SOME RESULTS ON COINCIDENCE AND FIXED POINT THEOREMS FOR GENERALIZED CONTRACTION TYPE MAPPINGS

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ABSTRACT. Some coincidence and fixed point theorems are proved for certain generalized contraction type single-valued and set-valued compatible mappings.

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1. Introduction. Jungck [1] generalized the Banach contraction principle using the commuting map concept, which is extended by Sessa [4] giving weakly commuting map concept; this again modified in [2] by compatibility condition. Several authors [3, 5, 6] discussed various results on coincidence and fixed point theorem for compatible single-valued and multi-valued maps. Here we develop some coincidence and fixed point theorems for compatible single-valued and multi-valued maps satisfying some generalized contraction type condition. Henceforth, we denote by $\mathbb{N}$ and $\mathbb{R}_+$, the set of naturals and nonnegative reals, respectively, and $\omega = \mathbb{N} \cup \{0\}$ and (X, d) , a metric space, unless otherwise stated.

2. Preliminaries

DEFINITION 2.1 (see [3]). Two mappings $f, g : X \rightarrow X$ are compatible if and only if $d(fgx_n, gfx_n) \rightarrow 0$ whenever $\{x_n\}$ is a sequence in X such that $fx_n \rightarrow t$, $gx_n \rightarrow t$, $t \in X$.

Let $C(X)$ = class of closed subsets of X , $CB(X)$ = class of closed bounded subsets of X , $\text{co}(K)$ = convex hull of $K \subset X$. The Hausdorff metric H on $CB(X)$ is defined as $H(A, B) = \max\{\sup_{x \in A} D(x, B), \sup_{x \in B} D(x, A)\}$, for all $A, B \in CB(X)$, where $D(x, A) = \inf_{y \in A} d(x, y)$.

DEFINITION 2.2 (see [3]). The maps $f : X \rightarrow X$ and $T : X \rightarrow CB(X)$ are compatible if and only if $fTx \in CB(X)$ for all $x \in X$ and $H(fTx_n, Tfx_n) \rightarrow 0$ whenever $\{x_n\}$ is a sequence in X such that $Tx_n \rightarrow M \in CB(X)$, $fx_n \rightarrow t \in M$, where H is the Hausdorff metric on X .

We now recall the following lemmas.

LEMMA 2.3 (see [7]). Let $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be a nondecreasing upper semi-continuous (u.s.c.) function. Then $h(t) < t$ if and only if $h^n(t) \rightarrow 0$ for each $t > 0$ where h^n denotes the composition of h with itself n times.

LEMMA 2.4 (see [3]). Let $T : X \rightarrow CB(X)$ and $f : X \rightarrow X$ be compatible. If $fz \in Tz$ for some $z \in X$, then $fTz = Tfz$.

3. Coincidence and fixed point theorems for single-valued maps

THEOREM 3.1. *Let X be any nonempty set and (Y, d) be a complete metric space. Let $f, g, T : X \rightarrow Y$ satisfy*

- (i) $f(X), g(X) \subseteq T(X)$;
- (ii) $T(X)$ is closed in Y ;
- (iii) for all $x, y \in X$,

$$d(fx, gy) \leq \varphi[\max\{d(Tx, Ty), d(Tx, fx), d(Tx, gy), d(Ty, fx), d(Ty, gy)\}], \quad (3.1)$$

where $h(t) = \varphi[\max\{t, t, at, bt, t\}] < t$, for each $t > 0$, $a, b \in \{0, 1, 2\}$ with $a + b = 2$ and $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is nondecreasing u.s.c function. Then f, g, T have a coincidence point in X .

Further if

- (iv) f or g is injective, then the coincidence point is unique in X .

PROOF. Choose any $x_0 \in X$. From (i), we define an iteration $y_{2n} = fx_{2n} = Tx_{2n+1}$, $y_{2n+1} = gx_{2n+1} = Tx_{2n+2}$. Let $d_n = d(Tx_n, Tx_{n+1})$. Then from (iii), we have

$$\begin{aligned} d_{2n+1} &= d(Tx_{2n+1}, Tx_{2n+2}) = d(y_{2n}, y_{2n+1}) = d(fx_{2n}, gx_{2n+1}) \\ &\leq \varphi \left[\max \left\{ d(Tx_{2n}, Tx_{2n+1}), d(Tx_{2n}, fx_{2n}), \right. \right. \\ &\quad \left. \left. d(Tx_{2n}, gx_{2n+1}), d(Tx_{2n+1}, fx_{2n}), d(Tx_{2n+1}, gx_{2n+1}) \right\} \right] \\ &\leq \varphi \left[\max \left\{ d_{2n}, d_{2n}, (d_{2n} + d_{2n+1}), 0, d_{2n} \right\} \right]. \end{aligned} \quad (3.2)$$

If $d_{2n+1} > d_{2n}$ then contradiction arises; so taking $d_{2n+1} \leq d_{2n}$, we have $d_{2n+1} \leq h(d_{2n})$. Similarly, $d_{2n+2} \leq d_{2n+1}$, $d_{2n+2} \leq h(d_{2n+1})$. Hence $d_{n+1} \leq d_n$ and $d_n \leq h(d_{n-1}) \leq \dots \leq h^n(d_0)$, for all $n \in \omega$.

This yields, by Lemma 2.3, $\lim_n d_n = 0 = \lim_n d(y_n, y_{n+1})$. Now, the sequence $\{y_n\}$ is a Cauchy sequence in $f(X)$, which can be proved using the same technique as used in [6, Theorem 2.1] so from (ii), $\exists u \in X \ni \lim_n y_n = Tu$, that is, $\lim_n Tx_n = Tu$ and $\lim_n fx_{2n} = Tu = \lim_n gx_{2n+1}$. Suppose that $fu \neq Tu \neq gu$. Then

$$\begin{aligned} d(fu, Tu) &\leq d(fu, gx_{2n+1}) + d(gx_{2n+1}, Tu) \\ &\leq \varphi \left[\max \left\{ d(Tu, Tx_{2n+1}), d(Tu, fu), d(Tu, gx_{2n+1}), d(Tx_{2n+1}, fu), \right. \right. \\ &\quad \left. \left. d(Tx_{2n+1}, gx_{2n+1}) \right\} \right] + d(gx_{2n+1}, Tu) \Rightarrow d(fu, Tu) \\ &\leq \varphi \left[\max \left\{ 0, d(Tu, fu), 0, d(Tu, fu), 0 \right\} \right], \end{aligned} \quad (3.3)$$

as $n \rightarrow \infty$; hence $d(fu, Tu) < d(fu, Tu)$ which is absurd. Hence $fu = Tu$. Similarly, $gu = Tu$. Thus, $fu = Tu = gu$ and uniqueness of u follows from (iii) and (iv). $\square$

LEMMA 3.2. *Let $f, g : X \rightarrow X$ be compatible. If $fz = gz$ for some $z \in X$, then $fgz = gfgz$.*

PROOF. The proof is similar to that of Kaneko and Sessa [3]. $\square$

THEOREM 3.3. *Let (X, d, δ) be a bimetric space such that X is complete with respect to δ . Let $f, g, T : X \rightarrow X$ satisfy conditions (i)–(iii) of [Theorem 3.1](#) with respect to d , and*

(v) *(f, T) and (g, T) are compatible pairs;*

(vi) *$\delta(x, y) \leq k(d(x, y))$ for all $x, y \in X$,*

where $k : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is continuous with $k(0) = 0$. Then f, g, T have a unique common fixed point in X .

PROOF. By [Theorem 3.1](#), $\{Tx_n\}$ is Cauchy with respect to d and hence from (vi) it is Cauchy with respect to δ . Since X is complete with respect to δ , from [Theorem 3.1\(ii\)](#), there exists $z \in X \ni fz = Tz = gz$. Thus, by [Lemma 3.2](#) and (v), $Tfz = fTz$ and $gTz = Tgz$. So $TTz = Tfz = fTz = ffgz = fgz = gfgz = ggz = gTz = Tgz$. Now, from [Theorem 3.1\(iii\)](#) it is easy to show that $fz = gfgz$. Thus, $fz = gfgz = Tfz = ffgz$ is a common fixed point of f, T and g in X . The uniqueness part follows from [Theorem 3.1\(iii\)](#). $\square$

COROLLARY 3.4. *Let (X, d) be a complete metric space $f, g, T : X \rightarrow X$ satisfying (i)–(iv) of [Theorem 3.1](#) and (v) of [Theorem 3.3](#). Then f, g , and T have a unique common fixed point in X .*

COROLLARY 3.5. *Let (X, d) be a complete metric space and let $\mathfrak{S}$ be a family of self maps of X . If there is a map T in $\mathfrak{S}$ such that for each pair f, g in $\mathfrak{S}$ satisfying (i)–(iv) of [Theorem 3.1](#) and (v) of [Theorem 3.3](#), then each member of $\mathfrak{S}$ has a unique fixed point in X which is a unique common fixed point of the family $\mathfrak{S}$.*

THEOREM 3.6. *Let (X, d) be a complete metric space. Then $f, g, T : X \rightarrow X$ satisfying [Theorem 3.1\(iii\)](#) have a unique common fixed point if and only if there is $u \in X$ such that $fu = gu = Tu$ and $f^2u = g^2u = T^2u$.*

PROOF. The necessary part is trivial. To prove the sufficient part, let there be a $u \in X \ni$ (a) $fu = gu = Tu$, (b) $f^2u = g^2u = T^2u$. Let $y = fu = gu = Tu$. Then from [Theorem 3.1\(iii\)](#) and (b), we can show that $y = fy = Ty = gy$, that is, y is a common fixed point of f, g, T in X . Further, from (iii) of [Theorem 3.1](#), the uniqueness of y follows at once. $\square$

THEOREM 3.7. *Let X be a set and Y a Banach space. Let $f, g : X \rightarrow Y$ be such that*

(i) *$\text{co}(f(X)) \subset g(X)$;*

(ii) *$g(X)$ is closed in Y ;*

(iii) *$\|fx - fy\| \leq \varphi[\max\{\|gx - gy\|, \|gx - fx\|, \|gy - fy\|\}]$ for all $x, y \in X$ where $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is nondecreasing u.s.c. function with $\varphi(qt) < t$, $1 \leq q \leq 2$. Then there is a $u \in X$ such that $fu = gu$. Further, if f or g is injective, then u is unique.*

PROOF. Choose $x \in X$. From (i) of [Theorem 3.7](#), we define $\{x_n\}$ in X as $fx_n = gx_{n+1}$, for all $n \in \omega$. Writing $d_n = \|fx_n - fx_{n+1}\|$ and using (iii) of [Theorem 3.7](#), we get

$$d_n < d_{n-1}, \quad d_n \leq \varphi(d_{n-1}) \leq \cdots \leq \varphi^n(d_0), \quad \forall n \in \omega. \quad (3.4)$$

Now, for each $p \in \mathbb{N}$,

$$\begin{aligned} \|fx_n - fx_{n+p}\| &\leq \sum_{i=1}^{p-1} \|fx_{n+i} - fx_{n+1+i}\| \\ &\leq \sum_{i=0}^{p-1} \varphi^{n+i}(d_0) = \varphi^n(d_0) \cdot \left\{ \frac{\varphi^p(d_0) - 1}{\varphi(d_0) - 1} \right\} \rightarrow 0 \end{aligned} \quad (3.5)$$

as $n \rightarrow \infty$ by Lemma 2.3 implies $\{fx_n\}$ is Cauchy in Y and by assumption, $\lim_n fx_n$ exists finitely in Y . From (i), define $gy_n = afx_n + (1-a)fx_{n+1}$, $0 \leq a \leq 1$ in $\mathcal{G}(X)$. We have

$$\begin{aligned} \|fy_n - gy_n\| &\leq a\|fx_n - fy_n\| + (1-a)\|fx_{n+1} - fy_n\| \\ &\leq a\varphi \left[\max \{ \|gx_n - gy_n\|, \|gx_n - fx_n\|, \|gy_n - fy_n\| \} \right] \\ &\quad + (1-a)\varphi \left[\max \{ \|gx_{n+1} - gy_n\|, \|gx_{n+1} - fx_{n+1}\|, \|gy_n - fy_n\| \} \right]. \end{aligned} \quad (3.6)$$

Also,

$$\begin{aligned} \|gx_n - gy_n\| &\leq \|fx_{n-1}fx_n\| + (1-a)\|fx_{n-1} - fx_{n+1}\| \\ &\leq \varphi^{n-1}(d_0) + (1-a)\varphi^n(d_0) \leq (2-a)\varphi^{n-1}(d_0) \quad (\text{using } \varphi(d_0) < d_0), \\ \|gx_{n+1} - gy_n\| &= (1-a)\|fx_n - fx_{n+1}\| \leq (1-a)\varphi^n(d_0). \end{aligned} \quad (3.7)$$

Thus, from (3.4) and (3.7), (3.6) reduces to

$$\begin{aligned} \|fy_n - gy_n\| &\leq a\varphi \left[\max \{ (2-a)\varphi^{n-1}(d_0), \varphi^{n-1}(d_0), \|fy_n - gy_n\| \} \right] \\ &\quad + (1-a)\varphi \left[\max \{ (1-a)\varphi^n(d_0), \varphi^n(d_0), \|fy_n - gy_n\| \} \right] \\ &\leq a\varphi \left[\max \{ (2-a)\varphi^{n-1}(d_0), \|fy_n - gy_n\| \} \right] \\ &\quad + (1-a)\varphi \left[\max \{ (2-a)\varphi^{n-1}(d_0), \|fy_n - gy_n\| \} \right], \end{aligned} \quad (3.8)$$

as $\varphi(d_0) < d_0, 1 \leq 2-a \leq 2$

$$\begin{aligned} &\leq \varphi \left[\max \{ (2-a)\varphi^{n-1}(d_0), \|fy_n - gy_n\| \} \right] \\ &\leq \varphi \left[(2-a)\varphi^{n-1}(d_0) \right] < \varphi^{n-1}(d_0), \end{aligned}$$

otherwise, if $\|fy_n - gy_n\|$ is maximum then a contradiction arises.

Now, for any $p \in \mathbb{N}$, writing $K_p = (\varphi^p(d_0) - 1)/(\varphi(d_0) - 1)$ we get

$$\begin{aligned} \|gy_n - gy_{n+p}\| &\leq a\|fx_n - fx_{n+p}\| + (1-a)\|fx_{n+1} - fx_{n+1+p}\| \\ &\leq \left[a\varphi^n(d_0) + (1-a)\varphi^{n-1}(d_0) \right] K_p \rightarrow 0 \quad \text{as } n \rightarrow \infty \Rightarrow \{gy_n\} \end{aligned} \quad (3.9)$$

is Cauchy in $g(X) \subset Y$, and from (ii) of [Theorem 3.7](#) there exists $u \in X \ni \lim_n g y_n = gu$. So, from (3.4), (3.7), and (3.8) we have, $\|f x_n - f y_n\| \leq \|f x_n - g x_n\| + \|g x_n - g y_n\| + \|g y_n - f y_n\| \rightarrow 0$ as $n \rightarrow \infty$. Hence, $\lim_n f x_n = \lim_n f y_n = \lim_n g y_n = \lim_n g x_n = gu$.

Now, let $fu \neq gu$. Then from (iii) of [Theorem 3.7](#), we have $\|fu - f x_n\| \leq \varphi[\max\{\|gu - g x_n\|, \|gu - fu\|, \|g x_n - f x_n\|\}]$; taking limit as $n \rightarrow \infty$, we have $\|fu - gu\| \leq \varphi[\max\{0, \|fu - gu\|, 0\}] < \|fu - gu\|$ which is a contradiction. Hence $fu = gu$. The second part follows from (iii) of [Theorem 3.7](#) and injectiveness of f or g . $\square$

4. Coincidence point for multivalued mappings

THEOREM 4.1. *Let X be a Banach space; and let $S, T : X \rightarrow CB(X)$ and $f : X \rightarrow X$ be such that*

- (i) $S(X)UT(X) \subseteq f(X) \in C(X)$,
- (ii) *for all $x, y \in X$, $H(Sx, Ty) \leq \varphi\{\|fx - fy\|, D(fx, Sx), D(fy, Ty), D(fx, Ty), D(fy, Sx)\}$ where $\varphi : \mathbb{R}_+^5 \rightarrow \mathbb{R}_+$ is u.s.c. and nondecreasing in each coordinate variable with $\gamma(t) = \max[\varphi(t, t, t, at, bt) : a + b = 2, a, b \in \{0, 1, 2\}] \leq qt, 0 \leq q < 1, t > 0$. Then f, S and T have a coincidence point in X .*

PROOF. Choose $a \in (0, 1)$ such that $q^{1-a} < 1$. Let $x_0 \in X$. Form (i), we define a sequence $\{x_n\}$ in X as $fx_{2n+1} \in Sx_{2n}, fx_{2n+2} \in Tx_{2n+1}$ such that

$$\begin{aligned} \|fx_{2n+1} - fx_{2n+2}\| &< q^{-a}H(Sx_{2n}, Tx_{2n+1}), \\ \|fx_{2n+2} - fx_{2n+3}\| &< q^{-a}H(Tx_{2n+1}, Sx_{2n+2}), \end{aligned} \quad (4.1)$$

for all $n \in \omega$, writing $d_n = \|fx_n - fx_{n+1}\|$, we have from (ii) by routine calculations that $d_{2n+1} \leq d_{2n}$ and $d_{2n+1} \leq q^{1-a}d_{2n}$. Similarly, $d_{2n+2} \leq d_{2n+1}$ and $d_{2n+2} \leq q^{1-a}d_{2n+1}$. Thus, combining these we can write

$$d_{n+1} \leq d_n, \quad d_n \leq q^{1-a}d_{n-1} \leq \dots \leq q^{(1-a)n}d_0, \quad \forall n \in \omega, 0 \leq q^{1-a} < 1. \quad (4.2)$$

This shows that $\{fx_n\}$ is Cauchy in $f(X)$ and from (i) of [Theorem 4.1](#), there exists $z \in X \ni \lim fx_n = fz$,

$$\begin{aligned} D(fz, Sz) &\leq \|fz - fx_{2n+2}\| + D(fx_{2n+2}, Sz) \leq \|fz - fx_{2n+2}\| + H(Sz, Tx_{2n+1}) \\ &\leq \varphi\{\|fz - fx_{2n+1}\|, D(fz, Sz), D(fx_{2n+1}, Tx_{2n+1}), D(fz, Tx_{2n+1}), \\ &\quad D(fx_{2n+1}, Sz)\} + \|fz - fx_{2n+2}\| \\ &\leq \varphi\{\|fz - fx_{2n+1}\|, D(fz, Sz), \|fx_{2n+1} - fx_{2n+2}\|, \|fz - fx_{2n+2}\|, \\ &\quad (\|fx_{2n+1} - fz\| + D(fz, Sz))\} + \|fz - fx_{2n+2}\|. \end{aligned} \quad (4.3)$$

As $n \rightarrow \infty$, we have $D(fz, Sz) \leq \varphi\{0, D(fz, Sz), 0, 0, D(fz, Sz)\} \leq \varphi\{t, t, t, t, t\} \leq qt$ (where $t = D(fz, Sz)$) which implies that $fz \in \bar{S}z = Sz$. Similarly $fz \in Tz$.

Hence z is a coincidence point of f, S and T in X . $\square$

In [3, Theorem 2] the continuity of the involved maps are taken; but in [Theorem 4.1](#) instead of the continuity condition of the maps we take only $f(X) \in C(X)$ for the existence of a coincidence point; to support this we give the following example.

EXAMPLE 4.2. Let $X = [0, 1]$. Define $S, T : X \rightarrow CB(X)$ and $f : X \rightarrow X$ as follows:

$$Sx = \begin{cases} \{0\}, & 0 \leq x \leq \frac{1}{2}, \\ \{\frac{1}{4}\}, & \frac{1}{2} < x \leq 1, \end{cases} \quad Tx = \begin{cases} \{0\}, & 0 \leq x \leq \frac{1}{2}, \\ \{\frac{1}{4}\}, & \frac{1}{2} < x \leq 1, \end{cases} \quad fx = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2}, \\ \frac{1}{4}, & \frac{1}{2} < x < 1, \\ \frac{2}{3}, & x = 1. \end{cases} \quad (4.4)$$

Then $SX = \{0, 1/4\} = TX$, $fX = \{0, 1/4, 2/3\} \in C(X)$; S, T , and f are discontinuous. Let $\varphi : \mathbb{R}_+^5 \rightarrow \mathbb{R}_+$ be given by $\varphi(t_1, t_2, t_3, t_4, t_5) = \sqrt{t_1}/2$, $t_i > 0$; then $\gamma(t) = \sqrt{t_1}/2$. Clearly S, T, f and φ, γ satisfy all the conditions of [Theorem 4.1](#) with $q = 1/2$ and $0 = f0 \in S0 = T0$, that is, 0 is a coincidence point of S, T , and f .

THEOREM 4.3. Let X be a Banach space and $f : X \rightarrow X$, $S, T : X \rightarrow C(X)$ satisfy (i)-(ii) of [Theorem 4.1](#) and (iii) (f, S) and (f, T) are compatible pairs. Then there is a point $z \in X$ such that $fz \in Sz \cap Tz$. Suppose that $\{z_n = f^n z\}$ is a sequence of iterate in X for z and $\{S_n\}, \{T_n\}$ are sequences of multifunctions on X where $S_n z = S f^{n-1} z$,

$$T_n z = T f^{n-1} z, \quad f^n z \in S_n z \cap T_n z, \quad \forall n \in \mathbb{N}. \quad (4.5)$$

If $z_n \rightarrow z$ and $\{S_n\}, \{T_n\}$ converge, respectively, to S and T on X pointwise, then z is a common fixed point of S and T .

PROOF. From [Theorem 4.1](#), there is $z \in X \ni fz \in Sz \cap Tz$. Again from (ii) of [Theorem 4.1](#), it is easy to show that $Sz = Tz$. Again, from (iii) of [Theorem 4.3](#) and [Lemma 2.4](#), we have $fz \in Sz = Tz \Rightarrow f^2 z \in fSz = Sfz$, $f^2 z \in fTz = Tfz$, and $Sfz = Tfz$. Continuing this process, we get $S_n z = S f^{n-1} z = T f^{n-1} z = T_n z$ where $z_n = f^n z \in S f^{n-1} z = T f^{n-1} z$. By hypothesis, $S_n z \rightarrow Sz$ and $T_n z \rightarrow Tz$. Then

$$\begin{aligned} D(z, Sz) &\leq \|z - z_n\| + D(z_n, Sz) \\ &\leq \|z - z_n\| + H(S_n z, Sz) \rightarrow 0 \quad \text{as } n \rightarrow \infty, \text{ which implies that } z \in \overline{Sz} = Sz. \end{aligned} \quad (4.6)$$

As $Sz = Tz$, hence z is a common fixed point of S and T in X . $\square$

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