ABSTRACT. Let $f \in H(B_n)$, $f^{[\beta]}$ denotes the $\beta$th fractional derivative of $f$. If $f^{[\beta]} \in A^{p,q,\alpha}(B_n)$, we show that

(I) If $\beta < \frac{\alpha + 1}{p} + \frac{n}{q} = \delta$, then $f \in A^{p,q,\alpha}(B_n)$, and \( \left\| f^{[\beta]} \right\|_{p,q,\alpha} \leq C \left\| f^{[\beta]} \right\|_{p,q,\alpha}^{\beta} \left\| f \right\|_{p,q,\alpha}^{t} \), $t = \frac{\delta q}{\delta - \beta}$

(II) If $\beta > \frac{\alpha + 1}{p} + \frac{n}{q}$, then $f \in B(B_n)$ and $\left\| f \right\|_B \leq C \left\| f^{[\beta]} \right\|_{p,q,\alpha}$

(III) If $\beta > \frac{\alpha + 1}{p} + \frac{n}{q}$, then $f \in \Lambda^{\beta - \frac{\alpha + 1}{p} - \frac{n}{q}}(B_n)$ especially if $\beta = 1$ then $\left\| f \right\|_{\Lambda^{\frac{1}{1 - \alpha q}}} \leq C \left\| f \right\|_{p,q,\alpha}$, where $B_n$ is the unit ball of $C^n$.

KEY WORDS AND PHRASES. Fractional derivative, Bergman space, Bloch space, Lipschitz space

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Let $\Omega$ be a bounded symmetric domain in the complex vector space $C^n$, $\omega \in \Omega$, with Bergman-Silov boundary $b$, $\Gamma$ the group of holomorphic automorphisms of $\Omega$ and $\Gamma_0$ its isotropy group. It is known that $\Omega$ is circular and star-shaped with respect to $\omega$ and $b$ is circular. The group $\Gamma_0$ is transitive on $b$ and $b$ has a unique normalized $\Gamma_0$-invariant measure $\sigma$ with $\sigma(b) = 1$. Hua [2] constructed by group representation theory a system $\{f_{kv}\}$ of homogeneous polynomials, $k = 0, 1, \ldots, v = 1, \ldots, m_k, m_k = \binom{n + k - 1}{k}$, complete and orthogonal on $\Omega$ and orthonormal on $b$.

By $H(\Omega)$ we denote the class of all holomorphic functions on $\Omega$. Every $f \in H(\Omega)$ has a series expansion

$$f(z) = \sum_{k,v} a_{kv} f_{kv}(z), \quad a_{kv} = \lim_{r \to 1} \int_0^r f(r\xi) \frac{\phi_{kv}(\xi)}{d\sigma(\xi)}$$

(0)

where $\sum_{k,v} = \sum_{k=0}^{\infty} \sum_{v=1}^{m_k}$ and the convergence is uniform on a compact subset of $\Omega$.

Let $f \in H(\Omega)$ with the expansion (0) and $\beta > 0$. The $\beta$th fractional derivatives of $f$ are defined, respectively, by

$$f^{[\beta]}(z) = \sum_{k,v} \frac{\Gamma(k + 1 + \beta)}{\Gamma(k + 1)} a_{kv} f_{kv}(z)$$

$$f_{[\beta]}(z) = \sum_{k,v} \frac{\Gamma(k + 1 + \beta)}{\Gamma(k + 1)} a_{kv} f_{kv}(z)$$

It is known that $f^{[\beta]}, f_{[\beta]} \in H(\Omega)$ and

$$f(r\xi) = \frac{1}{\Gamma(\beta)} \int_0^1 (1 - \rho)^{\beta - 1} f^{[\beta]}(\tau \rho \xi) d\rho$$

(1)

Let $f \in H(\Omega)$. It will be said that $f$ belongs to the Bergman spaces $A^{p,q,\alpha}(\Omega), 0 < p, q \leq \infty, \alpha > -1$ if

$$\left\| f \right\|_{p,q,\alpha} = \begin{cases} \left( \int_0^1 (1 - r)^{\alpha} M_q(r,f)^p d\lambda \right)^{\frac{1}{p}}, & p < \infty \\ \sup_{0 < r < 1} (1 - r)^{\alpha} M_q(r,f), & p = \infty \end{cases}$$

is finite, where
\[ M_q(r, f) = \left( \int_0^r |f(r\xi)|^q \, d\alpha(\xi) \right)^{1/q}, \quad 0 < q < \infty \]

and

\[ M_\infty(r, f) = \sup_{\xi \in \mathbb{B}} |f(r\xi)| \]

see [1, 3, 5, 6, 7] for more on \( A^{p,q,0}(\Omega) \) For \( 0 < p \leq \infty \), let \( A^p(\Omega) \) denote \( A^{p,0,0}(\Omega) \) (see [10, 12]), \( H^p(\Omega) \) denote \( A^{\infty,0,0}(\Omega) \) (see [9]).

Let \( B_n \) denote the unit ball in \( C^n \). A function \( f \in H(B_n) \) is called a Bloch function, that is, \( f \in B(B_n) \), if (\( \|f\|_B = \sup_{z \in B_n} (1 - |z|^2) |f'(z)| < \infty \))

For \( 0 < \alpha < \infty \), the definition of Lipschitz space \( A^\alpha(B_n) \) can be found in [4, §8.8].

In [10] and [12], Watanable and Stojan considered the problem \( \text{If } f' \in A^p(D) (D \text{ is the unit disc of } C^1), \text{then } q = 5 \text{ such that } f \in A^q(D) \) In this paper we consider and solve the same problem in \( A^{p,q,0}(\Omega) \).

The main results of this paper are the following

**THEOREM 1.** Let \( 0 < p, q \leq \infty, \alpha > -1, 0 < \beta < \delta \leq \frac{\alpha + 1}{p} + \frac{n}{q} \), if \( f^{[\beta]} \in A^{p,q,0}(\Omega) \) and \( f^{[\beta]}(r\xi) = O\left(\|f^{[\beta]}\|_{p,q,0}(1 - r)^{-\delta}\right) \), then \( f \in A^{s,t,0}(\Omega) \) and \( \|f\|_{s,t,0} \leq C\|f^{[\beta]}\|_{p,q,0} \), where \( s = \frac{\beta p}{\delta - \beta}, t = \frac{\beta n}{\delta - \beta} \).

**THEOREM 2.** Let \( 0 < p, q \leq \infty, \alpha > -1, 0 < \beta < \infty, f^{[\beta]} \in A^{p,q,0}(B_n) \).

- (I) If \( \beta < \frac{\alpha + 1}{p} + \frac{n}{q} \), then \( f \in B(B_n) \) and \( \|f\|_B \leq C\|f^{[\beta]}\|_{p,q,0} \), where \( s, t \) are the same as above.
- (II) If \( \beta = \frac{\alpha + 1}{p} + \frac{n}{q} \), then \( f \in B(B_n) \) and \( \|f\|_B \leq C\|f^{[\beta]}\|_{p,q,0} \), especially if \( \beta = 1 \), then \( \|f\|_{1,1} \leq C\|f^{[1]}\|_{p,q,0} \).

**REMARK.** (i) Theorem 2(I) \( (p = q, \alpha = 0, \beta = n = 1) \) extends the results of Watanable's and Stojan's (ii) Theorem 1 \( (p = \infty) \) extends the results of Shi's ([9]) and Lou's ([6, 7]).

**REFERENCES**

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