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SEMIPRIME SF-RINGS WHOSE ESSENTIAL LEFT IDEALS ARE TWO-SIDED

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ABSTRACT. It is proved that if R is a semiprime ELT-ring and every simple right R-module is flat then R is regular. Is R regular if R is a semiprime ELT-ring and every simple right R-module is flat? In this note, we give a positive answer to the question.

KEY WORDS AND PHRASES. (Von Neumann) regular ring, SF-ring, ELT-ring.
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1. INTRODUCTION.

In [1] Yue Chi Ming proposed the following question: Is R regular if R is a semiprime ELT-ring and every simple right R-module is flat? In this note, we give a positive answer to the question.

All rings considered in this paper are associative with identity, and all modules are unital. A ring R is (Von Neumann) regular provided that for every $a \in R$ there exists $b \in R$ such that $a = aba$ (see [2]). R is called a strongly regular ring if for each $a \in R, a \in a^2R$. Following [1], call R and ELT-ring if every essential left ideal is an ideal of R. We call R a right SF-ring if every simple right R-module is flat (see [3]).

2. MAIN RESULTS.

We begin by stating following lemmas which will be used in proof of our main result.

LEMMA 1. ([4], p.30, Exercise 19) If R is a semiprime ring, then $Soc({}_RR) = Soc(R_R)$.

LEMMA 2. ([5], Corollary 8.5) If R is a semiprime ring, then every minimal left (right) ideal is generated by an idempotent.

LEMMA 3. ([3], Proposition 3.2) Let R be a left (right) SF-ring. If I is an ideal of R, then R/I also is a left (right) SF-ring.

LEMMA 4. ([3], Theorem 4.10) Let R be a left (right) SF-ring. If every maximal right (left) ideal of R is an ideal, then R strongly regular.

LEMMA 5. If R is a semiprime ELT and right SF-ring, then R is fully left (right) idempotent.

PROOF. From Lemma 1, $Soc({}_RR) = Soc(R_R)$. Now we write S instead of $Soc({}_RR)$. By Lemma 2, S is fully left (right) idempotent. Since R is an ELT-ring, and every maximal left ideal of R/S is an image of a maximal essential left ideal of R under the natural map $v: R \to R/S$, hence every maximal left ideal of R/S is an ideal. By Lemma 3, R/S is a right SF-ring. It follows from Lemma 4 that R/S is strongly regular, whence R/S is fully left (right) idempotent. Since S is fully left (right) idempotent, then R is fully left (right) idempotent.

Now we prove our main result which gives a positive answer to the question raised in [1].

THEOREM 2.1. If R is a semiprime ELT and right SF-ring, then R is regular.

PROOF. From Lemma 5, R is a fully left (right) idempotent ring. If P is a prime ideal of R, then it is easy to know that R/P is a fully right idempotent ring. Since R is ELT, this implies that R/P is an ELT-ring. By (see [6], Corollary 6), R/P is regular. Considering that R is fully idempotent, thus R is a regular ring (see [2], Corollary 1.18).

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REFERENCES

1. YUE CHI MING, R., On Von Neumann regular rings V, Math. J. Okayama Univ. 22 (1980), 151-160.
2. GOODEARL, K.R., Von Neumann Regular Rings, Pitman, London, 1979.
3. REGE, M.B., On Von Neumann regular rings and SF-rings, Math. Japonica 31 No. 6 (1986), 927-936.
4. GOODEARL, K.R., Ring Theory: Non-singular Rings and Modules, Dekkar, New York, 1974.
5. FAITH, C., Algebra I: Rings, Modules, and Categories, Springer-Verlag, 1981.
6. HIRANO, Y. & TOMINAGA, H., Regular rings, V-rings and their generalizations, Hiroshima Math. J. 9 (1979), 137-149.