CORRIGENDUM

to the paper

A REMARK ON THE WEIGHTED AVERAGES FOR SUPERADDITIVE PROCESSES


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The proof of Theorem 3.2 in [1] contains an error. It is wrongly stated in [1] that a T-
superadditive process is decomposed into the difference of a T-additive process G and a positive,
purely T-superadditive process \( H = \{H_n\} = \{\Sigma_{k=0}^{\infty} h_k\} \), and \( h_k = f_k - T^k \delta \). Actually, the
equality \( h_k = T^k \delta - f_k \) holds for all \( k \), and hence \( H \) must be a positive, purely T-subadditive
process. Consequently, \( h_k \)'s need not be positive, and the inequality

\[
\limsup_{n \to \infty} S_n(A, H) \leq M \limsup_{n \to \infty} \frac{1}{n} H_n,
\]

which is asserted to be true in [1], need not be valid in general. On the other hand, if \( h_k \)'s are
nonnegative for all \( k \), then this inequality still holds. Nonetheless, the decomposition results of
Section 2 of [1] are still true, with the forementioned corrections, whereas Theorem 3.2 is not true
as it stands, but is true if all \( h_k \)'s are assumed to be nonnegative. Hence, the corrected statement
of Theorem 3.2 should read as:

THEOREM 3.2. Let \( T \) be a positive Dunford-Schwartz operator on \( L_1 \), or a positive \( L_p \)-
contraction for \( 1 < p < \infty \), and \( F \) be a T-superadditive process with \( h_k \geq 0 \), for all \( k \), where
\( \{\Sigma_{k=0}^{\infty} h_k\}_n \) is the purely subadditive part of the process. Assume also that

\[
T \text{ is Markovian and } \sup_{n \geq 1} \| \frac{1}{n} F_n \| < \infty, \text{ when } p = 1, \text{ or}
\]

\[
\lim\inf_{n} \left( \frac{1}{n} \Sigma_{i=1}^{n} (F_i - F_{i-1}) \right)_{p} < \infty, \text{ when } 1 < p < \infty.
\]

If \( A \) is a bounded sequence such that \((A, T)\) is Birkhoff, then \( \lim_{n} \frac{1}{n} S_n(F, A) \) exists a.e.

It must be noted here that, when \( h_k \)'s are not necessarily nonnegative, \( \lim_{n} \frac{1}{n} S_n(H, A) \) may
not exist as the following simple example shows.

EXAMPLE. Let \( h_k = (-1)^k \), \( k \geq 0 \). Then \( \{H_n\} = \{\Sigma_{k=0}^{n-1} h_k\} \) is a subadditive sequence
(assuming \( H_0 = 0 \)). Define the sequence \( A = \{a_k\} \) of weights as \( a_0 = a_1 = 1 \), and for \( i \geq 0 \),

\[
a_k = (-1)^k, \text{ when } 3^i.2 \leq k < 3^i.4,
\]

\[
a_k = (-1)^{k+1}, \text{ when } 3^i.4 \leq k < 3^{i+1}.2.
\]

Then \( \frac{1}{n} S_n(H, A) = 0 \), when \( n = 3^i.2 \), and is equal to \( 1/2 \) when \( n = 3^i.4 \), for all \( i \geq 0 \). Therefore
we see that

\[
\limsup S_n(H, A) = \frac{1}{2}, \text{ whereas } \liminf S_n(H, A) = 0.
\]

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REFERENCE

1. COMEZ, D., A remark on the weighted averages for superadditive processes, Internat. J.