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ON REAL HYPERSURFACES IN QUATERNIONIC PROJECTIVE SPACE WITH $\mathfrak{D}^{\perp}$-RECURRENT SECOND FUNDAMENTAL TENSOR

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ABSTRACT. In this paper, we give a complete classification of real hypersurfaces in a quaternionic projective space QP^m with $\mathfrak{D}^{\perp}$-recurrent second fundamental tensor under certain condition on the orthogonal distribution $\mathfrak{D}$.

Keywords and phrases. Quaternionic projective space, $\mathfrak{D}^{\perp}$-recurrent second fundamental tensor, orthogonal distribution.

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1. Introduction. Throughout this paper M denotes a connected real hypersurface of the quaternionic projective space QP^m, $m \geq 3$, endowed with the metric g of constant quaternionic sectional curvature 4. Let N be a unit local normal vector field on M and $U_i = -J_iN$, $i = 1,2,3$, where $\{J_i\}_{i=1,2,3}$ is a local basis of the quaternionic structure of QP^m, [5]. Several examples of such real hypersurfaces are well known. See, for instance, [2, 1, 5, 8, 9, 13].

Now, let us define a distribution $\mathfrak{D}$ by $\mathfrak{D}(x) = \{X \in T_xM : X \perp U_i(x), i = 1,2,3\}$, $x \in M$, of a real hypersurface M in QP^m, which is orthogonal to the structure vector fields $\{U_1,U_2,U_3\}$ and invariant with respect to structure tensors $\{\phi_1,\phi_2,\phi_3\}$, and by $\mathfrak{D}^{\perp} = \mathrm{Span}\{U_1,U_2,U_3\}$ its orthogonal complement in TM.

There exist many studies about real hypersurfaces of quaternionic projective space QP^m. Among them, Martinez and Perez [9] have classified real hypersurfaces of QP^m with constant principal curvatures when the distribution $\mathfrak{D}$ is invariant by the second fundamental tensor, that is, the shape operator A. It was shown that these real hypersurfaces of QP^m could be divided into three types which are said to be of type A_1, A_2, and B, where a real hypersurface of type B denotes a tube over a complex projective space CP^m. Hereafter, let us say *A-invariant* when the distribution $\mathfrak{D}$ is invariant by the shape operator A.

Without the additional assumption of constant principal curvatures and as a further improvement of this result, Berndt [2] showed recently that all real hypersurfaces of QP^m could be divided into the above three types when the distributions $\mathfrak{D}$ and $\mathfrak{D}^{\perp}$ satisfy $g(A\mathfrak{D},\mathfrak{D}^{\perp}) = 0$, that is, the distribution $\mathfrak{D}$ is *A-invariant*.

On the other hand, in [7], Kobayashi and Nomizu have introduced the notion of recurrent tensor field of type (r,s) on a manifold M with a linear connection. That is, a nonzero tensor field K of type (r,s) on M is said to be recurrent if there exists a 1-form α such that $\nabla K = K \otimes \alpha$. Moreover, they gave some geometric interpretations of a manifold M with recurrent curvature tensor in terms of the holonomy group.

Now, let us consider a real hypersurface M with recurrent second fundamental tensor A in a quaternionic projective space QP^m. Then from the definition, we have

$$\nabla A = A \otimes \alpha, \tag{1.1}$$

where ∇ denotes the induced connection defined on M. Then (1.1) means

$$[\nabla_X A, A] = \alpha(X)[A, A] = 0 \tag{1.2}$$

for any tangent vector field X defined on M. We can interpret its geometrical meaning in such a way that *the eigen spaces of the shape operator A of M are parallel along any curve γ in M*. Here, the eigenspaces of the shape operator A are said to be *parallel* along γ if they are *invariant* with respect to parallel translation along γ.

Recently, Hamada [4] has applied this notion to real hypersurfaces in a complex projective space P_nC and asserted that there did not exist any real hypersurface in P_nC which had recurrent second fundamental tensor. Moreover, in [4] he defined the notion of *η-recurrent* second fundamental form.

Now, in this paper, let us introduce the notion of $\mathfrak{D}^\perp$-recurrent second fundamental form defined by

$$g((\nabla_X A)Y, Z) = \alpha(X)g(AY, Z) \tag{1.3}$$

for a certain 1-form α defined on the distribution $\mathfrak{D}$ and any vector fields X, Y, Z in $\mathfrak{D}$. Then the geometrical meaning of $\mathfrak{D}^\perp$-recurrency can be interpreted as *the eigen spaces of the shape operator A are parallel along the curve γ orthogonal to the distribution* $\mathfrak{D}^\perp = \operatorname{Span}\{U_1, U_2, U_3\}$.

In this paper, let us consider another condition on the distribution $\mathfrak{D}$ defined by

$$g((A\phi_i - \phi_i A)X, Y) = 0 \tag{1.4}$$

for any X and Y in $\mathfrak{D}$, which is weaker than the condition that the structure tensors ϕ_i and the second fundamental tensor A commute with each other. Then under this condition (1.4), we can give a complete classification of $\mathfrak{D}^\perp$-recurrency of the second fundamental tensor. That is, we have the following.

Theorem. *Let M be a real hypersurface in QP^m, $m \geq 3$, with $\mathfrak{D}^\perp$-recurrent second fundamental tensor. If it satisfies (1.4), then M is congruent to one of the following spaces:*

(A_1) *a tube of radius r over a hyperplane QP^{m-1}, where $0 < r < \pi/2$,*

(A_2) *a tube of radius r over a totally geodesic QP^k $(1 \leq k \leq m-2)$, where $0 < r < \pi/2$.*

(R) *a ruled real hypersurface foliated by totally geodesic quaternionic hyperplanes QP^{m-1}.*

When the above 1-form α in (1.3) vanishes, that is, for any X, Y and Z in $\mathfrak{D}$

$$g((\nabla_X A)Y, Z) = 0, \tag{1.5}$$

then the second fundamental form A is said to be $\mathfrak{D}^\perp$-parallel. About a ruled real hypersurface of QP^m some properties are investigated by Martinez [8] and Perez [10].

It is shown in Section 3 that the second fundamental form of a ruled real hypersurface is $\mathfrak{D}^\perp$-*parallel*. Moreover, for real hypersurfaces of type A_1, A_2, and B in QP^m, it can be easily seen that its second fundamental tensors are $\mathfrak{D}^\perp$-parallel. Thus, by virtue of the Theorem, we can, also, give the following (see [12]).

Corollary. *Let M be a real hypersurface in QP^m, $m \geq 3$, with $\mathfrak{D}^\perp$-parallel second fundamental tensor. If it satisfies (1.4), then M is congruent to one of the following spaces:*

(A_1) *a tube of radius r over a hyperplane QP^{m-1}, where $0 < r < \pi/2$,*

(A_2) *a tube of radius r over a totally geodesic $QP^k (1 \leq k \leq m-2)$, where $0 < r < \pi/2$.*

(R) *a ruled real hypersurface foliated by totally geodesic quaternionic hyperplanes QP^{m-1}.*

Under the condition $g((A\phi_i - \phi_i A)X, Y) = 0$, $X, Y \in \mathfrak{D}$, we know that $\mathfrak{D}^\perp$-recurrent implies $\mathfrak{D}^\perp$-parallel. That is, by virtue of the above Theorem and Corollary, it can be seen that there do not exist real hypersurfaces satisfying (1.4) in QP^m with their second fundamental tensors $\mathfrak{D}^\perp$-recurrent but not $\mathfrak{D}^\perp$-parallel.

2. Preliminaries. Let X be a tangent field to M. We write $J_iX = \phi_iX + f_i(X)N, i = 1,2,3$, where ϕ_iX is the tangent component of J_iX and $f_i(X) = g(X, U_i), i = 1,2,3$. As $J_i^2 = -\mathrm{id}$, $i = 1,2,3$, where id denotes the identity endomorphism on TQP^m, we get

$$\phi_i^2 X = -X + f_i(X)U_i, \qquad f_i(\phi_iX) = 0, \qquad \phi_iU_i = 0, \quad i = 1,2,3 \tag{2.1}$$

for any X tangent to M. As $J_iJ_j = -J_jJ_i = J_k$, where (i,j,k) is a cyclic permutation of $(1,2,3)$, we obtain

$$\phi_iX = \phi_j\phi_kX - f_k(X)U_j = -\phi_k\phi_jX + f_j(X)U_k \tag{2.2}$$

and

$$f_i(X) = f_j(\phi_kX) = -f_k(\phi_jX) \tag{2.3}$$

for any vector field X tangent to M, where (i,j,k) is a cyclic permutation of $(1,2,3)$. It is, also, easy to see that, for any X, Y tangent to M and $i = 1,2,3$,

$$g(\phi_iX, Y) + g(X, \phi_iY) = 0, \qquad g(\phi_iX, \phi_iY) = g(X,Y) - f_i(X)f_i(Y) \tag{2.4}$$

and

$$\phi_iU_j = -\phi_jU_i = U_k, \tag{2.5}$$

(i,j,k) being a cyclic permutation of $(1,2,3)$. From the expression of the curvature tensor of QP^m, $m \geq 2$, we have the equations of Gauss and Codazzi, respectively, given by

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)X - g(X,Z)Y \\ &\quad + \textstyle\sum_{i=1}^3 \left\{ g(\phi_iY, Z)\phi_iX - g(\phi_iX, Z)\phi_iY + 2g(X, \phi_iY)\phi_iZ \right\} \\ &\quad + g(AY,Z)AX - g(AX,Z)AY, \end{aligned} \tag{2.6}$$

and

$$(\nabla_X A)Y - (\nabla_Y A)X = \sum_{i=1}^{3} \left\{ f_i(X)\phi_i Y - f_i(Y)\phi_i X + 2g(X,\phi_i Y)U_i \right\} \tag{2.7}$$

for any X, Y, Z tangent to M, where R denotes the curvature tensor of M. See [9].

From the expressions of the covariant derivatives of J_i, $i = 1,2,3$, it is easy to see that

$$\nabla_X U_i = -p_j(X)U_k + p_k(X)U_j + \phi_i AX \tag{2.8}$$

and

$$(\nabla_X \phi_i)Y = -p_j(X)\phi_k Y + p_k(X)\phi_j Y + f_i(Y)AX - g(AX,Y)U_i \tag{2.9}$$

for any X, Y tangent to M, (i,j,k) being a cyclic permutation of $(1,2,3)$ and p_i, $i = 1,2,3$, local 1-forms on QP^m.

3. $\mathfrak{D}^\perp$-recurrent second fundamental form. Let M be a real hypersurface in a quaternionic projective space QP^m and let $\mathfrak{D}$ be a distribution defined by $\mathfrak{D}(x) = \{X \in T_xM : X \perp U_i(x),\ i = 1,2,3\}$. Then a real hypersurface M in QP^m is said to be $\mathfrak{D}^\perp$-*recurrent* if there is a 1-form α such that

$$g((\nabla_X A)Y, Z) = \alpha(X)g(AY,Z) \tag{3.1}$$

for any X,Y and $Z \in \mathfrak{D}$.

The second fundamental tensor A of real hypersurfaces of type A_1 or A_2 in QP^m must satisfy

$$(\nabla_X A)Y = -\sum_{i=1}^{3} \left\{ f_i(Y)\phi_i X + g(\phi_i X, Y)U_i \right\} \tag{3.2}$$

for any tangent vector fields X and Y of M (see [12]). From this expression, we know that its second fundamental form is $\mathfrak{D}^\perp$-*recurrent*, in particular, $\mathfrak{D}^\perp$-*parallel*. Moreover, also in [12], we have proved that the second fundamental tensor of real hypersurfaces of type B in QP^m is $\mathfrak{D}^\perp$-*parallel*. Then, naturally, we say $\mathfrak{D}^\perp$-*recurrent*.

As another example which has $\mathfrak{D}^\perp$-*recurrent* second fundamental form, we have consructed ruled real hypersurfaces of QP^m in [12]. Then from the construction, its expression of the shape operator A can be given by

$$AU_i = \Sigma_j \alpha_{ij} U_j + \epsilon_i X_i, \qquad AX_i = \Sigma_j \epsilon_j g_{ij} U_j, \qquad AX = 0 \tag{3.3}$$

for any vector X orthogonal to U_i and X_i, where $g_{ij} = g(X_i, X_j)$ and $X_i, i = 1,2,3$, denote unit vector fields in $\mathfrak{D}$, and $\epsilon_i\,(\epsilon_i \neq 0)$, α_{ij} are smooth functions on M. By investigating some fundamental properties of these ruled real hypersurfaces and the formula (3.3), we have, also, proved in [12] that their second fundamental forms are $\mathfrak{D}^\perp$-*parallel*. Then, naturally, it should be $\mathfrak{D}^\perp$-*recurrent*.

Now, in order to prove our theorem in the introduction, we need the following lemma which was proved in [6].

Lemma 3.1. *Let M be a real hypersurface of QP^m. If it satisfies the condition (1.4) for any $i = 1,2,3$ and for any vector fields X, Y in $\mathfrak{D}$, then we have*

$$g((\nabla_X A)Y, Z) = \mathfrak{S} g(AX, Y) g(Z, V_i), \quad i = 1,2,3, \tag{3.4}$$

where $\mathfrak{S}$ denotes the cyclic sum with respect to X, Y and Z in $\mathfrak{D}$ and V_i stands for the vector field defined by $\phi_i A U_i$.

Remark 3.2. For real hypersurfaces of type B in QP^m, it can be easily seen that they do not satisfy the condition (1.4). In fact, when $i = 2$, we have

$$A\phi_2 e_k - \phi_2 A e_k = -(\tan r + \cot r)\phi_2 e_k, \tag{3.5}$$

so that $g(A\phi_2 e_k - \phi_2 A e_k, \phi_2 e_k) = -(\tan r + \cot r) \neq 0$ for $0 < r < \pi/4$ or $\pi/4 < r < \pi/2$.

4. Proof of the Theorem. Now, we prove the theorem in the introduction. In this section, we give a complete classification of real hypersurfaces in QP^m, $m \geq 3$, with $\mathfrak{D}^\perp$-*recurrent* second fundamental tensor under condition (1.4) on the distribution $\mathfrak{D}$, where $\mathfrak{D}^\perp = \operatorname{Span}\{U_1, U_2, U_3\}$. From (3.4) and the $\mathfrak{D}^\perp$-recurrency of the second fundamental form, it follows that

$$g(AX, Y)g(Z, V_1) + \{g(X, V_1) - \alpha(X)\}g(AY, Z) + g(AZ, X)g(Y, V_1) = 0 \tag{4.1}$$

for any X, Y, Z in $\mathfrak{D}$, where we have put $V_1 = \phi_1 A U_1$.

Putting $Z = V_1$ in (4.1), we get

$$g(AX, Y)g(V_1, V_1) + \{g(X, V_1) - \alpha(X)\}g(AY, V_1) + g(AV_1, X)g(Y, V_1) = 0. \tag{4.2}$$

From this and, also, by putting $Y = V_1$, we get

$$2g(AX, V_1)g(V_1, V_1) + \{g(X, V_1) - \alpha(X)\}g(AV_1, V_1) = 0. \tag{4.3}$$

So taking $X = V_1$, we get

$$\{3g(V_1, V_1) - \alpha(V_1)\}g(AV_1, V_1) = 0. \tag{4.4}$$

Similarly, we can, also, find

$$\{3g(V_i, V_i) - \alpha(V_i)\}g(AV_i, V_i) = 0, \quad i = 1,2,3. \tag{4.5}$$

If the structure vector fields U_1, U_2, and U_3 are principal on M, then, $g(A\mathfrak{D}, \mathfrak{D}^\perp) = 0$. Then by a theorem of Berndt [2], M is locally congruent to one of either type A_1, A_2 or B.

Now, let us consider the case where at least one of them is not principal. For convenience sake, let us say U_1 is not principal. Then there exists an open subset of M such that

$$\mathcal{U}_1 = \{p \in M \mid AU_1 - g(AU_1, U_1)U_1 \neq 0\}, \tag{4.6}$$

on which AU_1 can be expressed in such a way that

$$AU_1 = \alpha_1 U_1 + \beta_1 X_1, \tag{4.7}$$

for some vector field X_1 in $\mathcal{D}$. Moreover, on this $\mathcal{U}_1$, we know that

$$V_1 = \phi_1 AU_1 = \beta_1 \phi_1 X_1. \tag{4.8}$$

Now, let us consider the following cases

Case (1). Let $\mathcal{V} = \{p \in \mathcal{U}_1 : 3g(V_1, V_1) \neq \alpha(V_1)\}$. Then, on this open subset $\mathcal{V}$ of $\mathcal{U}_1$, formula (4.4) gives

$$g(AV_1, V_1) = 0. \tag{4.9}$$

From this together with (4.3), it follows that $g(AX, V_1) = 0$ for any $X \in \mathcal{D}$. Thus, (4.2) implies $g(AX, Y) = 0$ for any $X, Y \in \mathcal{D}$.

Case (2). Let $\mathcal{W} = \operatorname{Int}(\mathcal{U}_1 - \mathcal{V})$. Then, on $\mathcal{W}$, we have

$$3g(V_1, V_1) = \alpha(V_1). \tag{4.10}$$

Unless otherwise stated, let us continue our discussion on $\mathcal{W}$. Now, formula (3.4) gives

$$(\nabla_X A)Y = g(AX, Y)V_1 + g(X, V_1)AY + g(Y, V_1)AX + \sum\nolimits_j k_j(X, Y)U_j, \tag{4.11}$$

where k_j denotes a certain real valued function defined on the product distribution $\mathcal{D} \times \mathcal{D}$.

On the other hand, from the $\mathcal{D}^\perp$-recurrency of the second fundamental form, we have

$$(\nabla_X A)Y = \alpha(X)AY + \sum\nolimits_j h_j(X, Y)U_j, \tag{4.12}$$

where h_j, also, denotes a real valued function defined on $\mathcal{D} \times \mathcal{D}$.

Putting $X = Y = V_1$ in (4.11) and (4.12) and using (4.10), we get

$$g(AV_1, V_1)V_1 + \sum_j k_j(V_1, V_1)U_j = g(V_1, V_1)AV_1 + \sum_j h_j(V_1, V_1)U_j. \tag{4.13}$$

Thus, by virtue of $V_1 = \beta_1 \phi_1 X_1$, (4.13) can be written as follows.

$$A\phi_1 X_1 = \gamma \phi_1 X_1 + \sum_i \delta_i U_i. \tag{4.14}$$

From this, taking the inner product with $\phi_1 Y$ for any $Y \in \mathcal{D}$ and using the condition (1.4), we get $g(AX_1, Y) = \gamma g(X_1, Y)$, so that

$$AX_1 = \gamma X_1 + \sum_i \epsilon_i U_i. \tag{4.15}$$

Putting $X = V_1$ in (4.1), we have, for any Y and Z in $\mathcal{D}$,

$$g(AV_1, Y)g(Z, V_1) + \{g(V_1, V_1) - \alpha(V_1)\}g(AY, Z) + g(AZ, V_1)g(Y, V_1) = 0. \tag{4.16}$$

From this together with the fact $3g(V_1,V_1) = \alpha(V_1)$ and (4.14), it follows that

$$g(AY,Z) = \gamma g(\phi_1 X_1, Y) g(\phi_1 X_1, Z). \tag{4.17}$$

Thus, for any $Y, Z \in \mathfrak{D}$ and orthogonal to $\phi_1 X_1$, we have

$$g(AY,Z) = 0. \tag{4.18}$$

Now, let us show that the function γ in (4.17) identically vanishes. For this, let us combine (4.11) and (4.12). Then, for any $X, Y \in \mathfrak{D}$,

$$\begin{aligned} g(AX,Y)V_1 + \{g(X,V_1) - \alpha(X)\}AY + g(Y,V_1)AX \\ + \sum_j \{f_j(X,Y) - h_j(X,Y)\}U_j = 0. \end{aligned} \tag{4.19}$$

From this, putting $X = \phi_1 X_1$ and using (4.10) and (4.14), we get

$$\begin{aligned} 2\beta_1 \gamma g(\phi_1 X_1, Y)\phi_1 X_1 - 2\beta_1 AY + \sum\nolimits_j g(Y, \beta_1 \phi_1 X_1)\delta_j U_j \\ + \sum\nolimits_j \Big\{k_j(\phi_1 X_1, Y) - h_j(\phi_1 X_1, Y)\Big\}U_j = 0, \end{aligned} \tag{4.20}$$

where we have used the fact $3\beta_1 = \alpha(\phi_1 X_1)$. From this together with (4.15) and by putting $Y = X_1$, we get

$$\beta_1 \gamma X_1 = 0. \tag{4.21}$$

This implies that $\gamma = 0$ on $\mathcal{W}$. On this open set $\mathcal{W}$, we can, also, assert that $g(AX,Y) = 0$ for any X, Y in $\mathfrak{D}$. Thus, summing up the above two Cases (1) and (2) and using the continuity of the above functions, we can assert the following.

$$g(AX,Y) = 0 \tag{4.22}$$

for any X, Y in $\mathfrak{D}$ defined on $\mathcal{U}_1$. If there exist open subsets such that $\mathcal{U}_2 = \{p \in M \mid \beta_2(p) \neq 0\}$ and $\mathcal{U}_3 = \{p \in M \mid \beta_3(p) \neq 0\}$, then on these open subsets we can, also, apply the same method. Thus, on $\mathcal{U}_1 \cup \mathcal{U}_2 \cup \mathcal{U}_3$, we can assert that $g(AX,Y) = 0$.

Now, let us suppose $\mathcal{V} = \operatorname{Int}\{M - (\mathcal{U}_1 \cup \mathcal{U}_2 \cup \mathcal{U}_3)\}$ is not empty. Then almost contact 3 structure vector fields U_1, U_2 and U_3 are principal on $\mathcal{V}$. This implies that $g(A\mathfrak{D}, \mathfrak{D}^\perp) = 0$ on $\mathcal{V}$. So, by a theorem of Berndt [2], the open subset $\mathcal{V}$ is congruent to an open part of real hypersurfaces of type A_1, A_2 or B in a quaternionic projective space QP^m.

Now, let us consider the case of $\mathcal{V}$ being congruent to real hypersurfaces of type B in a quaternionic projective space QP^m. Then the principal curvatures on the distributions $\mathfrak{D}^\perp$ and $\mathfrak{D}$ of such a tube are given by

$$\alpha_1 = 2\cot 2r, \quad \alpha_2 = \alpha_3 = -2\tan 2r, \quad \lambda = \cot r \quad \text{and} \quad \mu = -\tan r, \tag{4.23}$$

with multiplicities 1, 2, 2(m-1), and 2(m-1), respectively. Moreover, it is, also, known that

$$A\phi_i X = \frac{\lambda \alpha_i + 2}{2\lambda - \alpha_i} \phi_i X, \quad i = 1,2,3, \tag{4.24}$$

for a principal vector X in $\mathcal{D}$ with principal curvature λ.

When we consider the case where $\alpha_2 = \alpha_3 = -2\tan 2r$, we have

$$(A\phi_i - \phi_i A)X = -(\cot r + \tan r)\phi_i X, \quad i = 2,3, \tag{4.25}$$

for any X in $\mathcal{D}$ with principal curvatures $\cot r$. Then from (1.4), we have $-\cot r - \tan r = 0$. This implies that $\cot^2 r = -1$, which is impossible. Thus, real hypersurfaces of type B cannot occur. But among them, real hypersurfaces of type A_1 and A_2 satisfy $A\phi_i - \phi_i A = 0$ on $\mathcal{V}$. Moreover, for real hypersurfaces of these types all of their principal curvatures are nonzero constant on $\mathcal{V}$. By continuity of principal curvatures again, $M - \mathcal{V} = M$ and then the subset $\mathcal{V}$ is empty. That is, structure vector fields U_1, U_2 and U_3 are principal on M. This implies that $g(A\mathcal{D}, \mathcal{D}^\perp) = 0$ on M. Thus, M is locally congruent to real hypersurfaces of type A_1 and A_2.

When we suppose that the open set $\mathcal{V} = \operatorname{Int}\{M - \mathcal{U}_1 \cup \mathcal{U}_2 \cup \mathcal{U}_3\}$ is empty, then the open subset $\mathcal{U}_1 \cup \mathcal{U}_2 \cup \mathcal{U}_3$ becomes a dense subset of M. By continuity of principal curvatures, the shape operator satisfies

$$g(AX, Y) = 0 \tag{4.26}$$

on the whole set M. From this, we know that the distribution $\mathcal{D}$ is integrable on M.

In fact, for any $X, Y \in \mathcal{D}$, we have $[X, Y] = \nabla_X Y - \nabla_Y X \in \mathcal{D}$, because

$$g(\nabla_X Y, U_i) = -g(Y, \nabla_X U_i) = -g(Y, -p_j(X)U_k + p_k(X)U_j + \phi_i AX) = 0. \tag{4.27}$$

Thus, its integral manifold can be regarded as the submanifold of codimension 4 in QP^m whose normal vectors are U_1, U_2, U_3 and C. Moreover, the integral manifold of $\mathcal{D}$ is totally geodesic in QP^m. In fact, for any $X, Y \in \mathcal{D}$, if we put

$$D_X Y = \nabla'_X Y + \sum_i \sigma_i(X, Y)U_i + \rho(X, Y)N, \tag{4.28}$$

where D and ∇' denote the connection of QP^m and the induced connection from ∇ defined on an integral manifold of the distribution $\mathcal{D}$, respectively.

For this, if we take the inner product with U_i, we get

$$\bar{g}(D_X Y, U_i) = g(\nabla_X Y, U_i) = -g(Y, \phi_i AX) = 0. \tag{4.29}$$

This means that $\sum_i \sigma_i(X, Y) = 0$. Also, taking an inner product with the unit normal N, we obtain $\rho(X, Y) = 0$. Moreover, it can be easily verified that $\mathcal{D}$ is J_i-invariant, $i = 1, 2$, and 3, and its integral manifold is a quaternionic manifold and, therefore, a quaternionic hyperplane QP^{m-1} of QP^m. Thus, M is locally congruent to a ruled real hypersurface. From this, we complete the proof of our theorem. □

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References

[1] J. Berndt, Personal communications.

[2] ———, *Real hypersurfaces in quaternionic space forms*, J. Reine Angew. Math. **419** (1991), 9–26. MR 92i:53048. Zbl 718.53017.

[3] T. E. Cecil and P. J. Ryan, *Focal sets and real hypersurfaces in complex projective space*, Trans. Amer. Math. Soc. **269** (1982), no. 2, 481–499. MR 83b:53049. Zbl 492.53039.

[4] T. Hamada, *On real hypersurfaces of a complex projective space with η-recurrent second fundamental tensor*, Nihonkai Math. J. **6** (1995), no. 2, 153–163. MR 96k:53083.

[5] S. Ishihara, *Quaternion Kahlerian manifolds*, J. Differential Geometry **9** (1974), 483–500. MR 50 1184. Zbl 297.53014.

[6] U-Hang Ki, Y. J. Suh, and J. D. Pérez, *Real hypersurfaces of type A in quaternionic projective space*, Internat. J. Math. Math. Sci. **20** (1997), no. 1, 115–122. CMP 97 07. Zbl 878.53018.

[7] S. Kobayashi and K. Nomizu, *Foundations of differential geometry. I*, Interscience Publishers, a division of John Wiley & Sons, New York, 1963. MR 27#2945.

[8] A. Martinez, *Ruled real hypersurfaces in quaternionic projective space*, Şti. Univ. "Al. I. Cuza" Iaşi Secţ. I a Mat. **34** (1988), no. 1, 73–78. MR 89k:53052. Zbl 659.53042.

[9] A. Martinez and J. D. Pérez, *Real hypersurfaces in quaternionic projective space*, Ann. Mat. Pura Appl. (4) **145** (1986), 355–384. MR 89a:53062. Zbl 615.53012.

[10] J. D. Pérez, *A characterization of real hypersurfaces of quaternionic projective space*, Tsukuba J. Math. **15** (1991), no. 2, 315–323. MR 93d:53075. Zbl 766.53005.

[11] ———, *Real hypersurfaces of quaternionic projective space satisfying $\nabla_{U_i} A = 0$*, J. Geom. **49** (1994), no. 1-2, 166–177. MR 94j:53068. Zbl 799.53018.

[12] J. D. Pérez and Y. J. Suh, *On real hypersurfaces in quaternionic projective space with $\mathcal{D}^\perp$-parallel second fundamental form*, Nihonkai Math. J. **7** (1996), no. 2, 185–195. MR 97i:53068.

[13] ———, *Real hypersurfaces of quaternionic projective space satisfying $\nabla_{U_i} R = 0$*, Differ. Geom. Appl. **7** (1997), no. 3, 211–217. Zbl 980.26709.

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