ON LOCALLY S-CLOSED SPACES

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ABSTRACT. In the present paper, the concepts of s-closed subspaces, locally s-closed spaces have been introduced and characterized. We have seen that local s-closedness is a semi-regular property; the concept of s-θ-closed mapping has been introduced here and the following important properties are established:--

Let \( f : X \to Y \) be an s-θ-closed surjection with s-set (Malo and Noiri [8]) point inverses. Then:

(a) If \( f \) is completely continuous (Arya and Gupta [1]) and \( Y \) is a locally compact \( T_\sigma \)-space, then, \( X \) is locally s-closed.
(b) If \( f \) is \( \gamma \)-continuous (Ganguly and Basu [5]) and \( X \) is a locally compact \( T_\sigma \)-space, then, \( Y \) is locally s-closed.

KEY WORDS AND PHRASES. s-closed subspace, s-set, locally s-closed, s-θ-closed mapping, \( \gamma \)-continuous and completely continuous mapping, regular open set, s-θ-open set, local compactness.


1. INTRODUCTION. S-closed spaces (Thompson [14]) and s-closed (Malo and Noiri [8]) spaces originated from almost compact spaces by the use of semi-open sets as introduced by Levine [7]. Ganster and Reilly [6] had shown, towards the distinction between these concepts, that every infinite topological space can be embedded as a closed connected subspace of an S-closed space which is not an s-closed space. Noiri [13] further generalized S-closed spaces to locally S-closed spaces. In this paper we generalize s-closed spaces to locally s-closed spaces and study s-closed subspaces. Certain important characterizations and properties of locally s-closed spaces have also been established. s-θ-closed mapping is introduced and characterized and we have seen, under certain conditions on the domain and co-domain spaces, that an s-θ-closed mapping would be a continuous mapping. Completely continuous and \( \gamma \)-continuous mappings were introduced respectively by Arya and Gupta [1] and Ganguly and Basu [5]; by the help of these mappings we have been able to establish certain properties which correlate locally compact \( T_\sigma \)-spaces with locally s-closed spaces.
Throughout the present paper, by \((X,T)\) or simply by \(X\) we shall mean a topological space. A subset \(A\) of a topological space is said to be regular open (resp. regular closed) if \(\text{int}(\text{cl}(A))=A\) (resp. \(\text{cl}(\text{int}(A))=A\)), where \(\text{cl}(A)\) (resp. \(\text{int}(A)\)) denotes the closure (resp. interior) of \(A\). A subset \(A\) of a space \(X\) is said to be semi-open \([7]\) if there exists an open set \(O\) such that \(O \subseteq A \subseteq \text{cl}(O)\). The complement of a semi-open set is called semi-closed (Crossley and Hildebrand \([3]\)). The semi-closure of a subset \(A\) of a space, denoted by \(\text{scl}(A)\), is the intersection of all semi-closed sets containing \(A\) (Crossley and Hildebrand \([3]\)). A set \(A\) which is both semi-open as well as semi-closed is called a semi-regular set (Maio and Noiri \([8]\)). The collection of all semi-open (resp. semi-regular, regular open) sets containing a point \(x\) of \(X\) will be denoted by \(\text{SO}(x)\) (resp. \(\text{SR}(x), \text{RO}(x)\)) and for the whole space \(X\) these will be denoted by \(\text{SO}(X)\) (resp. \(\text{SR}(X), \text{RO}(X)\)). A point \(x\) of \(X\) is said to be \(s\)-\(\theta\)-cluster \([8]\) point of a subset \(A\) of \(X\) if for every \(U \in \text{SO}(x)\), \(\text{scl}(U) \cap A \neq \emptyset\). Since, for a semi-open set \(U\), \(\text{scl}(U)\) is a semi-regular set \([8]\), a point \(x\) of \(X\) is said to be an \(s\)-\(\theta\)-cluster point of \(A\) if \(\forall R \in \text{SR}(x), R \cap A \neq \emptyset\). The collection of all \(s\)-\(\theta\)-cluster points of \(A\) will be denoted by \(\text{scl}(A)\) (short for \(s\)-\(\theta\)-closed). A complement of an \(s\)-\(\theta\)-closed set is called an \(s\)-\(\theta\)-open set. For a space \((X,T)\), \(\text{RO}(X,T)\) is a base for a topology \(T_s\) on \(X\) coarser than \(T\) and \((X,T_s)\) is called the semi-regularization space of \((X,T)\). A topological property \(P\) is said to be semi-regular property if whenever a space \((X,T)\) possesses that property \(P\) so does its semi-regularization space \((X,T_s)\). A subset \(A\) of \(X\) is \(s\)-closed \([8]\) (resp. \(s\)-closed (Noiri \([11]\))) relative to \(X\) or simply an \(s\)-set (resp. \(S\)-set) if every cover \(\mathcal{U}\) of \(A\) by sets of \(\text{SO}(X)\) admits a finite subfamily \(\mathcal{U}_0\) such that \(A \subseteq \bigcup \mathcal{U}_0\) (resp. \(\text{cl}(A) \subseteq \bigcup \mathcal{U}_0\)). In case \(A = X\) and \(A\) is an \(s\)-set (resp. \(S\)-set), then \(X\) is called \(s\)-closed \([8]\) (resp. \(S\)-closed \([14]\)). A subset \(A\) is called nearly compact (NC-set (Carnahan \([2]\)), for short) if every cover \(\mathcal{U}\) of \(A\) by means of open sets of \(X\) has a finite subfamily \(U_1, \ldots, U_n\) (say) such that \(A \subseteq \bigcup_{i=1}^n \text{int}(U_i)\). Clearly every \(s\)-set (resp. compact) set, is an NC-set, but not conversely. A subset \(A\) of a space \(X\) is said to be an \(\alpha\)-set (Noiri \([10]\)) if \(A \subseteq \text{int}(\text{cl}(\text{int}(A)))\).

2. \(s\)-CLOSED SUBSPACES. At the very outset, an example is given to assert that, every set, \(s\)-closed relative to \(X\), is not necessarily an \(s\)-closed subspace of \(X\).

**EXAMPLE 1.** Every countable set in an uncountable set \(X\) with co-countable topology \(T\) is \(s\)-closed relative to \((X,T)\), but is not even an \(s\)-closed subspace.

**DEFINITION 1.** A subset \(A\) of \(X\) is said to be pre-open (Mashour et al. \([9]\)) if \(A \subseteq \text{int}(\text{cl}(A))\). This collection includes all open sets and, even more, all \(\alpha\)-open sets.

**LEMMA 1.** (See Dorsett \([4]\)) Let \((X,T)\) be a topological space and let \(A\) be pre-open set in \((X,T)\), then \(\text{SR}(A,T_A)=\text{SR}(X,T) \cap A\), where \(T_A\) is the subspace topology on \(A\).

**THEOREM 1.** A pre-open set \(A\) of \(X\) is \(s\)-closed as a subspace iff it is \(s\)-closed relative to \(X\).

**PROOF.** Let \(A\) be \(s\)-closed relative to \(X\) and also let \(\{V_\alpha : \alpha \in I\}\) be a cover of \(A\) by semi-regular sets of the subspace \(A\). Then by Lemma 1, there exists a semi-regular set \(U_\alpha\) in \(X\), for each \(\alpha \in I\), such that \(V_\alpha = U_\alpha \cap A\). Therefore, \(A \subseteq \bigcup_{\alpha \in I} U_\alpha\). Since \(A\) is \(s\)-closed relative to \(X\), there exists a finite subset \(\alpha_0\) of \(I\) such that \(A \subseteq \bigcup_{\alpha \in \alpha_0} U_\alpha\), which shows that \(A \subseteq \bigcup_{\alpha \in \alpha_0} (U_\alpha \cap A)\) i.e., \(A \subseteq \bigcup_{\alpha \in \alpha_0} V_\alpha\). Therefore, \(A\) is \(s\)-closed as a subspace.
Conversely, let $A$ be $s$-closed as a subspace. Let $\{V_\alpha : \alpha \in I\}$ be a cover of $A$ by semi-regular sets of $X$. Then $A = \bigcup \limits_{\alpha \in I} (V_\alpha \cap A)$. Since $A$ is $s$-closed as a subspace, there exists a finite subset $I_0$ of $I$ such that $A = \bigcup \limits_{\alpha \in I_0} (V_\alpha \cap A)$, which shows that $A \subseteq \bigcup \limits_{\alpha \in I_0} V_\alpha$. Therefore $A$ is $s$-closed relative to $X$.

**Theorem 2.** Let $B$ be a pre-open set in $(X,T)$. Then a subset $A$ of $B$ is $s$-closed relative to the subspace $B$ iff $A$ is $s$-closed relative to $X$.

**Proof.** The proof follows by Lemma 1.

**Corollary 1.** Let $A$ and $B$ be open sets of a space $X$ such that $A \subseteq B$. Then $A$ is an $s$-closed subspace of $B$ iff $A$ is an $s$-closed subspace of $X$.

**Proof.** Applying Theorem 1 and Theorem 2, we get the result.

**Definition 2.** Let $(X,T)$ be a topological space, then $SR(X,T)$ forms a sub-base for a topology called $T_{SR}$-topology on $X$.

**Lemma 2.** A subset $A$ of a space $(X,T)$ is $s$-closed relative to $(X,T)$ iff $A$ is compact in $(X,T_{SR})$.

**Proof.** Let $A$ be $s$-closed relative to $(X,T)$. Then every cover of $A$ by sets of $SR(X,T)$ has a finite subcover. But $SR(X,T)$ forms a sub-base for $(X,T_{SR})$. So every sub-basic open cover of $(X,T_{SR})$ has a finite subcover. Therefore by Alexander sub-base theorem $A$ is compact in $(X,T_{SR})$.

Conversely, if $A$ is compact in $(X,T_{SR})$ then every sub-basic open cover has a finite subcover. So every cover by sets of $SR(X,T)$ has a finite subcover. Therefore $A$ is $s$-closed relative to $(X,T)$.

**Theorem 3.** Let $B$ be a $T_{SR}$-closed set in $X$ and let $A$ be any subset of $X$ which is $s$-closed relative to $(X,T)$. Then $A \cap B$ is $s$-closed relative to $(X,T)$.

**Proof.** Let $\{U_\alpha : \alpha \in I\}$ be a $T_{SR}$-open cover of $A \cap B$. Then clearly $\{U_\alpha : \alpha \in I\} \cup (X-B)$ is a $T_{SR}$-open cover of $A$. By Lemma 2, $A$ is compact relative to $(X,T_{SR})$; and so, there exists a finite subset $I_0$ of $I$ such that $A \subseteq \bigcup \limits_{\alpha \in I_0} U_\alpha \cup (X-B)$, which implies that $A \cap B \subseteq \bigcup \limits_{\alpha \in I_0} U_\alpha$. Therefore $A \cap B$ is compact in $(X,T_{SR})$. Then by Lemma 2, $A \cap B$ is $s$-closed relative to $(X,T)$.

**Corollary 2.** If $B$ is regular open or regular closed and $A$ is any subset of $X$ which is $s$-closed relative to $X$, then $A \cap B$ is $s$-closed relative to $X$.

**Proof.** Since every regular closed or regular open set is semi-regular, the corollary follows from Theorem 2.

**Corollary 3.** If $X$ is an $s$-closed space and $A$ is a regular open set of $X$, then $A$ is an $s$-closed subspace of $X$.

**Proof.** The proof follows from Theorem 1 and Theorem 3.

**Corollary 4.** If $A$ is $s$-closed open subspace of $X$ and $B$ is a regular open set of $X$, then $A \cap B$ is an $s$-closed subspace of $X$ and (hence of $A$ and $B$).

**Proof.** The proof follows from Corollary 2 and Theorem 1 and second part follows from Corollary 1.

**Theorem 4.** If $A_i$, $i = 1,2,\ldots,n$ are $s$-sets i.e., $s$-closed relative to $X$, then $\bigcup \limits_{i=1}^n A_i$ is $s$-closed relative to $X$.

**Proof.** Straightforward.

**Theorem 5.** Let $X$ be an $s$-closed space and let $A$ be a closed set of $X$ and let frontier of $A$, denoted by $Fr(A)$, be $s$-closed relative to $X$. Then $A$ is $s$-closed relative to $X$. 
PROOF. Since $X$ is s-closed, by Corollary 3 and Theorem 1, $\text{int}A$ is s-closed relative to $X$ whenever $A$ is a closed set. Since $A = \text{int}A \cup \text{Fr}(A)$, by Theorem 4, $A$ is s-closed relative to $X$.

3. LOCALLY s-CLOSED SPACES

DEFINITION 3. A space $X$ is said to be locally s-closed iff each point belongs to a regular open neighbourhood (nbd. for short) which is an s-closed subspace of $X$.

REMARK 1. Clearly every s-closed space is a locally s-closed space. However, the converse is not true, in general, because any uncountable set with discrete topology is locally s-closed but not s-closed.

THEOREM 6. A topological space $(S, T)$ is locally s-closed iff for each point $x \in X$, there exists a regular open set $U$ containing $x$ such that $U$ is locally s-closed.

PROOF. Sufficiency: At first we prove that if $A$ is a regular-open set in $(X, T)$ then every regular-open set in the subspace $(A, T_A)$ is also regular-open in $(X, T)$. Let $V \subset A$ be regular-open in the subspace $(A, T_A)$. Then $V = \text{int} \text{cl} V = \text{int} (A \cap \text{cl} V) = \text{int} X (A \cap \text{cl} V) = A \cap \text{int} \text{cl} V = A \cap \text{int} \text{cl} V = \text{int} \text{cl} V$ (as $V \subset A$ implies $\text{int} \text{cl} V \subset \text{int} \text{cl} A = A$). Therefore $V$ is regular open in $(X, T)$. Now let $x$ be any point of $X$. Then, by hypothesis, there exists a regular-open set $U$ of $(X, T)$ containing $x$ such that $U$ is locally s-closed. Then there exists a regular open set $V$ in $U$ such that $x \in V$ and $V$ is an s-closed subspace of $U$. Therefore $V$ is a regular-open set in $(X, T)$ and by Corollary 1, $V$ is s-closed subspace of $X$. Therefore $(X, T)$ is locally s-closed.

Necessity: The proof is straightforward.

THEOREM 7. Let $(X, T)$ be a topological space. The following are equivalent:

(i) $X$ is locally s-closed;
(ii) every point has a regular-open set which is s-closed relative to $X$;
(iii) every point $x$ of $X$ has an open nbd $U$ such that $\text{int} \text{cl} U$ is s-closed relative to $X$;
(iv) every point $x$ of $X$ has an open nbd $U$ such that $\text{scl} U$ is s-closed relative to $X$;
(v) for every point $x$ of $X$, there exists an $\alpha$-open set $V$ containing $x$ such that $\text{scl} V$ is s-closed relative to $X$;
(vi) for every point $x$ of $X$, there exists an $\alpha$-open set $V$ containing $x$ such that $\text{int} \text{cl} V$ is s-closed relative to $X$;
(vii) for each $x \in X$, there exists a pre-open set $V$ containing $x$ such that $\text{scl} V$ is s-closed relative to $X$;
(viii) for every $x$ of $X$, there exists a pre-open set $V$ containing $x$ such that $\text{int} \text{cl} V$ is s-closed relative to $X$;
(ix) for every $x \in X$, there exists a pre-open set $V$ containing $x$ such that $\text{int} \text{cl} V$ is an s-closed subspace of $X$.

PROOF. (i) $\Rightarrow$ (ii): Follows from Theorem 1 and from the fact that every regular-open set is pre-open set. (ii) $\Rightarrow$ (iii) is obvious.

(iii) $\Rightarrow$ (iv): Follows from the fact that for an open set $U$, $\text{scl} U = \text{int} \text{cl} U$ (Maio and Noiri [8]). (iv) $\Rightarrow$ (v) is evident, since every open set is $\alpha$-open.

(v) $\Rightarrow$ (vi), (vi) $\Rightarrow$ (vii), (vii) $\Rightarrow$ (viii) and (viii) $\Rightarrow$ (ix) are straightforward because of the facts that every $\alpha$-open set is pre-open and a set $V$ is pre-open iff $\text{scl} V = \text{int} \text{cl} V$ (Dorsett [4]). (ix) $\Rightarrow$ (i) follows from Theorem 1.
THEOREM 8. A topological space \((X,T)\) is locally \(s\)-closed if, its semi-
regularization space \((X,T_S)\) is locally \(s\)-closed.

PROOF. Let \((X,T)\) be locally \(s\)-closed. Dorsett [4] proved that \(SR(X,T)=SR(X,T_S)\) and hence a subset \(A\) of \(X\) is \(s\)-closed relative to \((X,T)\) iff \(A\) is \(s\)-closed relative to \((X,T_S)\). We know that if \(U\) is an open and \(V\) a closed subset of \((X,T)\), then \(cl_T(U) = cl_T S(V) = int_T(U) = int_T S(V)\). Using these results we can easily prove that for a regular-open set \(U\) of \((X,T)\), \(int cl_T(U) = int cl_T S(U)\). Therefore every regular-open set in \((X,T)\) is regular open in \((X,T_S)\) and vice-versa. So \((X,T)\) and \((X,T_S)\) have the same collection of regular-open sets. Therefore, by definition, \((X,T)\) is locally \(s\)-closed iff \((X,T_S)\) is locally \(s\)-closed.

REMARK 2. Local \(s\)-closedness is a semi-regular property.

DEFINITION 4. A function \(f : X \to Y\) is said to be \(s\)-\(\Theta\)-closed if image of each \(s\)-\(\Theta\)-closed set \(A\) of \(X\) is closed in \(Y\).

THEOREM 9. A function \(f : X \to Y\) is \(s\)-\(\Theta\)-closed iff \(cl(f(A)) \subseteq f([A]_{s-\Theta})\) for any subset \(A\) of \(X\).

PROOF. Let \(f\) be \(s\)-\(\Theta\)-closed and \(A\) be any subset of \(X\). Then \([A]_{s-\Theta}\) is closed in \(Y\) (since \([A]_{s-\Theta}\) is \(s\)-\(\Theta\)-closed set). Clearly \(f(A) \subseteq cl(f([A]_{s-\Theta}))\), hence \(cl(f(A)) \subseteq f([A]_{s-\Theta})\). Conversely, let \(A\) be an arbitrary \(s\)-\(\Theta\)-closed set in \(X\). By hypothesis \(f(A) \subseteq cl(f([A]_{s-\Theta}))\), hence \(f(A) = cl(f(A))\) and hence \(f(A)\) is closed in \(Y\).

THEOREM 10. A surjective function \(f : X \to Y\) is \(s\)-\(\Theta\)-closed iff for each subset \(A\) of \(Y\) and each \(s\)-\(\Theta\)-open set \(U\) of \(X\) containing \(f^{-1}(A)\), there exists an open set \(V\) in \(Y\) containing \(A\) such that \(f^{-1}(V) \subseteq U\).

PROOF. Sufficiency: Suppose that the given hypothesis holds. Let \(A\) be any \(s\)-\(\Theta\)-closed set in \(X\). Let \(y\) be an arbitrary point in \(Y\) \(f(A)\); then \(X-A\) is an \(s\)-\(\Theta\)-open set containing \(f^{-1}(y)\); by hypothesis, there exists an open set \(V\) containing \(y\) such that \(f^{-1}(V) \subseteq X-A\). This shows that \(y \in V \subseteq Y-f(A)\). Therefore \(Y-f(A) = \bigcup \{V : y \in Y-f(A)\}\). Hence \(Y-f(A)\) is an open set i.e., \(f(A)\) is closed in \(Y\).

Necessity: Let \(V = Y - f(X-U)\). Since \(f^{-1}(A) \subseteq U\), it shows that \(A \subseteq V\). As \(f\) is \(s\)-\(\Theta\)-closed, \(f(X-U)\) is closed and hence \(V\) is open in \(Y\). Therefore, \(f^{-1}(V) \subseteq X-f^{-1}(f(X-U)) \subseteq U\).

LEMMA 3. A subset \(A\) of a space \(X\) is an \(s\)-set iff every cover of \(A\) by \(s\)-\(\Theta\)-open sets has a finite subfamily which covers \(A\).

PROOF. Sufficiency part is straightforward.

Necessity: Let \(A\) be an \(s\)-set. Let \(\{U \alpha : \alpha \in I\}\) be an \(s\)-\(\Theta\)-open cover of \(A\) and also let \(x \in A\). Then there exists \(\alpha \in I\) such that \(x \in U \alpha\). But \(U \alpha\) being an \(s\)-\(\Theta\)-open set, there exists a semi-open set \(V \alpha\) such that \(x \in V \alpha \subseteq X-cl X \alpha \subseteq U \alpha\). Therefore the family \(\{V \alpha : x \in A\}\) is a cover of \(A\) by semi-open sets of \(X\). Hence there exist points say \(x_1, \ldots, x_n\) such that \(A \subseteq \bigcup_{i=1}^{n} X_{x_i}\). Hence \(A \subseteq \bigcup_{\alpha \in I} U \alpha\). Therefore \(\{U \alpha : \alpha \in I\}\) has a finite subfamily which covers \(A\).

THEOREM 11. Let \(f : X \to Y\) be an \(s\)-\(\Theta\)-closed surjection with \(s\)-\(\Theta\)-point inverses; if \(A\) is any compact set in \(Y\) then \(f^{-1}(A)\) is an \(s\)-set in \(X\).

PROOF. Let \(\{U \alpha : \alpha \in I\}\) be any cover of \(f^{-1}(A)\) by \(s\)-\(\Theta\)-open sets of \(X\). For each point \(y \in A\), \(f^{-1}(y) \subseteq \bigcup_{\alpha \in I} U \alpha\). By hypothesis \(f^{-1}(y)\) is an \(s\)-set, by Lemma 3,
there exists a finite subfamily \( I_0 \) of \( I \) such that \( f^{-1}(y) \subseteq \bigcup_{\alpha \in I_0} U_{\alpha} \). Since we know that \( \text{Union of any collection } s-\Theta \text{-open sets is } s-\Theta \text{-open} \) and since \( f \) is an \( s-\Theta \)-closed function, by Theorem 10, there exists an open set \( V_y \) of \( Y \) containing \( y \) such that \( f^{-1}(V_y) \subseteq \bigcup_{\alpha \in I_0} U_{\alpha} \). \( \{ V_y : y \in A \} \) is a cover of a compact set \( A \) and hence there exist points \( y_1, \ldots, y_n \) of \( A \) such that \( A \subseteq \bigcup_{i=1}^{n} V_{y_i} \) which shows that \( f^{-1}(A) \) is covered by a finite number of \( s-\Theta \)-open sets from \( q \) and hence \( f^{-1}(A) \) is an \( s \)-set.

**COROLLARY 5.** Let \( f : X \to Y \) be an \( s-\Theta \)-closed surjection with \( s \)-set point inverses; if \( X \) is \( T_2 \) and \( Y \) is compact then \( f \) is continuous.

**PROOF.** Let \( A \) be a closed set in \( Y \). Therefore \( A \) is also compact; by Theorem 11, \( f^{-1}(A) \) is an \( s \)-set in \( X \). Since every \( s \)-set is an \( NC \)-set and \( X \) is \( T_2 \), by Theorem 2.1 of T. Noiri [12], \( f^{-1}(A) \) is closed and hence \( f \) is continuous.

**DEFINITION 5.** A function \( f : X \to Y \) is said to be completely continuous (Arya and Gupta [1]) if inverse image of each open set in \( Y \) is regular-open in \( X \).

**THEOREM 12.** Let \( f : X \to Y \) be a completely-continuous \( s-\Theta \)-closed surjection with \( s \)-set point inverses. If \( Y \) is locally compact \( T_2 \), \( X \) is locally \( s \)-closed.

**PROOF.** Since \( Y \) is locally compact \( T_2 \), for each point \( x \in X \), there exists a closed compact nbhd. \( U \) of \( f(x) \). Since \( f \) is completely continuous, \( f^{-1}(\text{int } U) \) is a regular open set containing \( x \). But it is easy to see that every regular-open set is semi-regular and hence an \( s-\Theta \)-closed set (see Malo and Noiri [8]). Since \( U \) is compact and \( f \) is an \( s-\Theta \)-closed function, by Theorem 11, \( f^{-1}(U) \) is an \( s \)-set in \( X \) and \( x \in f^{-1}(\text{int } U) \subseteq f^{-1}(U) \). Hence, by Corollary 2, \( f^{-1}(\text{int } U) \) is an \( s \)-set in \( X \). Therefore \( X \) is locally \( s \)-closed.

**DEFINITION 6.** A function \( f : X \to Y \) is said to be \( \gamma \)-continuous (Ganguly and Basu [5]) if for each \( x \in X \) and each \( W \in \text{SO}(f(x)) \), there is an open set \( V \) containing \( x \) such that \( f(V) \supseteq W \). Equivalently \( f \) is \( \gamma \)-continuous iff the inverse image of each semi-regular set is clopen.

**LEMMA 4.** If \( f : X \to Y \) is \( \gamma \)-continuous and \( K \subseteq X \) is compact; then \( f(K) \) is an \( s \)-set in \( Y \).

**PROOF.** Let \( \{ U_{\alpha} : \alpha \in I \} \) be a cover of \( f(K) \) by semi-regular sets of \( Y \). Then \( \{ f^{-1}(U_{\alpha}) : \alpha \in I \} \) is a cover of \( K \) by open sets of \( X \). Since \( K \) is compact, there exists a finite subset \( I_0 \) of \( I \) such that \( K \subseteq \bigcup_{\alpha \in I_0} f^{-1}(U_{\alpha}) \), i.e., \( f(K) \subseteq \bigcup_{\alpha \in I_0} U_{\alpha} \). So \( f(K) \) is an \( s \)-set in \( Y \).

**LEMMA 5.** (See [12]) Let \( X \) be a \( T_2 \)-space. Then for any disjoint \( NC \)-sets \( A \) and \( B \), there exist disjoint regular open sets \( U \) and \( V \) such that \( A \subseteq U \) and \( B \subseteq V \).

**THEOREM 13.** If \( f : X \to Y \) is an \( s-\Theta \)-closed, \( \gamma \)-continuous surjection with \( s \)-set point inverses and if \( X \) is locally compact \( T_2 \), then \( Y \) is locally \( s \)-closed.

**PROOF.** We shall first prove that \( Y \) is \( T_2 \). Let \( y_1 \) and \( y_2 \) be two distinct points of \( Y \). Then \( f^{-1}(y_1) \) and \( f^{-1}(y_2) \) are disjoint \( s \)-sets and hence disjoint \( NC \)-sets. By Lemma 5, there exist disjoint regular-open sets \( U_1 \) and \( U_2 \) such that \( f^{-1}(y_1) \subseteq U_1 \) and \( f^{-1}(y_2) \subseteq U_2 \). But every regular-open set is an \( s-\Theta \)-open set and so, by Theorem 10, there exist open sets \( V_j, j = 1,2 \) containing \( y_j \) in \( Y \) such that \( f^{-1}(V_j) \subseteq U_j \) where \( j = 1,2 \). Thus \( Y \) is \( T_2 \). Let \( X \) be locally compact \( T_2 \); for each point \( x \) of \( f^{-1}(y) \), there exists a compact closed nbhd. \( U_x \) of \( x \) in \( X \). Since interior of a closed nbhd. is a regular-open set, it is semi-regular as well. Therefore the family \( \{ \text{int } U_x : x \in f^{-1}(y) \} \) is a cover of an \( s \)-set \( f^{-1}(y) \) by semi-regular sets. By Proposition 4.1
of Maio and Noiri [8], there exist points $x_1, \ldots, x_n$ in $t^{-1}(y)$ such that $f^{-1}(y) \subseteq \bigcup_{i=1}^{n} x_i \subseteq \bigcup_{i=1}^{n} x_i \subseteq \text{int} U$. Let $U = \bigcup_{i=1}^{n} x_i$. Then $f^{-1}(y) \subseteq \bigcup_{i=1}^{n} x_i \subseteq \text{int} U$. Since $\text{int} U$ is clearly an $s$-open set containing $t^{-1}(y)$ and since, $f$ is an $s$-closed function by Theorem 10, there exists an open set $V$ containing $y$ such that $f^{-1}(V) \subseteq \text{int} U$. But $f$ being $\mathcal{Y}$-continuous, $f(U)$ is an $s$-set by Lemma 4. Since $Y$ is $T_2$, $f(U)$ is closed by Theorem 2.1 of Noiri [12]. Therefore $Y \in \mathcal{V}$ is an $s$-set and hence by Corollary 2, $\text{intcl} V$ is an $s$-set. Hence $Y$ is locally $s$-closed.

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