SEMIPRIME SF-RINGS WHOSE ESSENTIAL LEFT IDEALS ARE TWO-SIDED

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ABSTRACT. It is proved that if $R$ is a semiprime ELT-ring and every simple right $R$-module is flat then $R$ is regular. Is $R$ regular if $R$ is a semiprime ELT-ring and every simple right $R$-module is flat? In this note, we give a positive answer to the question.

KEY WORDS AND PHRASES. (Von Neumann) regular ring, SF-ring, ELT-ring.

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1. INTRODUCTION.

In [1] Yue Chi Ming proposed the following question: Is $R$ regular if $R$ is a semiprime ELT-ring and every simple right $R$-module is flat? In this note, we give a positive answer to the question.

All rings considered in this paper are associative with identity, and all modules are unital. A ring $R$ is (Von Neumann) regular provided that for every $a \in R$ there exists $b \in R$ such that $a = aba$ (see [2]). $R$ is called a strongly regular ring if for each $a \in R, a \in a^2 R$. Following [1], call $R$ and ELT-ring if every essential left ideal is an ideal of $R$. We call $R$ a right SF-ring if every simple right $R$-module is flat (see [3]).

2. MAIN RESULTS.

We begin by stating following lemmas which will be used in proof of our main result.

LEMMA 1. ([4], p.30, Exercise 19) If $R$ is a semiprime ring, then $\text{Soc}(R) = \text{Soc}(R)$.

LEMMA 2. ([5], Corollary 8.5) If $R$ is a semiprime ring, then every minimal left (right) ideal is generated by an idempotent.

LEMMA 3. ([3], Proposition 3.2) Let $R$ be a left (right) SF-ring. If $I$ is an ideal of $R$, then $R/I$ also is a left (right) SF-ring.

LEMMA 4. ([3], Theorem 4.10) Let $R$ be a left (right) SF-ring. If every maximal right (left) ideal of $R$ is an ideal, then $R$ strongly regular.

LEMMA 5. If $R$ is a semiprime ELT and right SF-ring, then $R$ is fully left (right) idempotent.

PROOF. From Lemma 1, $\text{Soc}(R) = \text{Soc}(R)$. Now we write $S$ instead of $\text{Soc}(R)$. By Lemma 2, $S$ is fully left (right) idempotent. Since $R$ is an ELT-ring, and every maximal left ideal of $R/S$ is an image of a maximal essential left ideal of $R$ under the natural map $v: R \rightarrow R/S$, hence every maximal left ideal of $R/S$ is an ideal. By Lemma 3, $R/S$ is a right SF-ring. It follows from Lemma 4 that $R/S$ is strongly regular, whence $R/S$ is fully left (right) idempotent.

Now we prove our main result which gives a positive answer to the question raised in [1].
THEOREM 2.1. If $R$ is a semiprime ELT and right SF-ring, then $R$ is regular.

PROOF. From Lemma 5, $R$ is a fully left (right) idempotent ring. If $P$ is a prime ideal of $R$, then it is easy to know that $R/P$ is a fully right idempotent ring. Since $R$ is ELT, this implies that $R/P$ is an ELT-ring. By (see [6], Corollary 6), $R/P$ is regular. Considering that $R$ is fully idempotent, thus $R$ is a regular ring (see [2], Corollary 1.18).

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REFERENCES

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