Internat. J. Math. & Math. Sci.
VOL. 17 NO. 1 (1993) 204

CORRIGENDUM

to the paper

A REMARK ON THE WEIGHTED AVERAGES FOR SUPERADDITIVE PROCESSES (INTERNAT. J. MATH. & MATH. SCI. VOL. 14, NO. 3 (1991) 435-438)

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The proof of Theorem 3.2 in [1] contains an error. It is wrongly stated in [1] thate a T-superadditive process is decomposed into the difference of a T-additive process G and a positive, purely T-superadditive process $H = \{H_n\} = \{\Sigma_{k=0}^{n-1} \ h_k\}$, and $h_k = f_k - T^k\delta$. Actually, the equality $h_k = T^k\delta - f_k$ holds for all k, and hence H must be a positive, purely T-subadditive process. Consequently, h_k's need not be positive, and the inequality

$$lim_n sup \mid S_n(A,H) \mid \ \leq M \ lim_n sup \ \frac{1}{n} \ H_n,$$

which is asserted to be true in [1], need not be valid in general. On the other hand, if h_k's are nonnegative for all k, then this inequality still holds. Nonetheless, the decomposition results of Section 2 of [1] are still true, with the forementioned corrections, whereas Theorem 3.2 is not true as it stands, but is true if all h_k's are assumed to be nonnegative. Hence, the corrected statement of Theorem 3.2 should read as:

THEOREM 3.2. Let T be a positive Dunford-Schwartz operator on L_1, or a positive L_p-contraction for $1 < p < \infty$, and F be a T-superadditive process with $h_k \geq 0$, for all k, where $\{\Sigma_{k=0}^{n-1} \ h_k\}_n$ is the purely subadditive part of the process. Assume also that

$$T \text{ is Markovian and } sup_{n \geq 1} \| \frac{1}{n} \ F_n \| \ < \infty, \text{ when } p = 1, \text{ or}$$

$$lim \ inf_n \| \frac{1}{n} \Sigma_{i=1}^{n} \ (F_i - F_{i-1}) \|_p < \infty, \text{ when } 1 < p < \infty.$$

If A is a bounded sequence such that (A,T) is Birkhoff, then $lim_n \ \frac{1}{n} \ S_n(F,A)$ exists a.e.

It must be noted here that, when h_k's are not necessarily nonnegative, $lim_n \ \frac{1}{n} S_n(H,A)$ may not exist as the following simple example shows.

EXAMPLE. Let $h_k = (-1)^k$, $k \geq 0$. Then $\{H_n\} = \{\Sigma_{k=0}^{n-1} h_k\}$ is a subadditive sequence (assuming $H_0 = 0$). Define the sequence $A = \{a_k\}$ of weights as $a_0 = a_1 = 1$, and for $i \geq 0$,

$$a_k = (-1)^k, \text{ when } 3^i.2 \leq k < 3^i.4,$$

$$a_k = (-1)^{k+1}, \text{ when } 3^i.4 \leq k < 3^{i+1}.2.$$

Then $\frac{1}{n} \ S_n(H,A) = 0$, when $n = 3^i.2$, and is equal to $1/2$ when $n = 3^i.4$, for all $i \geq 0$. Therefore we see that

$$lim \ sup \ S_n(H,A) = \frac{1}{2} \qquad \text{whereas } lim \ inf \ S_n(H,A) = 0.$$

ACKKNOWLEDGEMENT. The author is grateful to Professor M. Lin of the Ben Gurion University of the Negev, Israel, for bringing the error in [1] to his attention.

REFERENCE

1. COMEZ, D., A remark on the weighted averages for superadditive processes, *Internat. J. Math. & Math. Sci.* 14 (1991), 435-438.