Internat. J. Math. & Math. Sci.
VOL. 21 NO. 1 (1998) 33-40

ON CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

SELMA ÖZÇAĞ
Department of Mathematics
Hacettepe University,
Beytepe
06532-Ankara / TURKEY

DOĞAN ÇOKER
Department of Mathematics Education
Hacettepe University,
Beytepe
06532-Ankara / TURKEY

(Received May 1, 1996 and in revised form July 7, 1996)

ABSTRACT. The aim of this paper is to construct the basic concepts related to connectedness in intuitionistic fuzzy special topological spaces. Here we introduce the concepts of C_5-connectedness, connectedness, C_S-connectedness, C_M-connectedness, strong connectedness, super connectedness, C_i-connectedness (i=1,2,3,4), and, obtain several preservation properties and some characterizations concerning connectedness in these spaces.

KEY WORDS AND PHRASES. Intuitionistic fuzzy special set; intuitionistic fuzzy special topology, intuitionistic fuzzy special topological space, continuity; C_5-connectedness; connectedness; C_S-connectedness; C_M-connectedness; strong connectedness; super connectedness; C_i-connectedness (i=1,2,3,4).

1992 AMS SUBJECT CLASSIFICATION CODE. 04A99.

1. INTRODUCTION

After the introduction of the concept of fuzzy sets by Zadeh [1] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set was first published by Krassimir Atanassov [2] and many works by the same author appeared in the literature (see Atanassov [2,3]) Later this concept is used to define intuitionistic fuzzy special sets by Coker [4] and intuitionistic fuzzy topological spaces are introduced by Çoker [5], Coker-Es [6].In this direction some preliminary concepts are also defined by Coþkun-Çoker[7].Here we shall give the classical version of this kind of fuzzy topological space in the framework of connectedness;

especially, we shall make use of several types of fuzzy connectedness in intuitionistic fuzzy topological spaces in Turanli-Coker [8].

2. PRELIMINARIES

First we shall present the fundamental definitions. The following one is obviously inspired by K. Atanassov [2,3]:

DEFINITION 2.1. (see Çoker [4] Let X be a nonempty fixed set. An intuitionistic fuzzy special set (IFSS for short) A is an object having the form $A = < x, A_1 , A_2 >$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A.

Obviously every set A on a nonempty set X is obviously an IFSS having the form $<x,A, A^c>$ One can define several relations and operations between IFSS's as follows:

DEFINITION 2.2. (see Çoker [4,5]) Let X be a nonempty set, and the IFSS's A and B be in the form $A=<x, A_1 , A_2 >$, $B=<x, B_1 , B_2 >$, respectively. Furthermore, let $\{ A_i : i\in J\}$ be an arbitrary family of IFSS's in X, where $A =<x,A_1^{(1)},A_1^{(2)}>$. Then

(a) $A\subseteq B$ iff $A_1\subseteq B_1$ and $A_2\supseteq B_2$; (b) $A=B$ iff $A\subseteq B$ and $B\subseteq A$;

(c) $\overline{A}=<x, A_2 , A_1 >$; (d) $[]A =<x, A_1, A_1^c>$,

(e) $<>A=<x, A_2^c, A_2 >$, (f) $\cup Ai=<x, \cup A_1^{(1)},\cap A_1^{(2)}>$,

(g) $\cap Ai = <x, \cap A_1^{(1)},\cup A_1^{(2)}>$; (h) $\underset{\sim}{\emptyset}= <x, \emptyset ,X >$ and $\underset{\sim}{X}=<x,X, \emptyset>$.

We shall define the image and preimage of IFSS's. Let X and Y be two nonempty sets and $f : X \to Y$ a function.

DEFINITION 2.3. (see Çoker [4,5]) (a) If $B=<y,B_1 ,B_2 >$ is an IFSS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFSS in X defined by $f^{-1}(B) = <x, f^{-1}(B_1), f^{-1}(B_2)>$

(b) If $A=<x,A_1 ,A_2 >$ is an IS in X, then the image of A under f, denoted by f(A), is the IFSS in Y defined by $f(A) = <y, f(A_1), f\_(A_2)>$, where $f\_(A_2)=(f(A_2^c))^c$

COROLLARY 2.1. Let A, A_i ($i\in J$) be IFSS's in X, B, B_j ($j\in K$) IFSS's in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1)\subseteq f(A_2)$ (b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$

(c) $A\subseteq f^{-1}(f(A))$ and if f is injective, then $A=f^{-1}(f(A))$.

(d) $f(f^{-1}(B))\subseteq B$, and if f is surjective, then $f(f^{-1}(B))=B$

(e) $f^{-1}(\cup B_j)=\cup f^{-1}(B_j)$ (f) $f^{-1}(\cap B_j)=\cap f^{-1}(B_j)$

(g) $f(\cup A_i)=\cup f(A_i)$ (h) $f(\cap A_i)\subseteq\cap f(A_i)$, and if f is injective, then $f(\cap A_i)=\cap f(A_i)$.

(i) $f^{-1}(\underset{\sim}{Y})=\underset{\sim}{X}$ (j) $f^{-1}(\underset{\sim}{\emptyset})=\underset{\sim}{\emptyset}$

(k) $f(\underset{\sim}{X})=\underset{\sim}{Y}$ if f is surjective. (l) $f(\underset{\sim}{\emptyset})=\underset{\sim}{\emptyset}$

(m) If f is surjective, then $\overline{f(A)}\subseteq f(\overline{A})$; and if, furthermore, f is injective, we have $\overline{(f(A))} = f(\overline{A})$.

(n) $f^{-1}(\overline{B})=\overline{f^{-1}(B)}$

DEFINITION 2.4 (see Coker [5,9], Coker-Es [6]) An intuitionistic fuzzy special topology (IFST for short) on a nonempty set X is a family τ of IFSS's in X containing $\underset{\sim}{\emptyset}$, $\underset{\sim}{X}$, and closed under finite infima and arbitrary suprema. In this case the pair (X,τ) is called an intuitionistic fuzzy special topological space (IFSTS for short) and any IFSS in τ is known as an intuitionistic fuzzy special open set (IFSOS for short) in X.

Any topological space can be obviously treated as an IFSTS in a usual manner.

PROPOSITION 2.1. Let (X,τ) be an IFSTS on X. Then, we can also construct several IFSTS's on X in the following way.

(a) $\tau_{0,1}=\{[]G:G\in\tau\}$, (b) $\tau_{0,2}=\{\langle\rangle G:G\in\tau\}$.

REMARK 2.1 Let (X, τ) be an IFSTS $\tau_1=\{ G_1 \quad G=\langle x,G_1 ,G_2 \rangle\in\tau \}$ is a topological space on X $\tau_2^* = \{ G_2 : G=\langle x,G_1 ,G_2 \rangle\in\tau \}$ is the family of all closed sets of the topological space $(\tau_2 = \{G_2^c$ $G=\langle x,G_1 ,G_2 \rangle\in\tau \}$ on X.

The complement $\overline{A}$ of an IFSOS A in an IFSTS (X, τ) is called an intuitionistic fuzzy special closed set (IFSCS for short) in X, and the interior and closure of an IFSS A are defined by

$cl(A)=\cap\{K : K$ is an IFSCS in X and $A\subseteq K\}$,

$int(A)=\cup\{G : G$ is an IFSOS in X and $G\subseteq A\}$

DEFINITION 2.5. Let (X, τ) be an IFSTS on X. If $A=int(cl(A))$, then A is called an intuitionistic fuzzy special regular open set in X

DEFINITION 2.6. Let (X, τ) and (Y, ψ) be two IFSTS's and let $f:X\rightarrow Y$ be a function. Then f is said to be continuous iff the preimage of each IFSS in ψ is an IFSS in τ

Here we obtain some characterizations of continuity.

PROPOSITION 2.2 The following are equivalent to each other:

(a) $f . (X, \tau) \rightarrow (Y, \psi)$ is continuous.

(b) The preimage of each IFSCS in Y is an IFSCS in X

(c) $f^{-1} (int(B))\subseteq int(f^{-1} (B))$ for each IFSS B in Y.

(d) $cl(f^{-1} (B))\subseteq (f^{-1} (cl(B))$ for each IFSS B in Y

3. TYPES OF CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

Throughout this section (X, τ) and (Y, ψ) will always denote IFSTS's We shall define several types of connectedness in IFSTS's

DEFINITION 3.1. (see Chaudhuri-Das [10], Turanli-Coker [8])

(a)X is called C_5 -disconnected, if there exists an IFSS A which is both intuitionistic fuzzy special open and intuitionistic fuzzy special closed, such that $\underset{\sim}{\emptyset}\neq A\neq \underset{\sim}{X}$.

(b) X is called C_5 -connected, if X is not C_5 -disconnected.

(c)X is called disconnected, if there exist IFSOS's $A\neq\underset{\sim}{\emptyset}$ and $B\neq\underset{\sim}{\emptyset}$ such that $A\cup B=\underset{\sim}{X}$ and $A\cap B=\underset{\sim}{\emptyset}$

(d) X is called connected, if X is not disconnected.

PROPOSITION 3.1. C_5 -connectedness implies connectedness.

PROOF. Suppose that there exist nonempty IFSOS's A and B such that $A \cup B = \underset{\sim}{X}$, $A \cap B = \underset{\sim}{\emptyset}$, from which we get $A_1 \cup B_1 = X$, $A_2 \cap B_2 = \emptyset$, $A_1 \cap B_1 = \emptyset$, $A_2 \cup B_2 = X$, in other words, $A = \overline{B}$. Hence A is intuitionistic fuzzy special clopen, i.e. (X,τ) is C_5 -disconnected. ■

COUNTEREXAMPLE 3.1. Consider the IFTS τ on $X=\{a,b,c,d\}$, where $\tau=\{\underset{\sim}{\emptyset}, \underset{\sim}{X}, A_1, A_2, A_3, A_4\}$, $A_1 = \langle x,\{a\},\{b,c\}\rangle$, $A_2 = \langle x,\{b,c\},\{a\}\rangle$, $A_3 = \langle x, \emptyset,\{a,b,c\}\rangle$, $A_4 = \langle x,\{a,b,c\},\emptyset\rangle$ (X,τ) is connected, but not C_5 -connected (namely, A_4 is intuitionistic fuzzy special clopen in X).

PROPOSITION 3.2. Let $f: (X,\tau) \to (Y, \psi)$ be a continuous surjection. If X is connected, then so is Y.

PROOF. Assume that Y is disconnected Thus there exist IFSOS's $C \neq \underset{\sim}{\emptyset}$, $D \neq \underset{\sim}{\emptyset}$ in Y such that $C \cup D = \underset{\sim}{Y}$, $C \cap D = \underset{\sim}{\emptyset}$. Now we see that $A = f^{-1}(C)$, $B = f^{-1}(D)$ are IFSOS's in X, since f is continuous From $C \neq \underset{\sim}{\emptyset}$, we get $A = f^{-1}(C) \neq \underset{\sim}{\emptyset}$ (If $f^{1}(C) = \underset{\sim}{\emptyset}$, then $C = ff^{1}(C) = f(\underset{\sim}{\emptyset}) = \underset{\sim}{\emptyset}$, which is a contradiction.) Similarly, we obtain $B \neq \underset{\sim}{\emptyset}$. Now $C \cup D = \underset{\sim}{Y} \Rightarrow f^{-1}(C) \cup f^{-1}(D) = f^{1}(\underset{\sim}{Y}) = \underset{\sim}{X} \Rightarrow A \cup B = \underset{\sim}{X}$, $C \cap D = \underset{\sim}{\emptyset} \Rightarrow f^{1}(C) \cap f^{-1}(D) = f^{1}(\underset{\sim}{\emptyset}) = \underset{\sim}{\emptyset} \Rightarrow A \cap B = \underset{\sim}{\emptyset}$. But this is a contradiction to our hypothesis, thus Y is connected. ■

PROPOSITION 3.3. If (X,τ) is disconnected, then so are the IFSTS's $(X,\tau_{0,1})$ and $(X,\tau_{0,2})$

PROOF. Let there exist IFSOS's $A \neq \underset{\sim}{\emptyset}$ and $B \neq \underset{\sim}{\emptyset}$ such that $A \cup B = \underset{\sim}{X}$, $A \cap B = \underset{\sim}{\emptyset}$. In this case we obtain

$$\underset{\sim}{X} = [\,]\,\underset{\sim}{X} = [\,](A \cup B) = ([\,]A) \cup ([\,]B) \Rightarrow ([\,]A) \cup ([\,]B) = \underset{\sim}{X};$$

$$\underset{\sim}{\emptyset} = [\,]\,\underset{\sim}{\emptyset} = [\,](A \cap B) = ([\,]A) \cap ([\,]B) \Rightarrow ([\,]A) \cap ([\,]B) = \underset{\sim}{\emptyset},$$

which is a contradiction. ■

PROPOSITION 3.4. (X,τ) is C_5-connected iff there exist no nonempty IFSOS's A and B in X such that $A = \overline{B}$.

PROOF. ($\Rightarrow$:) Suppose that A and B are IFSOS's in X such that $A \neq \underset{\sim}{\emptyset} \neq B$ and $A = \overline{B}$ Since $A = \overline{B}$, B is an IFSCS, and $A \neq \underset{\sim}{\emptyset} \Rightarrow B \neq \underset{\sim}{X}$. But this is a contradiction to the fact that X is C_5 -connected

($\Leftarrow$:) Let A be both an IFSOS and IFSCS such that $\underset{\sim}{\emptyset} \neq A \neq \underset{\sim}{X}$. Now take $B = \overline{A}$. In this case B is an IFSOS and $A \neq \underset{\sim}{X} \Rightarrow B = \overline{A} \neq \underset{\sim}{\emptyset}$, which is a contradiction. ■

PROPOSITION 3.5. (X,τ) is C_5 -connected iff there exist no nonempty IFSS's A and B in X such that $B = \overline{A}$, $B = \overline{cl(A)}$, $A = \overline{cl(B)}$.

PROOF. ($\Rightarrow$:) Assume that there exist IFSS's A and B such that $A \neq \underset{\sim}{\emptyset} \neq B$, $B = \overline{A}$, $B = \overline{cl(A)}$, $A = \overline{cl(B)}$. Since $\overline{cl(A)}$ and $\overline{cl(B)}$ are IFSOS's in X, A and B are IFSOS's in X, which is a contradiction (

$\Leftarrow$:) Let A be both an IFSOS and IFSCS in X such that $\underset{\sim}{\emptyset} \neq A \neq \underset{\sim}{X}$. Taking $B=\overline{A}$, we obtain a contradiction. ■

Here we generalize the concepts of C_S-connectedness and C_M-connectedness given by Chaudhuri - Das [10] to the intuitionistic case:

LEMMA 3.1. (a) $A \cap B = \underset{\sim}{\emptyset} \Rightarrow A \subseteq \overline{B}$, (b) $A \not\subseteq \overline{B} \Rightarrow A \cap B \neq \underset{\sim}{\emptyset}$

DEFINITION 3.2. Let A and B be nonzero IFSS's in (X,τ). A and B are said to be weakly separated, if $cl(A) \subseteq \overline{B}$ and $cl(B) \subseteq \overline{A}$; and q-separated, if $cl(A) \cap B = \underset{\sim}{\emptyset} = A \cap cl(B)$.

DEFINITION 3.3. (see Turanli-Çoker [8]) (a) An IFSTS (X,τ) is said to be C_S-disconnected, if there exist weakly separated nonzero IFSS's A and B in (X,τ) such that $\underset{\sim}{X} = A \cup B$

(b) (X,τ) is called C_S-connected, if (X,τ) is not C_S-disconnected.

(c) X is said to be C_M-disconnected, if there exist q-separated nonzero IFSS's A and B in X such that $\underset{\sim}{X} = A \cup B$.

(d) X is called C_M-connected, if X is not C_M-disconnected.

Let us give the connection between these two types of connectedness in IFSTS's:

COROLLARY 3.1. If the IFSTS X is C_S -connected, then X is also C_M -connected.

DEFINITION 3.4. (see Turanli-Çoker [8]) An IFSTS (X,τ) is said to be strongly connected, if there exit no nonempty IFSCS's A and B in X such that $A \cap B = \underset{\sim}{\emptyset}$.

PROPOSITION 3.6. X is strongly connected iff there exist no IFSOS's A and B in X such that $A \neq \underset{\sim}{X} \neq B$ and $A \cup B = \underset{\sim}{X}$.

PROOF. ($\Rightarrow$:) Let A and B be IFSOS's in X such that $A \neq \underset{\sim}{X} \neq B$ and $A \cup B = \underset{\sim}{X}$. If we take $C=\overline{A}$ and $D=\overline{B}$, then C and D become IFSCS's in X and $C \neq \underset{\sim}{\emptyset} \neq D$, $C \cap D = \underset{\sim}{\emptyset}$, a contradiction.

($\Leftarrow$.) Use a similar technique as above ■

PROPOSITION 3.7. Let $f : (X,\tau) \to (Y, \psi)$ be a continuous surjection. If X is strongly connected, then so is Y

PROOF. Suppose that Y is not strongly connected. In this case there exist IFSCS's C and D in Y such that $C \neq \underset{\sim}{\emptyset} \neq D$, $C \cap D = \underset{\sim}{\emptyset}$. Since f is continuous, $f^{1}(C)$ and $f^{1}(D)$ are IFSCS's in X, and $f^{1}(C) \cap f^{1}(D) = \underset{\sim}{\emptyset}$, $f^{-1}(C) \neq \underset{\sim}{\emptyset}$, $f^{1}(D) \neq \underset{\sim}{\emptyset}$. (If $f^{-1}(C) = \underset{\sim}{\emptyset}$, then $f(f^{-1}(C)) = C \Rightarrow f(\underset{\sim}{\emptyset}) = C \Rightarrow \underset{\sim}{\emptyset} = C$, a contradiction.) But this is a contradiction, hence Y is strongly connected, too. ■

Strong connectedness does not imply C_5 -connectedness, and the same is true for IFSTS converse, i.e. C_5 connectedness does not imply strong connectedness. For this purpose see the following counterexamples:

COUNTEREXAMPLES 3.2. Let $X=\{a,b,c,d\}$ (a) If $\tau=\{\underset{\sim}{\emptyset}, \underset{\sim}{X}, A_1, A_2, A_3, A_4\}$, where $A_1 = \langle x,\{b,c\},\{d\}\rangle$, $A_2 = \langle x,\{d\},\{b,c\}\rangle$, $A_3 = \langle x,\emptyset,\{b,c,d\}\rangle$, $A_4 = \langle x,\{b,c,d\},\emptyset\rangle$, then the IFSTS (X,τ) is strongly connected, but not C_5-connected.

(b) If $\tau=\{\underset{\sim}{\emptyset},\underset{\sim}{X},A_1,A_2,A_3,A_4,A_5\}$, where $A_1=\langle x,\{b,c\},\{d\}\rangle$, $A_2=\langle x,\{a\},\{c\}\rangle$, $A_3=\langle x,\{a,d\},\{c\}\rangle$, $A_4=\langle x,\{a,b,c\},\emptyset\rangle$, $A_5=\langle x,\emptyset,\{c,d\}\rangle$, then the IFSTS (X,τ) is C_5-connected, but not strongly connected

DEFINITION 3.5. (see Turanli-Çoker [8]) (a) If there exists an intuitionistic fuzzy special regular open set A in X such that $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$, then X is called super disconnected

(b) X is called super connected, if X is not super disconnected.

Now we give some characterizations of super connectedness:

PROPOSITION 3.8. The following assertions are equivalent:

(a) X is super connected. (b) For each IFSOS $A\neq\underset{\sim}{\emptyset}$ in X we have $cl(A)=\underset{\sim}{X}$

(c) For each IFSCS $A\neq\underset{\sim}{X}$ in X we have $int(A)=\underset{\sim}{\emptyset}$

(d) There exist no IFSOS's A and B in such that $A\neq\underset{\sim}{\emptyset}\neq B$, $A\subseteq\overline{B}$.

(e) There exist no IFSOS's A and B in X such that $A\neq\underset{\sim}{\emptyset}\neq B$, $B=\overline{cl(A)}$, $A=\overline{cl(B)}$

(f) There exist no IFSCS's A and B in X such that $A\neq\underset{\sim}{\emptyset}\neq B$, $B=\overline{int(A)}$, $A=\overline{int(B)}$

PROOF. (a)$\Rightarrow$(b) : Assume that there exists an IFSOS $A\neq\underset{\sim}{\emptyset}$ such that $cl(A)\neq\underset{\sim}{X}$. Now take $B=int(cl(A))$. Then B is a proper intuitionistic fuzzy special regular open set in X, and this is in contradiction with the super connectedness of X.

(b) $\Rightarrow$(c) : Let $A\neq\underset{\sim}{X}$ be an IFSCS in X. If we take $B=\overline{A}$, then B is an IFSOS in X and $B\neq\underset{\sim}{\emptyset}$ Hence $cl(B)=\underset{\sim}{X}\Rightarrow\overline{cl(B)}=\underset{\sim}{\emptyset}\Rightarrow int(\overline{B})=\underset{\sim}{\emptyset}\Rightarrow int(A)=\underset{\sim}{\emptyset}$ follows.

(c) $\Rightarrow$ (d) : Let A and B be IFSOS's in X such that $A\neq\underset{\sim}{\emptyset}\neq B$ and $A\subseteq\overline{B}$. Since $\overline{B}$ is an IFCS in X and $B\neq\underset{\sim}{\emptyset}\Rightarrow\overline{B}\neq\underset{\sim}{X}$, we obtain $int(\overline{B})=\underset{\sim}{\emptyset}$ But, from $A\subseteq\overline{B}$, we see that $\underset{\sim}{\emptyset}\neq A=int(A)\subseteq int(\overline{B})=\underset{\sim}{\emptyset}$, which is a contradiction.

(d) $\Rightarrow$ (a) : Let $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$ be an intuitionistic fuzzy special regular open set in X. If we take $B=\overline{cl(A)}$, we get $B\neq\underset{\sim}{\emptyset}$. (Because, otherwise we have $B=\underset{\sim}{\emptyset}\Rightarrow\overline{cl(A)}=\underset{\sim}{\emptyset}\Rightarrow cl(A)=\underset{\sim}{X}\Rightarrow A=int(cl(A))=int(\underset{\sim}{X})=\underset{\sim}{X}$, but the last result contradicts the fact $A\neq\underset{\sim}{X}$.) We also have $A\subseteq\overline{B}$, and this is a contradiction, too.

(a) $\Rightarrow$ (e) : Let A and B be IFSOS's in X such that $A\neq\underset{\sim}{\emptyset}\neq B$ and $B=\overline{cl(A)}$, $A=\overline{cl(B)}$ Now we have $int(cl(A))=int(\overline{B})=\overline{cl(B)}=A$ and $A\neq\underset{\sim}{\emptyset}$, $A\neq\underset{\sim}{X}$. (If not, i.e. if $A=\underset{\sim}{X}$, then $\underset{\sim}{X}=\overline{cl(B)}\Rightarrow\underset{\sim}{\emptyset}=cl(B)\Rightarrow B=\underset{\sim}{\emptyset}$.) But this is a contradiction .

(e) $\Rightarrow$ (a) : Let A be an IFSOS in X such that $A=int(cl(A))$, $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$. Now take $B=\overline{cl(A)}$. In this case we get $B\neq\underset{\sim}{\emptyset}$ and B is an IFSOS in X and $B=\overline{cl(A)}$ and $\overline{cl(B)}=\overline{cl(\overline{cl(A)})}=\overline{\overline{int(cl(A))}}=int(cl(A))=A$, which is a contradiction.

(e) $\Rightarrow$ (f) : Let A and B IFSCS's in X such that $A \neq \underset{\sim}{X} \neq B$, $B=\overline{int(A)}$, $A=\overline{int(B)}$ Taking $C=\overline{A}$ and $D=\overline{B}$, C and D become IFSOS's in X and $C \neq \underset{\sim}{\varnothing} \neq D$, $\overline{cl(C)} = \overline{cl(\overline{A})} = \overline{\overline{int(A)}} = int(A) = \overline{B} = D$, and similarly $\overline{cl(D)}=C$. But this is an obvious contradiction.

(f) $\Rightarrow$ (e) : One can use a similar technique as in (e) $\Rightarrow$ (f). ■

PROPOSITION 3.9. Super connectedness implies C_5-connectedness.

PROOF. Obvious. ■

But the reverse implication to Proposition 3.9 does not hold in general·

COUNTEREXAMPLE 3.3. Let X={a,b,c,d} and the IFST $\tau=\{\underset{\sim}{\varnothing}, \underset{\sim}{X}, A_1, A_2, A_3, A_4\}$ on X,

where $A_1 = \langle x,\{a\},\{c,d\}\rangle$, $A_2=\langle x,\{d\},\{a,c\}\rangle$, $A_3 = \langle x,\{a,d\},\{c\}\rangle$, $A_4 = \langle x,\varnothing,\{a,c,d\}\rangle$. Then the IFSTS (X,τ) is C_5 -connected, but not super connected

PROPOSITION 3.10. Let $f:(X,\tau) \to (Y, \psi)$ be a continuous surjection. If X is super connected, then so is Y.

PROOF. Suppose that Y is super disconnected. In this case there exist IFSOS's C and D in Y such that $C \neq \underset{\sim}{\varnothing} \neq D$, $C \subseteq \overline{D}$ Since f is continuous, $f^{1}(C)$ and $f^{1}(D)$ are IFSOS's in X, and $C \subseteq \overline{D} \Rightarrow f^{1}(C) \subseteq f^{1}(\overline{D}) = \overline{f^{-1}(D)}$, $f^{-1}(C) \neq \underset{\sim}{\varnothing} \neq f^{-1}(\overline{D})$, which means that X is super disconnected ■

Now we shall summarize the interrelations between several types of connectedness in IFSTS's.

super connectedness	C_S -connectedness
↓	↓
C_5 -connectedness	C_M -connectedness
↓	
connectedness	

Here we generalize the idea of fuzzy C_i-connectedness in fuzzy topological spaces and in intuitionistic fuzzy topological spaces (see Ajmal-Kohli [11], Chaudhuri-Das [10] and Turanli-Çoker [8] to the intuitionistic case:

DEFINITION 3.6. Let N be an IFSS in (X,τ)

(a) If there exist IFSOS's M and W in X satisfying the following properties, then N is called C_i-disconnected (i=1,2,3,4) :

C_1:$N \subseteq M \cup W, M \cap W \subseteq \overline{N}, N \cap M \neq \underset{\sim}{\varnothing}, N \cap W \neq \underset{\sim}{\varnothing}$, C_2:$N \subseteq M \cup W, N \cap M \cap W = \underset{\sim}{\varnothing}, N \cap M \neq \underset{\sim}{\varnothing}, N \cap W \neq \underset{\sim}{\varnothing}$,

C_3:$N \subseteq M \cup W, M \cap W \subseteq \overline{N}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N}$, C_4:$N \subseteq M \cup W, N \cap M \cap W = \underset{\sim}{\varnothing}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N}$.

(b) N is said to be C_i -connected (i=1,2,3,4), if N is not C_i -disconnected (i=1,2,3,4)

Obviously, one can obtain the following implications between several types of C_i -connectedness (i=1,2,3,4) :

C_1 -connectedness	→	C_2 -connectedness
↓		↓
C_3-connectedness	→	C_4 -connectedness

None of these implications are reversible, as the following counterexamples state:

COUNTEREXAMPLES 3.4. Consider the IFST τ on X={a,b},where $\tau=\{\underset{\sim}{\emptyset},\underset{\sim}{X},A_1,A_2,A_3,A_4,A_5,A_6,A_7\}$, $A_1=\langle x,\{a\},\emptyset\rangle$, $A_2=\langle x,\{b\},\emptyset\rangle$, $A_3=\langle x,\emptyset,\{a\}\rangle$, $A_4=\langle x,\emptyset,\{b\}\rangle$, $A_5=\langle x,\{a\},\{b\}\rangle$, $A_6=\langle x,\{b\},\{a\}\rangle$, $A_7=\langle x,\emptyset,\emptyset\rangle$ and take the IFSS $N=\langle x,\emptyset,\{a\}\rangle$ in X.

(a) N is C_2-connected, but not C_1-connected. [Namely, A_2 and A_3 do satisfy the properties in (C_1)]

(b) N is C_3 -connected, but not C_1-connected

COUNTEREXAMPLE 3.5. Consider the IFST τ on X={a,b,c,d}, where $\tau=\{\underset{\sim}{\emptyset},\underset{\sim}{X},A_1,A_2,A_3,A_4\}$, $A_1=\langle x,\{a\},\{b,c\}\rangle$, $A_2=\langle x,\{b,c\},\{a\}\rangle$, $A_3=\langle x,\emptyset,\{a,b,c\}\rangle$, $A_4=\langle x,\{a,b,c\},\emptyset\rangle$ The IFSS $N=\langle x,\{a\},\{b\}\rangle$ in X is C_4-connected, but not C_3-connected [Namely, A_1 and A_2 do satisfy the properties in (C_3).]

COUNTEREXAMPLE 3.6. Consider the IFST τ on X={a,b,c}, where $\tau=\{\underset{\sim}{\emptyset},\underset{\sim}{X},A_1,A_2,A_3\}$,$A_1=\langle x,\emptyset,\{a\}\rangle$,$A_2=\langle x,\{a\},\{b,c\}\rangle$,$A_3=\langle x,\{a\},\emptyset\rangle$. The IFSS $N=\langle x,\{a\},\emptyset\rangle$ in X is C_4-connected, but not C_2-connected. [Namely, A_1 and A_2 do satisfy the properties in (C_2).]

REFERENCES

[1] ZADEH, L.A., Fuzzy sets, Information and Control 8 (1965) 338-353.

[2] ATANASSOV, K., Intuitionistic fuzzy sets, in: V. Sgurev, Ed., VII ITKR's Session, Sofia, June 1983 (Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984)

[3] ATANASSOV, K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

[4] ÇOKER, D., A note on intuitionistic sets and intuitionistic points, to appear in Doga TU..J. Math.

[5] ÇOKER, D., An introduction to intuitionistic fuzzy topological spaces, to appear in Fuzzy Sets and Systems.

[6] ÇOKER, D. and EŞ, A.H., On fuzzy compactness in intuitionistic fuzzy topological spaces, Journal of Fuzzy MathematIFSCS 3-4 (1995) 899-909.

[7] COŞKUN, E. and ÇOKER, D., On neighborhood structures in intuitionistic topological spaces, submitted to Mathematica Balkanica.

[8] TURANLI and ÇOKER, D., On fuzzy connectedness in intuitionistic topological spaces, submitted to Information Sciences.

[9] ÇOKER, D., An introduction to intuitionistic topological spaces, submitted to Doga TU.J.Math..

[10] CHAUDHURI, A.K. and DAS, P., Fuzzy connected sets in fuzzy topological spaces, Fuzzy Sets and Systems 49 (1992) 223-229.

[11] AJMAL, N. and KOHLI, J.K., Connectedness in fuzzy topological spaces, Fuzzy Sets and Systems 31 (1989) 369-388.