A characterization of intuitionistic fuzzy $\alpha$-open set is given, and conditions for an IFS to be an intuitionistic fuzzy $\alpha$-open set are provided. Characterizations of intuitionistic fuzzy precontinuous (resp., $\alpha$-continuous) mappings are given.

1. Introduction

After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them. Using the notion of intuitionistic fuzzy sets, Çoker [5] introduced the notion of intuitionistic fuzzy topological spaces. In this paper, we define the notion of intuitionistic fuzzy semiopen (resp., preopen and $\alpha$-open) mappings and investigate relation among them. We give a characterization of intuitionistic fuzzy $\alpha$-open set, and provide conditions for an IFS to be an intuitionistic fuzzy $\alpha$-open set. We discuss characterizations of intuitionistic fuzzy precontinuous (resp., $\alpha$-continuous) mappings. We give a condition for a mapping of IFTSs to be an intuitionistic fuzzy $\alpha$-continuous mapping.

2. Preliminaries

**Definition 2.1** (Atanassov [1]). An intuitionistic fuzzy set (IFS) $A$ in $X$ is an object having the form

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X \},$$

(2.1)

where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of nonmembership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

**Definition 2.2** (Atanassov [1]). Let $A$ and $B$ be IFSs of the forms $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X \}$ and $B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle \mid x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\tilde{A} = \{\langle x, y_A(x), \mu_A(x) \rangle \mid x \in X\}$,
(d) $A \cap B = \{\langle x, \mu_A(x) \land \mu_B(x), y_A(x) \lor y_B(x) \rangle \mid x \in X\}$,
(e) $A \cup B = \{\langle x, \mu_A(x) \lor \mu_B(x), y_A(x) \land y_B(x) \rangle \mid x \in X\}$.

For the sake of simplicity, we will use the notation $A = \langle x, \mu_A, \gamma_A \rangle$ instead of $A = \{\langle x, \mu_A(x), y_A(x) \rangle \mid x \in X\}$. A constant fuzzy set taking value $\alpha \in [0,1]$ will be denoted by $\alpha$. The IFSs $0_\sim$ and $1_\sim$ are defined to be $0_\sim = \langle x, 0_\sim, 1_\sim \rangle$ and $1_\sim = \langle x, 1_\sim, 0_\sim \rangle$, respectively. Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $p(\alpha, \beta)$, is defined to be an IFS of $X$ given by

$$p(\alpha, \beta)(x) := \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise}. \end{cases}$$

Let $f$ be a mapping from a set $X$ to a set $Y$. If

$$B = \{\langle y, \mu_B(y), y_B(y) \rangle : y \in Y\}$$

is an IFS in $Y$, then the preimage of $B$ under $f$, denoted by $f^{-1}(B)$, is the IFS in $X$ defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(y_B)(x) \rangle : x \in X\}$$

and the image of $A$ under $f$, denoted by $f(A)$, is an IFS of $Y$ defined by

$$f(A) = \langle y, f(\mu_A), f(\gamma_A) \rangle,$$

where

$$f(\mu_A)(y) := \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}, \end{cases}$$

$$f(\gamma_A)(y) := \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise}, \end{cases}$$

for each $y \in Y$. Çoker [5] generalized the concept of fuzzy topological space, first initiated by Chang [4], to the case of intuitionistic fuzzy sets as follows.
Definition 2.3 (Çoker [5, Definition 3.1]). An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

(T1) $0, 1 \in \tau$,
(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(T3) $\bigcup G_i \in \tau$ for any family $\{G_i | i \in J\} \subseteq \tau$.

In this case, the pair $(X, \tau)$ is called an intuitionistic fuzzy topological space (IFTS) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS) in X. The complement $\bar{A}$ of an IFOS $A$ in IFTS $(X, \tau)$ is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.4 (Çoker [5, Definition 3.13]). Let $(X, \tau)$ be an IFTS and let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of $A$ are defined by

$$\text{int}(A) = \bigcup \{G | G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$\text{cl}(A) = \bigcap \{K | K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$  \hspace{1cm} (2.8)

Note that for any IFS $A$ in $(X, \tau)$, we have

$$\text{cl} (\bar{A}) = \overline{\text{int}(A)}, \quad \text{int}(\bar{A}) = \overline{\text{cl}(A)}.$$ \hspace{1cm} (2.9)

3. Intuitionistic fuzzy openness

Definition 3.1 [7]. An IFS $A$ in an IFTS $(X, \tau)$ is called

(i) an intuitionistic fuzzy semiopen set (IFSOS) if

$$A \subseteq \text{cl} \left( \text{int}(A) \right),$$ \hspace{1cm} (3.1)

(ii) an intuitionistic fuzzy $\alpha$-open set (IF$\alpha$OS) [3] if

$$A \subseteq \text{int} \left( \text{cl} \left( \text{int}(A) \right) \right),$$ \hspace{1cm} (3.2)

(iii) an intuitionistic fuzzy preopen set (IFPOS) if

$$A \subseteq \text{int} \left( \text{cl}(A) \right),$$ \hspace{1cm} (3.3)

(iv) an intuitionistic fuzzy regular open set (IFROS) if

$$\text{int} \left( \text{cl}(A) \right) = A.$$ \hspace{1cm} (3.4)
An IFS $A$ is called an \textit{intuitionistic fuzzy semiclosed set}, \textit{intuitionistic fuzzy $\alpha$-closed set}, \textit{intuitionistic fuzzy preclosed set}, and \textit{intuitionistic fuzzy regular closed set}, respectively (IFS-SCS, IF$\alpha$CS, IFPCS, and IFRCS, resp.), if the complement of $A$ is an IFSOS, IF$\alpha$OS, IFPOS, and IFROS, respectively.

In the following diagram, we provide relations between various types of intuitionistic fuzzy openness (intuitionistic fuzzy closedness):

\begin{equation}
\text{IFSOS (IFSCS)} \quad \text{IFROS (IFRCS)} \quad \text{IFOS (IFCS)} \quad \text{IF$\alpha$OS (IF$\alpha$CS)} \quad \text{IFPOS (IFPCS)} \quad \text{IFS (IFSCS)} \quad \text{IF$\beta$OS (IF$\beta$CS)}
\end{equation}

The reverse implications are not true in the above diagram (see [7]). The following is a characterization of an IF$\alpha$OS.

\textbf{Theorem 3.2.} An IFS $A$ in an IFTS $(X, \tau)$ is an IF$\alpha$OS if and only if it is both an IFSOS and an IFPOS.

\textbf{Proof.} Necessity follows from the diagram given above. Suppose that $A$ is both an IFSOS and an IFPOS. Then $A \subseteq \text{cl}(\text{int}(A))$, and so

$$
\text{cl}(A) \subseteq \text{cl}(\text{cl}(\text{int}(A))) = \text{cl}(\text{int}(A)).
$$

(3.6)

It follows that $A \subseteq \text{int}(\text{cl}(A)) \subseteq \text{int}(\text{cl}(\text{int}(A)))$, so that $A$ is an IF$\alpha$OS. \hfill \Box

We give condition(s) for an IFS to be an IF$\alpha$OS.

\textbf{Theorem 3.3.} Let $A$ be an IFS in an IFTS $(X, \tau)$. If $B$ is an IFSOS such that $B \subseteq A \subseteq \text{int}(\text{cl}(B))$, then $A$ is an IF$\alpha$OS.
Proof. Since $B$ is an IFSOS, we have $B \subseteq \text{cl}(\text{int}(B))$. Thus,

$$A \subset \text{int}(\text{cl}(B)) \subseteq \text{int}(\text{cl}(\text{int}(B))) = \text{int}(\text{cl}(\text{int}(B))) = \text{int}(\text{cl}(\text{int}(A))),$$

and so $A$ is an IFαOS.

**Lemma 3.4.** Any union of IFαOSs (resp., IFPOSs) is an IFαOS (resp., IFPOS).

The proof is straightforward.

**Theorem 3.5.** An IFS $A$ in an IFTS $X$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen) if and only if for every IFP $p(\alpha,\beta) \in A$, there exists an IFαOS (resp., IFPOS) $B_{p(\alpha,\beta)}$ such that $p(\alpha,\beta) \in B_{p(\alpha,\beta)} \subseteq A$.

**Proof.** If $A$ is an IFαOS (resp., IFPOS), then we may take $B_{p(\alpha,\beta)} = A$ for every $p(\alpha,\beta) \in A$. Conversely assume that for every IFP $p(\alpha,\beta) \in A$, there exists an IFαOS (resp., IFPOS) $B_{p(\alpha,\beta)}$ such that $p(\alpha,\beta) \in B_{p(\alpha,\beta)} \subseteq A$. Then,

$$A = \bigcup \{ p(\alpha,\beta) \mid p(\alpha,\beta) \in A \} \subseteq \bigcup \{ B_{p(\alpha,\beta)} \mid p(\alpha,\beta) \in A \} \subseteq A,$$

and so $A = \bigcup \{ B_{p(\alpha,\beta)} \mid p(\alpha,\beta) \in A \}$, which is an IFαOS (resp., IFPOS) by Lemma 3.4.

**Definition 3.6.** Let $f$ be a mapping from an IFTS $(X,\tau)$ to an IFTS $(Y,\kappa)$. Then, $f$ is called

(i) an intuitionistic fuzzy open mapping if $f(A)$ is an IFOS in $Y$ for every IFOS $A$ in $X$,

(ii) an intuitionistic fuzzy $\alpha$-open mapping if $f(A)$ is an IFαOS in $Y$ for every IFOS $A$ in $X$,

(iii) an intuitionistic fuzzy preopen mapping if $f(A)$ is an IFPOS in $Y$ for every IFOS $A$ in $X$,

(iv) an intuitionistic fuzzy semiopen mapping if $f(A)$ is an IFSOS in $Y$ for every IFOS $A$ in $X$.

We have the following implications in which reverse implications are not valid, where “IF” means “intuitionistic fuzzy”:

$$\text{IF open mapping} \xRightarrow{\ \ } \text{IF } \alpha\text{-open mapping} \xRightarrow{\ \ } \text{IF semiopen mapping} \xRightarrow{\ \ } \text{IF preopen mapping}$$

(3.9)
Let $A = \langle x, \mu_A, \gamma_A \rangle$, $B = \langle x, \mu_B, \gamma_B \rangle$, and $C = \langle x, \mu_C, \gamma_C \rangle$ be IFSs in $I = [0,1]$ defined by

\[
\begin{align*}
\mu_A(x) &= \begin{cases} 
0, & 0 \leq x \leq \frac{1}{2}, \\
2x - 1, & \frac{1}{2} \leq x \leq 1,
\end{cases} & \gamma_A(x) &= \begin{cases} 
1, & 0 \leq x \leq \frac{1}{2}, \\
2(1-x), & \frac{1}{2} \leq x \leq 1,
\end{cases} \\
\mu_B(x) &= \begin{cases} 
1, & 0 \leq x \leq \frac{1}{4}, \\
2 - 4x, & \frac{1}{4} \leq x \leq \frac{1}{2}, \\
0, & \frac{1}{2} \leq x \leq 1,
\end{cases} & \gamma_B(x) &= \begin{cases} 
0, & 0 \leq x \leq \frac{1}{4}, \\
4x - 1, & \frac{1}{4} \leq x \leq \frac{1}{2}, \\
1, & \frac{1}{2} \leq x \leq 1,
\end{cases}
\end{align*}
\]

(3.10)

Then $\tau_1 = \{0_\sim, 1_\sim, B, A \cup B\}$, $\tau_2 = \{0_\sim, 1_\sim, \overline{C}\}$, and $\tau_3 = \{0_\sim, 1_\sim, C\}$ are IFTSs on $I$. Define a mapping $f : I \to I$ by $f(x) = \min\{2x, 1\}$ for each $x \in I$. Then $f(0_\sim) = 0_\sim$, $f(1_\sim) = 1_\sim$, $f(A) = 0_\sim$, and $f(B) = \overline{A} = f(A \cup B)$. It is easy to verify that $\overline{A}$ is an IF\alpha OS in $(I, \tau_2)$. Since $\overline{A} \notin \tau_2$, we know that the mapping $f : (I, \tau_1) \to (I, \tau_2)$ is intuitionistic fuzzy $\alpha$-open which is not intuitionistic fuzzy open. We also note that $\overline{A}$ is an IFSOS but not an IFPOS in $(I, \tau_1)$. Hence, $f : (I, \tau_1) \to (I, \tau_2)$ is an intuitionistic fuzzy semiopen mapping which is not intuitionistic fuzzy preopen, and so, also not intuitionistic fuzzy $\alpha$-open. Further, $\overline{A}$ is an IFPOS which is not an IFSOS in $(I, \tau_1)$. Therefore, $f : (I, \tau_1) \to (I, \tau_3)$ is an intuitionistic fuzzy preopen mapping which is not intuitionistic fuzzy semiopen, and thus, also not intuitionistic fuzzy $\alpha$-open.

**Theorem 3.7.** Let $f : (X, \tau) \to (Y, \kappa)$ and $g : (Y, \kappa) \to (Z, \delta)$ be mappings of IFTSs. If $f$ is intuitionistic fuzzy open and $g$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen), then $g \circ f$ is intuitionistic fuzzy $\alpha$-open (resp., intuitionistic fuzzy preopen).

The proof is straightforward.

**Theorem 3.8.** A mapping $f : (X, \tau) \to (Y, \kappa)$ is intuitionistic fuzzy $\alpha$-open if and only if it is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen.

**Proof.** Necessity follows from the above second diagram (3.9). Assume that $f$ is intuitionistic fuzzy preopen and intuitionistic fuzzy semiopen and let $A$ be an IFOS in $X$. Then, $f(A)$ is an IFPOS as well as an IFSOS in $Y$. It follows from Theorem 3.2 that $f(A)$ is an IF\alpha OS so that $f$ is an intuitionistic fuzzy $\alpha$-open mapping. □
4. Intuitionistic fuzzy continuity

Definition 4.1 [7]. Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then $f$ is called an intuitionistic fuzzy precontinuous mapping if $f^{-1}(B)$ is an IFPOS in $X$ for every IFOS $B$ in $Y$.

Theorem 4.2. For a mapping $f$ from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$, the following are equivalent.

(i) $f$ is intuitionistic fuzzy precontinuous.
(ii) $f^{-1}(B)$ is an IFPCS in $X$ for every IFCS $B$ in $Y$.
(iii) $\text{cl}(\text{int}(f^{-1}(A))) \subseteq f^{-1}(\text{cl}(A))$ for every IFS $A$ in $Y$.

Proof. (i) $\Rightarrow$ (ii). The proof is straightforward.

(ii) $\Rightarrow$ (iii). Let $A$ be an IFS in $Y$. Then $\text{cl}(A)$ is intuitionistic fuzzy closed. It follows from (ii) that $f^{-1}(\text{cl}(A))$ is an IFPCS in $X$ so that

$$\text{cl}(\text{int}(f^{-1}(A))) \subseteq \text{cl}(\text{int}(f^{-1}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A)). \quad (4.1)$$

(iii) $\Rightarrow$ (i). Let $A$ be an IFOS in $Y$. Then $\overline{A}$ is an IFCS in $Y$, and so

$$\text{cl}(\text{int}(f^{-1}(\overline{A}))) \subseteq f^{-1}(\text{cl}(\overline{A})) = f^{-1}(\overline{A}). \quad (4.2)$$

This implies that

$$\text{int}(\text{cl}(f^{-1}(A))) = \text{cl}(\text{int}(f^{-1}(A))) = \text{cl}(\text{int}(f^{-1}(\overline{A})))$$

$$= \text{cl}(\text{int}(f^{-1}(A))) \subseteq f^{-1}(\overline{A}) = f^{-1}(\overline{A}), \quad (4.3)$$

and thus $f^{-1}(A) \subseteq \text{int}(\text{cl}(f^{-1}(A)))$. Hence $f^{-1}(A)$ is an IFPOS in $X$, and $f$ is intuitionistic fuzzy precontinuous.

Definition 4.3 [9]. Let $p_{(a,\beta)}$ be an IFP of an IFTS $(X, \tau)$. An IFS $A$ of $X$ is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(a,\beta)}$ if there exists an IFOS $B$ in $X$ such that $p_{(a,\beta)} \in B \subseteq A$.

Theorem 4.4. Let $f$ be a mapping from an IFTS $(X, \tau)$ to an IFTS $(Y, \kappa)$. Then the following assertions are equivalent.

(i) $f$ is intuitionistic fuzzy precontinuous.
(ii) For each IFP $p_{(a,\beta)} \in X$ and every IFN $A$ of $f(p_{(a,\beta)})$, there exists an IFPOS $B$ in $X$ such that $p_{(a,\beta)} \in B \subseteq f^{-1}(A)$.
(iii) For each IFP $p_{(a,\beta)} \in X$ and every IFN $A$ of $f(p_{(a,\beta)})$, there exists an IFPOS $B$ in $X$ such that $p_{(a,\beta)} \in B$ and $f(B) \subseteq A$.

Proof. (i) $\Rightarrow$ (ii). Let $p_{(a,\beta)}$ be an IFP in $X$ and let $A$ be an IFN of $f(p_{(a,\beta)})$. Then there exists an IFOS $B$ in $Y$ such that $f(p_{(a,\beta)}) \subseteq B \subseteq A$. Since $f$ is intuitionistic fuzzy precontinuous,
we know that \( f^{-1}(B) \) is an IFPOS in \( X \) and

\[
p(a, \beta) \in f^{-1}(f(p(a, \beta))) \subseteq f^{-1}(B) \subseteq f^{-1}(A). \tag{4.4}
\]

Thus (ii) is valid.

(ii) \( \Rightarrow \) (iii). Let \( p(a, \beta) \) be an IFP in \( X \) and let \( A \) be an IFN of \( f(p(a, \beta)) \). The condition (ii) implies that there exists an IFPOS \( B \) in \( X \) such that \( p(a, \beta) \in B \subseteq f^{-1}(A) \) so that \( p(a, \beta) \in B \) and \( f(B) \subseteq f(f^{-1}(A)) \subseteq A \). Hence (iii) is true.

(iii) \( \Rightarrow \) (i). Let \( B \) be an IFOS in \( Y \) and let \( p(a, \beta) \in f^{-1}(B) \). Then \( f(p(a, \beta)) \in B \), and so \( B \) is an IFN of \( f(p(a, \beta)) \) since \( B \) is an IFOS. It follows from (iii) that there exists an IFPOS \( A \) in \( X \) such that \( p(a, \beta) \in A \) and \( f(A) \subseteq B \) so that

\[
p(a, \beta) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B). \tag{4.5}
\]

Applying Theorem 3.5 induces that \( f^{-1}(B) \) is an IFPOS in \( X \). Therefore, \( f \) is intuitionistic fuzzy precontinuous. \( \square \)

**Definition 4.5** [7]. Let \( f \) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\). Then \( f \) is called an **intuitionistic fuzzy \( \alpha \)-continuous mapping** if \( f^{-1}(B) \) is an IF\( \alpha \)-OS in \( X \) for every IFOS \( B \) in \( Y \).

**Theorem 4.6.** Let \( f \) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\) that satisfies

\[
\text{cl} \left( \text{int} \left( \text{cl} \left( f^{-1}(B) \right) \right) \right) \subseteq f^{-1} \left( \text{cl}(B) \right) \tag{4.6}
\]

for every IFS \( B \) in \( Y \). Then \( f \) is intuitionistic fuzzy \( \alpha \)-continuous.

**Proof.** Let \( B \) be an IFOS in \( Y \). Then \( \overline{B} \) is an IFCS in \( Y \), which implies from hypothesis that

\[
\text{cl} \left( \text{int} \left( \text{cl} \left( f^{-1}(\overline{B}) \right) \right) \right) \subseteq f^{-1}(\text{cl}(\overline{B})) = f^{-1}(\overline{B}). \tag{4.7}
\]

It follows that

\[
\overline{\text{int} \left( \text{cl} \left( f^{-1}(B) \right) \right)} = \text{cl} \left( \overline{\text{int} \left( f^{-1}(B) \right)} \right) = \text{cl} \left( \text{int} \left( \text{cl} \left( f^{-1}(B) \right) \right) \right) = \text{cl} \left( \text{int} \left( \text{cl} \left( f^{-1}(B) \right) \right) \right) \subseteq f^{-1}(\overline{B})
\]

so that \( f^{-1}(B) \subseteq \text{int}(\text{cl}(f^{-1}(B)))\). This shows that \( f^{-1}(B) \) is an IF\( \alpha \)-OS in \( X \). Hence, \( f \) is intuitionistic fuzzy \( \alpha \)-continuous. \( \square \)
Theorem 4.7. Let \( f \) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\). Then the following assertions are equivalent.

(i) \( f \) is intuitionistic fuzzy \( \alpha \)-continuous.
(ii) For each IFP \( p_{(\alpha, \beta)} \in X \) and every IFN \( A \) of \( f(p_{(\alpha, \beta)}) \), there exists an IF\( \alpha \)OS \( B \) such that \( p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A) \).
(iii) For each IFP \( p_{(\alpha, \beta)} \in X \) and every IFN \( A \) of \( f(p_{(\alpha, \beta)}) \), there exists an IF\( \alpha \)OS \( B \) such that \( p_{(\alpha, \beta)} \in B \) and \( f(B) \subseteq A \).

Proof. (i)\( \Rightarrow \) (ii). Let \( p_{(\alpha, \beta)} \) be an IFP in \( X \) and let \( A \) be an IFN of \( f(p_{(\alpha, \beta)}) \). Then there exists an IFOS \( C \) in \( Y \) such that \( f(p_{(\alpha, \beta)}) \in C \subseteq A \). Since \( f \) is intuitionistic fuzzy \( \alpha \)-continuous, \( B := f^{-1}(C) \) is an IF\( \alpha \)OS and

\[
p_{(\alpha, \beta)} \in f^{-1}(f(p_{(\alpha, \beta)})) \subseteq f^{-1}(C) = B \subseteq f^{-1}(A).
\]

Thus (ii) is valid.

(ii)\( \Rightarrow \) (iii). Let \( p_{(\alpha, \beta)} \) be an IFP in \( X \) and let \( A \) be an IFN of \( f(p_{(\alpha, \beta)}) \). Then there exists an IF\( \alpha \)OS \( B \) such that \( p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A) \) by (ii). Thus, we have \( p_{(\alpha, \beta)} \in B \) and \( f(B) \subseteq f(f^{-1}(A)) \subseteq A \). Hence (iii) is valid.

(iii)\( \Rightarrow \) (i). Let \( B \) be an IFOS in \( Y \) and take \( p_{(\alpha, \beta)} \in f^{-1}(B) \). Then \( f(p_{(\alpha, \beta)}) \in f(f^{-1}(B)) \subseteq B \). Since \( B \) is an IFOS, it follows that \( B \) is an IFN of \( f(p_{(\alpha, \beta)}) \) so from (iii), there exists an IF\( \alpha \)OS \( A \) such that \( p_{(\alpha, \beta)} \in A \) and \( f(A) \subseteq B \). This shows that

\[
p_{(\alpha, \beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B).
\]

Using Theorem 3.5, we know that \( f^{-1}(B) \) is an IF\( \alpha \)OS in \( X \), and hence \( f \) is intuitionistic fuzzy \( \alpha \)-continuous. \( \square \)

Combining Theorems 4.6, 4.7, and [8, Theorems 3.12 and 3.13], we have the following characterization of an intuitionistic fuzzy \( \alpha \)-continuous mapping.

Theorem 4.8. Let \( f \) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\). Then the following assertions are equivalent.

(i) \( f \) is intuitionistic fuzzy \( \alpha \)-continuous.
(ii) If \( C \) is an IFCS in \( Y \), then \( f^{-1}(C) \) is an IF\( \alpha \)CS in \( X \).
(iii) \( \text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \subseteq f^{-1}(\text{cl}(B)) \) for every IFS \( B \) in \( Y \).
(iv) For each IFP \( p_{(\alpha, \beta)} \in X \) and every IFN \( A \) of \( f(p_{(\alpha, \beta)}) \), there exists an IF\( \alpha \)OS \( B \) such that \( p_{(\alpha, \beta)} \in B \subseteq f^{-1}(A) \).
(v) For each IFP \( p_{(\alpha, \beta)} \in X \) and every IFN \( A \) of \( f(p_{(\alpha, \beta)}) \), there exists an IF\( \alpha \)OS \( B \) such that \( p_{(\alpha, \beta)} \in B \) and \( f(B) \subseteq A \).

Some aspects of intuitionistic fuzzy continuity, intuitionistic fuzzy almost continuity, intuitionistic fuzzy weak continuity, intuitionistic fuzzy \( \alpha \)-continuity, intuitionistic fuzzy precontinuity, intuitionistic fuzzy semicontinuity, and intuitionistic fuzzy \( \beta \)-continuity
are studied in [7] as well as in several papers. The relation among these types of intuitionistic fuzzy continuity is given in [7] as follows, where “IF” means “intuitionistic fuzzy”:

\begin{center}
\begin{tikzpicture}
  \node (w) at (0,0) {IF weak continuity};
  \node (a) at (0,-1) {IF almost continuity};
  \node (c) at (0,-2) {IF continuity};
  \node (as) at (0,-3) {IF \(\alpha\)-continuity};
  \node (s) at (0,-4) {IF semicontinuity};
  \node (f) at (0,-5) {IF \(\beta\)-continuity};
  \node (p) at (0,-6) {IF precontinuity};
  \node (fp) at (0,-7) {IF \(\beta\)-continuity};

  \draw [->] (w) -- (a);
  \draw [->] (a) -- (c);
  \draw [->] (c) -- (as);
  \draw [->] (as) -- (s);
  \draw [->] (s) -- (f);
  \draw [->] (f) -- (fp);
  \draw [->] (fp) -- (p);
\end{tikzpicture}
\end{center}

The reverse implications are not true in the above diagram in general (see [7]).

**Theorem 4.9.** Let \(f\) be a mapping from an IFTS \((X, \tau)\) to an IFTS \((Y, \kappa)\). If \(f\) is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, then it is intuitionistic fuzzy \(\alpha\)-continuous.

**Proof.** Let \(B\) be an IFOS in \(Y\). Since \(f\) is both intuitionistic fuzzy precontinuous and intuitionistic fuzzy semicontinuous, \(f^{-1}(B)\) is both an IFPOS and an IFSOS in \(X\). It follows from Theorem 3.2 that \(f^{-1}(B)\) is an IF\(\alpha\)OS in \(X\) so that \(f\) is intuitionistic fuzzy \(\alpha\)-continuous. \(\square\)

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