only if

$$g(h(X,JY) - h(JX,Y) + g(X,JY)B, FZ) = 0,$$

for any $X, Y \in D$ and $Z \in D^\perp$.

Let $M$ be a totally geodesic generic submanifold of a Kaehler manifold $\bar{M}$. Then it follows immediately that $P$ and $F$ are parallel, and furthermore $D$ is integrable. So, it is worthwhile to study generic submanifolds with parallel structures and also totally umbilical generic submanifolds in a l.c.K. manifold.

3. GENERIC SUBMANIFOLDS WITH PARELLEL STRUCTURES.

In this section, we consider generic submanifolds with parallel $P$ (resp. $F$) of a l.c.K. manifold.

**THEOREM 3.1.** Let $M$ be a generic submanifold of a l.c.K. manifold $\bar{M}$. If $P$ is parallel, then $D$ is integrable and $B^D = 0$ along $M$. Moreover, if $\dim D \geq 4$, then $B^T = 0$ along $M$.

**PROOF.** By (1.11) and (2.3), we get

$$-\omega(JY)X + \frac{1}{2}\omega(JX)Y + g(X,JY)B = 0,$$

for $X, Y \in D$. Putting $Y = JX$ in (3.2), we get

$$\omega(X)X + \omega(JX)JX - g(X,X)B + \frac{1}{2}\omega(X)JY = 0,$$

for any vector field $X$ on $M$. From (3.2), we get

$$(p - 1)g(B^D, B^D) + p g(B^D, B^D) = 0.$$

First, we assume $p \geq 2$. Then, by (3.3), we have

$$B^D = 0, \quad B^D = 0 \text{ (and hence $B^D = 0$)}.$$

Thus, by (2.3) and (3.4), we get

$$2h(X,Y) + g(X,Y)B = 0,$$

for $X, Y \in D$. On one hand, by (1.11) and (2.4), we get

$$-\frac{1}{2}\omega(JY)X + \frac{1}{2}\omega(JX)Y + h(X,JY)B = 0,$$

for $X, Y \in D$. By (1.11) and (3.6), we get

$$FP[X,Y] + f[h(X,JY) - h(JX,Y)] + g(X,JY)B = 0,$$

for $X, Y \in D$. From (3.5), we get also

$$t[h(X,JY) - h(JX,Y)] + g(X,JY)B = 0,$$

for $X, Y \in D$. Thus, by (3.7) and (3.8), we have

$$J[h(X,JY) - h(JX,Y)] + g(X,JY)B = -FP[X,Y],$$

for $X, Y \in D$. By (3.9), we have
for \( X, Y \in D \) and \( Z \in D^\perp \). Thus, from Proposition 3.1 and (3.10), it follows that \( D \) is integrable. Next, we assume that \( p = 1 \). Then, by (3.3), we have

\[
B^D = 0. 
\] (3.11)

By (2.3), we get

\[
\frac{1}{2} \omega(Y)X - \frac{1}{2} \omega(X)Y + \frac{1}{2} \omega(JY)JX - \frac{1}{2} \omega(JX)JY 
- t(h(X,JY) - h(JX,Y)) - g(X,JY)PB^T - g(X,JY)tB^T = 0,
\] (3.12)

for \( X, Y \in D \). On one hand, by (2.4) and (3.1), we get

\[
FP[X,Y] - f(h(X,JY) - h(JX,Y)) + g(X,JY)fB^\perp = 0,
\] (3.13)

for \( X, Y \in D \). By (3.12) and (3.13), we get

\[
J{h(X,JY) - h(JX,Y)} + g(X,JY)JB^T + g(X,JY)PB^T = 0,
\] (3.14)

for \( X, Y \in D \). From (3.11), it follows that \( PB^T = JB^T \).

Thus, (3.14) implies

\[
h(X,JY) - h(JX,Y) + g(X,JY)B = \frac{1}{2} \omega(X)JY - \frac{1}{2} \omega(Y)JX - \frac{1}{2} \omega(JY)JX + \frac{1}{2} \omega(JX)JY + 2 JFP \{X,Y\},
\] (3.15)

for \( X, Y \in D \). By (3.15), we have

\[
g(h(X,JY) - h(JX,Y) + g(X,JY)B,FZ) = g(FP[X,Y],Z) = 0,
\] (3.16)

for \( X, Y \in D \) and \( Z \in D^\perp \). Thus, from (3.16) and Proposition 3.1, it follows that \( D \) is integrable.

**THEOREM 3.2.** Let \( M \) be a generic submanifold of a l.c.K. manifold \( \tilde{M} \) such that \( F \) is parallel. Then the distribution \( D \) is integrable and each leaf of \( D \) is totally geodesic in \( M \).

**PROOF.** By (1.12), we have

\[
0 = (\nabla_K F)Y = F \nabla_X Y, \text{ for } X, Y \in D.
\] (3.17)

By (3.17), we have \( \nabla_X Y \in D \) for any \( X, Y \in D \), from which the theorem follows immediately.

4. GENERIC PRODUCT SUBMANIFOLDS.

Let \( M \) be a generic submanifold of an almost Hermitian manifold \( \tilde{M} \). If \( M \) is locally expressed in the form \( M = M_D \times M^\perp_D \), where \( M_D \) (resp. \( M_D^\perp \)) is a holomorphic submanifold (resp. a purely real submanifold) of \( \tilde{M} \), then \( M \) is called a generic product submanifold of \( \tilde{M} \). In this section, we consider generic product submanifold of a l.c.K. manifold \( \tilde{M} \).

**THEOREM 4.1.** Let \( M \) be a generic product submanifold of a l.c.K. manifold \( \tilde{M} \). If \( B^D = 0 \) along \( M \), then we have

\[
B^T = 0 \text{ along } M,
\] (4.1)

and

\[
\nabla_X P = 0, \quad (\nabla_Z P)X = 0,
\] (4.2)

for \( X \in D, Z \in D^\perp \).

**PROOF.** Since \( \nabla_X P \in D^\perp \), for \( X \in D, Z \in D^\perp \), by (2.5), we get

\[
g(h(X,Y),FZ) + \frac{1}{2} \omega(JZ)g(X,Y) - \frac{1}{2} \omega(Z)g(JX,Y) = 0,
\] (4.3)
for $X,Y \in D$, $Z \in D \perp$. By (4.3), we get immediately $B^D \perp = 0$, and hence (4.1). Since $(\nabla_X P)Y \in D$, for $X,Y \in D$, by (2.3) and (4.1), we get

$$\nabla_X P Y = 0, \quad \text{for } X,Y \in D. \quad (4.4)$$

Since $(\nabla_Z P) \in D \perp$, for $Z,W \in D \perp$, by (2.9) and (4.1), we get

$$g(h(X,Z), FW) = 0, \quad \text{for } X \in D, Z \in D \perp. \quad (4.5)$$

by (2.5), (4.1) and (4.5), we have

$$0 = g((\nabla_X P)Z, W) - g(h(X, W), FZ) - g(h(X, Z), FW) = g((\nabla_X P)Z, W) \quad (4.6)$$

for $X \in D, Z, W \in D \perp$. By (4.3) and (4.6), we have the first equality of (4.2). Since $(\nabla_Z P)X \in D$, for $X \in D, Z \in D \perp$, by (2.7), we have immediately the second equality of (4.2).

COROLLARY 4.2. Let $M$ be a $C^R$-product submanifold of a 1.c.K. manifold $\bar{M}$. If $B^D = 0$ along $M$, then $P$ is parallel.

PROOF. Since $PW = 0$, and $(\nabla_Z P)W \in D \perp$, for $Z, W \in D \perp$, we have immediately $(\nabla_Z P)W = 0$ for $Z, W \in D \perp$. Thus, from this together with (4.2), the corollary follows. Q.E.D.

5. TOTALLY UMBILICAL GENERIC SUBMANIFOLDS.

A Riemannian submanifold $M$ of a Riemannian manifold is called a totally umbilical submanifold if

$$\omega(U, V) = \omega(U, V)H, \quad (5.1)$$

for any vector fields $U, V$ tangent to $M$, where $H$ is the mean curvature vector. In this section, we consider some totally umbilical generic submanifolds of a 1.c.K. manifold.

THEOREM 5.1. Let $M$ be a totally umbilical generic submanifold of a 1.c.K. manifold $\bar{M}$ such that $\bar{P}$ is parallel. Then we have $B^D \perp = 0$ and $2H + B \perp = 0$ along $M$. In particular, if $\dim D \geq 4$, then $2H + B = 0$ along $M$.

PROOF. Since $\bar{P}$ is parallel, from Theorem 3.1 and (3.4), (3.11), it follows that $D$ is integrable and

$$B^D \perp = 0. \quad (5.2)$$

By (2.4), we have easily

$$2H + B \perp = 0. \quad (5.3)$$

By (3.1), we get

$$\omega(X)^2 + \omega(JX)^2 = g(X, X)g(B^T, B^T), \quad \text{for } X \in D. \quad (5.4)$$

By (5.2) and (5.4), we have

$$(p - 1)g(B^T, B^T) = 0. \quad (5.5)$$

By (5.5), if $p \geq 2$, we have $B^T = 0$. Therefore, the Theorem follows from (5.3). Q.E.D.

COROLLARY 5.2. Let $M$ be a totally umbilical generic submanifold of a 1.c.K. manifold $\bar{M}$ such that $B \in D$. If $\bar{P}$ is parallel, then $M$ is totally geodesic and $B = 0$ along $M$.

THEOREM 5.3. Let $M$ be a totally umbilical generic submanifold of a 1.c.K. manifold $\bar{M}$ such
that \( \dim FD^\perp < \dim D^\perp \) on a dense open subset in \( M \). If \( P \) is parallel and \( \dim D^\perp \geq 2 \), then \( 2H + B = 0 \) along \( M \).

**Proof.** By (1.5), (2.9), (2.1) and (2.2), we have

\[
0 = -\frac{1}{2}\omega(JW)g(Z, B^T) + \frac{1}{2}\omega(JZ)g(W, B^T) + \frac{1}{2}\omega(W)g(PZ, B^T) - \frac{1}{2}\omega(Z)g(PW, B^T) + g(Z, JW)g(B^T, B^T)
\]

for \( Z, W \in D^\perp \). From (5.2) and (5.6), taking account of Theorem 5.1, the theorem follows immediately. \( \quad \square \)

**Theorem 5.4.** Let \( M \) be a totally umbilical generic submanifold of a \( 1.c.K. \) manifold such that \( B \in D^\perp \). Then the purely real distribution \( D^\perp \) is totally geodesic in \( M \).

**Proof.** For \( X \in D, W \in D^\perp \) and \( N \in T^\perp (M) \), by (1.3), (1.5) and (5.1), we have

\[
0 = g(\overline{\nabla} W(J)N, X) = g(\overline{\nabla} W(JN), X) = g(\overline{\nabla} W(JN), X) + g(\overline{\nabla} W(N, JX)
\]

from which the theorem follows immediately. \( \quad \square \)

**Theorem 5.5.** Let \( M \) be a totally umbilical generic submanifold of a \( 1.c.K. \) manifold such that \( F \) is parallel. Then we have \( 2H + B^\perp = 0 \) along \( M \).

**Proof.** Since \( F \) is parallel, from Theorem 3.2, it follows that \( D^\perp \) is integrable and each leaf of \( D^\perp \) is totally geodesic in \( M \). Thus, by (2.4) and (5.1), we have immediately \( 2H + B^\perp = 0 \). \( \quad Q.E.D. \)

6. **Examples.**

In this section, we give some examples of generic submanifolds of Hopf manifolds which are not \( CR \)-submanifolds. Let \( \mathbb{R}^{2n+2} \) be a \((2n+2)\)-dimensional Euclidean space equipped with the canonical inner product \( (\cdot, \cdot) \) and \( \{e_1, \ldots, e_{2n+1}, e_{2n+2}\} \) the canonical orthonormal basis of \( \mathbb{R}^{2n+2} \). We denote by \( J_0 \) the complex structure on \( \mathbb{R}^{2n+2} \) defined by

\[
J_0 e_{2m} = -e_{2m-1}, \quad J_0 e_{2m-1} = e_{2m}, \quad 1 \leq m \leq n+1.
\]

Let \( S^{2n+1} = \{x \in \mathbb{R}^{2n+2}; (x, x) = 1\} \) be a \((2n+1)\)-dimensional unit sphere with the canonical Sasakian structure \((\varphi, \xi, \eta, h)\) induced from the Kaehler structure \((J_0, \cdot, \cdot))\) on \( \mathbb{R}^{2n+2} \). It is well known that the structure vector field \( \xi \) defines the Hopf fibration \( \rho: S^{2n+1} \rightarrow CP^n \), where \( CP^n \) is a \((complex)\ n\)-dimensional complex projective space equipped with the canonical Fibini-Study metric of constant holomorphic sectional curvature 4. Let \( S^1 = \{e^{it\sqrt{-1}}; t \in \mathbb{R}\} \) be a unit circle. We define an almost complex structure \( J \) on \( M = S^{2n+1} \times S^1 \) (resp. \( \tilde{M} = S^{2n+1} \times \mathbb{R} \)) by

\[
JT = J, \quad J\xi = -T \quad and \quad JU = \varphi U.
\]

for any vector field \( U \) on \( \tilde{M} \) such that \( \eta(U) = 0 \), where \( T = \frac{\partial}{\partial t} \) is the canonical unit vector field on \( S^1 \) (resp. \( \mathbb{R}^1 \)). Then \((S^{2n+1} \times S^1, J)\) (resp. \((S^{2n+1} \times \mathbb{R}^1, J)\)) is a \( 1.c.K. \) manifold (resp. a \( g.c.K. \) manifold) together with the product metric \( g = h + 1 \) on \( \tilde{M} = S^{2n+1} \times S^1 \) (resp. \( \tilde{M} = S^{2n+1} \times \mathbb{R}^1 \)). Then the Lee form \( \omega \) of \( \tilde{M} \) is given by \( \omega = 2dt \).

I. We denote by \( S_{pq} \) the Segre imbedding \( S_{pq} = CP^p \times CP^q \rightarrow CP^{p+q} \). Let \( M_1 \) be any \( q \)-dimensional purely real submanifold of \( CP^q \). Then \( M = CP^p \times M_1 \) is a generic product submanifold.
of $CP^p + q + p^q$ in which $CP^p$ is imbedded as a totally geodesic complex submanifold. We denote by
the immersion $i: M_1 \to CP^q$. Let $M = \{S_{pq} \circ (1 \times i) \circ (S^2(p + q + p^q) + 1)\}$ be pull-back of the Hopf
bundle $\pi: S^2(p + q + p^q) + 1$ by the immersion $S_{pq} \circ (1 \times i): CP^p \times M_1 \to CP^p + q + p^q$. Then we may
easily observe that $M$ is a generic submanifold of the Hopf manifold $\overline{M} = S^2(p + q + p^q) + 1 \times S^1$. For
example, let $M_1$ be the real submanifold of $CP^q$ ($q > 1$) defined by
$M_1 = \{(x_0, \ldots, x_{q-1}, x_q + \sqrt{-1} x_{q-1}) \in CP^q; (x_0, \ldots, x_{q-1}, x_q) \}$ be homogeneous coordinates of a q-
dimensional real projective space $RP^q$. Then $M_1$ is a purely real submanifold of $CP^q$ which is not
totally real.

In the following II, IV, we assume that $M = S^7 \times S^1$.

II. Let $M$ be the $5$-dimensional linear subspace of $\mathbb{R}^8$ given by $M = \text{span}\{e_1, \ldots, e_3\}$. We put
$S^4 = S^7 \cap \Pi$ and $M_4 = \{z = \sum_{i=1}^5 \xi_i \in S^4; 0 < x^5 < 1\}$. For each point $z \in M_4$, let $D_z$ be the subspace of
$T_z M_4$ defined by $D_z = \{u \in T_z M_4; (u, J_0 z) = 0, (u, e_5) = 0\}$. We put $M = M_4 \times S^1 \subset T^7 \times S^1$. For each
point $(z, e^{\sqrt{-1} t}) \in M$, let $D = \text{be the subspace of } T_{(z, e^{\sqrt{-1} t})} M$ defined by
$D = \{(u, 0) \in T_{(z, e^{\sqrt{-1} t})} M; u \in D_z\}$. Then we may easily observe that $M$ is a totally
geodesic generic submanifold of $\overline{M}$ with the holomorphic distribution $D$ which is not a CR-
submanifold of $\overline{M}$. We may easily check that the Lee form of $\overline{M}$ is tangent to $M$.

III. We put $M = M_4 \times \{1\} \subset S^7 \times S^1$. Then $M$ is also a totally geodesic generic submanifold of
$\overline{M}$ with holomorphic distribution $D$ as in II (restricted to $M_4 \times \{1\}$) which is not CR-submanifold of $\overline{M}$. In this case, we may easily check that the Lee form of $\overline{M}$ is normal to $M$.

IV. We put $M_3 = \{z = \sum_{i=1}^5 \xi_i e_i + \frac{1}{\sqrt{2}} e_7 \in S^7; 0 < \xi_5 < \frac{1}{\sqrt{2}}\}$. For each point $z \in M_3$, let $D_z$ be the
subspace of $T_z M_3$ defined by $D_z = \{u \in T_z M_3; (u, J_0 z) = 0, (u, e_5) = 0\}$. We put $M = M_3 \times \{1\}$.
For each point $(z, 1) \in M$, let $D_{(z, 1)}$ be the subspace of $T_{(z, 1)} M$ defined by
$D_{(z, 1)} = \{(u, 0) \in T_{(z, 1)} M; u \in D_z\}$. Then we may easily observe that $M$ is a totally umbilical generic
submanifold of $\overline{M}$ with holomorphic distribution $D$ which is not a CR-submanifold of $\overline{M}$ and is not
totally geodesic in $\overline{M}$.

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