Let $A$ be a unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over $\mathbb{C}$, in which all elements are bounded. It is shown that the center of $A$ is topologically isomorphic to $\mathbb{C}$.

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1. Introduction

(1) Let $A$ be an associative topological algebra over the field of complex numbers $\mathbb{C}$ with separately continuous multiplication. Then $A$ is an exponentially galbed algebra (see, e.g., [1–4, 19, 20]) if every neighbourhood $O$ of zero in $A$ defines another neighbourhood $U$ of zero such that

$$\left\{ \sum_{k=0}^{n} a_k : a_0, \ldots, a_n \in U \right\} \subset O$$

for each $n \in \mathbb{N}$. Herewith, $A$ is locally pseudoconvex, if it has a base $\{U_\lambda : \lambda \in \Lambda\}$ of neighbourhoods of zero consisting of balanced and pseudoconvex sets (i.e., of sets $U$ for which $\mu U \subset U$, whenever $|\mu| \leq 1$, and $U + U \subset \rho U$ for a $\rho \geq 2$). In particular, when every $U_\lambda$ in $\{U_\lambda : \lambda \in \Lambda\}$ is idempotent (i.e., $U_\lambda U_\lambda \subset U_\lambda$), then $A$ is called a locally $m$-pseudoconvex algebra, and when every $U_\lambda$ in $\{U_\lambda : \lambda \in \Lambda\}$ is $A$-pseudoconvex (i.e., for any $a \in A$ there is a $\mu > 0$ such that $a U_\lambda, U_\lambda a \subset \mu U_\lambda$), then $A$ is called a locally $A$-pseudoconvex algebra. It is well known (see [21, page 4] or [6, page 189]) that the locally pseudoconvex topology on $A$ is given by a family $\{p_\lambda : \lambda \in \Lambda\}$ of $k_\lambda$-homogeneous seminorms, where $k_\lambda \in (0, 1]$ for each $\lambda \in \Lambda$. The topology of a locally $m$-pseudoconvex ($A$-pseudoconvex) algebra $A$ is given by a family $\{p_\lambda : \lambda \in \Lambda\}$ of $k_\lambda$-homogeneous submultiplicative (i.e., $p_\lambda(ab) \leq p_\lambda(a)p_\lambda(b)$ for each $a, b \in A$ and $\lambda \in \Lambda$) (resp., $A$-multiplicative (i.e., for each $a \in A$ and each $\lambda \in \Lambda$ there are numbers $N(a, \lambda) > 0$ and $M(a, \lambda) > 0$ such that $p_\lambda(ab) \leq N(a, \lambda)p_\lambda(b)$ and $p_\lambda(ba) \leq M(a, \lambda)p_\lambda(b)$ for each $b \in A$)) seminorms, where $k_\lambda \in (0, 1]$ for each $\lambda \in \Lambda$. In particular, when $k_\lambda = 1$ for each $\lambda \in \Lambda$, then $A$ is a locally convex (resp., locally $m$-convex and locally $A$-convex) algebra and when the topology of $A$ has been defined by only one
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A $k$-homogeneous seminorm with $k \in (0, 1]$, then $A$ is a *locally bounded algebra*. It is easy to see that every locally pseudoconvex algebra is an exponentially galbed algebra.

Moreover, a complete locally bounded Hausdorff algebra $A$ is a *$p$-Banach algebra*; a complete metrizable algebra $A$ is a *Fréchet algebra*; a unital topological algebra $A$, in which the set of all invertible elements is open, is a *$Q$-algebra* (see, e.g., [14, page 43, Definition 6.2]) and a topological algebra $A$ is a *topologically primitive algebra* (see [5]), if

$$\{a \in A : aA \subset M\} = \{\theta_A\} \quad \{a \in A : Aa \subset M\} = \{\theta_A\} \quad (1.2)$$

for a closed maximal regular (or modular) left (resp., right) ideal $M$ of $A$ (here $\theta_A$ denotes the zero element of $A$).

An element $a$ in a topological algebra $A$ is *bounded*, if there exists an element $\lambda_a \in \mathbb{C}\setminus\{0\}$ such that the set

$$\left\{ \left( a \lambda_a \right)^n : n \in \mathbb{N} \right\} \quad (1.3)$$

is bounded in $A$ and *nilpotent*, if $a^m = \theta_A$ for some $m \in \mathbb{N}$. If all elements in $A$ are bounded (nilpotent), then $A$ is a *topological algebra with bounded elements* (resp., a *nil algebra*).

(2) It is well known that the center of a primitive ring (a ring (in particular, algebra) $R$ is *primitive* if it has a maximal left (right) regular ideal $M$ such that $\{a \in R : aR \subset M\} = \{\theta_R\}$ (resp., $\{a \in R : Ra \subset M\} = \{\theta_R\}$)) is an integral domain (a ring $R$ is an integral domain, if from $a, b \in R$ and $ab = \theta_R$ follows that $a = \theta_R$ or $b = \theta_R$) (see [12, Lemma 2.1.3]) and any commutative integral domain can be the center of a primitive ring (see [13, Chapter II.6, Example 3]). Herewith, every field is a commutative integral domain, but any commutative integral domain is not necessarily a field. In particular (see [5]), when $R$ is a unital primitive locally $A$-pseudoconvex Hausdorff algebra over $\mathbb{C}$ or a unital locally pseudoconvex Fréchet $Q$-algebra over $\mathbb{C}$, then the center $Z(R)$ of $R$ is topologically isomorphic to $\mathbb{C}$ (for Banach algebras a similar result has been given in [16, Corollary 2.4.5] (see also [8, page 127], [15, Theorem 4.2.11], and [9, Theorem 2.6.26 (ii)]); for $k$-Banach algebras in [6, Corollary 9.3.7]; for unital primitive locally $m$-convex $Q$-algebras in [17, Corollary 2], and for unital primitive locally $A$-convex algebras, in which all maximal ideals are closed, in [18, Theorem 3]). For topological algebras with all maximal regular one-sided or two-sided ideals closed see also [7, 10, 11, 14].

In the present paper we will show that a similar result will be true for any unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over $\mathbb{C}$ in which all elements are bounded.

2. Auxiliary results

For describing the center of primitive exponentially galbed algebras we need the following results.

**Proposition 2.1.** Let $A$ be a unital exponentially galbed Hausdorff algebra over $\mathbb{C}$ with bounded elements, $\lambda_0 \in \mathbb{C}$ and $a_0 \in A$. If $A$ is a sequentially complete or a nil algebra, then
there exists a neighbourhood $O(\lambda_0)$ of $\lambda_0$ such that

$$\sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k$$ (2.1)

converges in $A$ and

$$(e_A + (\lambda_0 - \lambda) a_0)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k$$ (2.2)

for each $\lambda \in O(\lambda_0)$.

Proof. Let $O$ be an arbitrary neighbourhood of zero in $A$. Then there is a closed and balanced neighbourhood $O'$ of zero in $A$ and a closed neighbourhood $O''$ of zero in $C$ such that $O''O' \subset O$. Now $O'$ defines a balanced neighbourhood $V$ of zero in $A$ such that

$$\left\{ \sum_{k=0}^{n} 2^k v_k : v_0, \ldots, v_n \in V \right\} \subset O'$$ (2.3)

for each $n \in \mathbb{N}$. Since every element in $A$ is bounded, then there is a number $\mu_0 = \mu_{a_0} \in \mathbb{C} \setminus \{0\}$ such that

$$\left\{ (a_0 \mu_0)^n : n \in \mathbb{N} \right\}$$ (2.4)

is bounded in $A$. Therefore, there exists a number $\rho_0 > 0$ such that

$$\left( \frac{a_0}{\mu_0} \right)^n \in \rho_0 V$$ (2.5)

for each $n \in \mathbb{N}$.

Let now $a_0 \in A$ and $\lambda_0 \in \mathbb{C}$ be fixed,

$$S_n(\lambda) = \sum_{k=0}^{n} (\lambda - \lambda_0)^k a_0^k$$ (2.6)

for each $n \in \mathbb{N}$ and $\lambda \in \mathbb{C}$,

$$U_C = \left\{ \lambda \in \mathbb{C} : |\lambda| < \frac{1}{3|\mu_0|} \right\}$$ (2.7)

and $U(\lambda_0) = \lambda_0 + U_C$. Then

$$S_m(\lambda) - S_n(\lambda) = \sum_{k=n+1}^{m} (\lambda - \lambda_0)^k a_0^k = \sum_{k=0}^{m-n-1} (\lambda - \lambda_0)^{n+k+1} a_0^{n+k+1}$$ (2.8)

for each $n, m \in \mathbb{N}$, whenever $m > n$ and $\lambda \in \mathbb{C}$. If we take

$$v_{n,k}(\lambda) = 2^k (\lambda - \lambda_0)^k a_0^{n+k+1} \rho_0 a_0^{n+k+1}$$ (2.9)
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for each \( n, k \in \mathbb{N} \) and \( \lambda \in \mathbb{C} \), then

\[
S_m(\lambda) - S_n(\lambda) = (\lambda - \lambda_0)^{n+1} \mu_0^{n+1} \rho_0 \sum_{k=0}^{m-n-1} v_{n,k}(\lambda) \frac{1}{2^k}
\]

(2.10)

for each \( n, m \in \mathbb{N} \), whenever \( m > n \) and \( \lambda \in \mathbb{C} \). Now,

\[
v_{n,k}(\lambda) = \frac{1}{\rho_0} (2(\lambda - \lambda_0) \mu_0)^k \left( \frac{a_0}{\mu_0} \right)^{n+k+1} \in \frac{1}{\rho_0} (2\mu_0 (\lambda - \lambda_0))^k \rho_0 V \subset V
\]

(2.11)

for each \( n, k \in \mathbb{N} \) and \( \lambda \in U(\lambda_0) \), because \(|2\mu_0(\lambda - \lambda_0)| < 2/3 < 1\). Hence,

\[
S_m(\lambda) - S_n(\lambda) \in \frac{(2\mu_0 (\lambda - \lambda_0))^{n+1}}{2^{n+1}} \rho_0 O',
\]

(2.12)

whenever \( m > n \) and \( \lambda \in U(\lambda_0) \). Since again \(|2\mu_0(\lambda - \lambda_0)| < 1\), then there exists a number \( n_0 \in \mathbb{N} \) such that

\[
(2\mu_0 (\lambda - \lambda_0))^{n+1} \in \frac{1}{\rho_0} O''
\]

(2.13)

for each \( n > n_0 \). Taking this into account,

\[
S_m(\lambda) - S_n(\lambda) \in \frac{1}{2^{n+1}} \frac{1}{\rho_0} O'' \rho_0 O' \subset O'' O' \subset O,
\]

(2.14)

whenever \( m > n > n_0 \) and \( \lambda \in U(\lambda_0) \), since \( O' \) is balanced. It means that \((S_n(\lambda))\) is a Cauchy complete, the sequence in \( A \) for each \( \lambda \in U(\lambda_0) \).

In the case when \( A \) is sequentially complete the sequence \((S_n(\lambda))\) converges in \( A \). Suppose now that \( A \) is a nil algebra. Then \( a_0^{m+1} = \theta_A \) for some \( m \in \mathbb{N} \). Hence,

\[
S_n(\lambda) = \sum_{k=0}^{m} (\lambda - \lambda_0)^k a_0^k
\]

(2.15)

for each \( \lambda \in \mathbb{C} \), whenever \( n \geq m \). Consequently, \((S_n(\lambda))\) converges in \( A \) for each \( \lambda \in O(\lambda_0) \) in both cases.

Since

\[
(e_A + (\lambda_0 - \lambda) a_0) \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k (e_A + (\lambda_0 - \lambda) a_0) = e_A,
\]

(2.16)

one gets

\[
(e_A + (\lambda_0 - \lambda) a_0)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a_0^k
\]

(2.17)

for each \( \lambda \in O(\lambda_0) \). \( \Box \)
Corollary 2.2. Let $A$ be a unital exponentially galbed algebra over $\mathbb{C}$ with bounded elements. If $A$ is a sequentially complete or a nil algebra, then for each $a_0 \in A$ there exists a number $R > 0$ such that

$$\sum_{k=0}^{\infty} \frac{a^k_0}{\mu^{k+1}}$$

converges in $A$, whenever $|\mu| > R$.

Proof. If we take $\lambda_0 = 0$ in the previous proposition, then we get that

$$\sum_{k=0}^{\infty} \lambda^k a^k_0$$

converges in $A$, whenever $|\lambda| < \delta$ for some $\delta > 0$. If now $\mu > R = \delta^{-1}$, then $|\mu^{-1}| < \delta$, which means that

$$\sum_{k=0}^{\infty} \frac{a^k_0}{\mu^{k+1}}$$

converges in $A$. Hence,

$$\sum_{k=0}^{\infty} \frac{a^k_0}{\mu^{k+1}} = \frac{1}{\mu} \sum_{k=0}^{\infty} \frac{a^k_0}{\mu^{k}}$$

converges in $A$, whenever $|\mu| > R$. □

3. Main result

Now, based on Proposition 2.1 and Corollary 2.2, we give a description of the center $Z(A)$ of such unital topologically primitive exponentially galbed Hausdorff algebras $A$ over $\mathbb{C}$ in which all elements are bounded.

Theorem 3.1. Let $A$ be a unital sequentially complete topologically primitive exponentially galbed Hausdorff algebra over $\mathbb{C}$ with bounded elements. Then $Z(A)$ is topologically isomorphic to $\mathbb{C}$.

Proof. Since $A$ is a topologically primitive algebra, there is a closed maximal left ideal (if $M$ is a closed maximal right ideal, then the proof is similar) $M$ in $A$ such that

$$\{a \in A : aA \subset M\} = \{\theta_A\}$$

(then $M \cap Z(A) = \{\theta_A\}$). Denote by $\pi_M$ the canonical homomorphism from $A$ onto the quotient space $A/M$ of $A$ with respect to $M$. For each $z \in Z(A) \setminus \{\theta_A\}$ consider the left ideal

$$K_z = \{a \in A : az \in M\}.$$ 

Since $mz = zm$ in $M$ for each $m \in M$ and $e_A z = z \notin M$, $M \subset K_z \neq A$. Hence, $K_z$ is a proper left ideal in $A$. Since the ideal $M$ is maximal, $M = K_z$ for each $z \in Z(A) \setminus \{\theta_A\}$. 

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We will show that every \( z \in Z(A) \) defines a number \( \lambda_z \in \mathbb{C} \) such that \( z = \lambda_z e_A \). If \( z = \theta_A \), then we take \( \lambda_z = 0 \). Suppose now that there exists a \( z \in Z(A) \setminus \{ \theta_A \} \) such that \( z(\lambda) = \lambda e_A - z \neq \theta_A \) for all \( \lambda \in \mathbb{C} \). Then \( z(\lambda) \in Z(A) \setminus \{ \theta_A \} \) means that \( z(\lambda) \notin M \) for each \( \lambda \in \mathbb{C} \).

\[ z(\lambda) = \theta_A + e_A z(\lambda) \in (M + Az(\lambda)) \setminus M \quad (3.3) \]

for each \( \lambda \in \mathbb{C} \). Since \( M \) is a maximal left ideal in \( A \), then \( M + Az(\lambda) = A \) for each \( \lambda \in \mathbb{C} \). Therefore, for each \( \lambda \in \mathbb{C} \) there are elements \( m(\lambda) \in M \) and \( a(\lambda) \in A \) such that \( e_A = m(\lambda) + a(\lambda) z(\lambda) \), because of which \( a(\lambda) z(\lambda) - e_A \in M \).

Let \( a'(\lambda) \in A \) be another element such that \( a'(\lambda) z(\lambda) - e_A \in M \). Then from

\[ [a(\lambda) - a'(\lambda)] z(\lambda) = a(\lambda) z(\lambda) - a'(\lambda) z(\lambda) \in M \quad (3.4) \]

it follows that \( [a(\lambda) - a'(\lambda)] \in K_z(\lambda) = M \). Therefore, \( \pi_M(a(\lambda)) = \pi_M(a'(\lambda)) \) for each \( \lambda \in \mathbb{C} \).

Let now \( \lambda_0 \in \mathbb{C} \) and

\[ d(\lambda) = e_A + (\lambda - \lambda_0) a(\lambda_0) \quad (3.5) \]

for each \( \lambda \in \mathbb{C} \). Then there is (by Proposition 2.1) a neighbourhood \( O(\lambda_0) \) of \( \lambda_0 \) such that

\[ \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a(\lambda_0)^k \quad (3.6) \]

converges in \( A \) and

\[ d(\lambda)^{-1} = \sum_{k=0}^{\infty} (\lambda - \lambda_0)^k a(\lambda_0)^k \quad (3.7) \]

for each \( \lambda \in O(\lambda_0) \).

Now,

\[ a(\lambda_0) d(\lambda)^{-1} z(\lambda) - e_A \]

\[ = a(\lambda_0) d(\lambda)^{-1} z(\lambda) - [a(\lambda_0) z(\lambda_0) + m(\lambda_0)] \]

\[ = -a(\lambda_0) d(\lambda)^{-1} [z(\lambda) + d(\lambda) z(\lambda_0)] - m(\lambda_0) \]

\[ = -a(\lambda_0) d(\lambda)^{-1} [(\lambda - \lambda e_A) + (e_A + (\lambda - \lambda_0) a(\lambda_0)) (\lambda_0 e_A - z)] - m(\lambda_0) \]

\[ = -a(\lambda_0) d(\lambda)^{-1} [(\lambda_0 - \lambda)(e_A - a(\lambda_0) z(\lambda_0))] - m(\lambda_0) \]

\[ = -a(\lambda_0) d(\lambda)^{-1} (\lambda_0 - \lambda) m(\lambda_0) - m(\lambda_0) \in M. \]

Therefore,

\[ \pi_M(a(\lambda)) = \pi_M(a(\lambda_0) d(\lambda)^{-1}) \quad (3.9) \]

for each \( \lambda \in O(\lambda_0) \).
Let now $\Psi(\lambda) = \pi_M(a(\lambda))$ for each $\lambda \in \mathbb{C}$. We will show that $\Psi$ is an $(A - M)$-valued analytic function (i.e., if $\lambda_0 \in \mathbb{C}$, then there are a number $\delta > 0$ and a sequence $(x_n)$ of elements of $A - M$ such that $\Psi(\lambda_0 + \lambda) = \sum_{k=0}^{\infty} x_k \lambda^k$, whenever $|\lambda| < \delta$, and a number $R > 0$ and a sequence $(y_n)$ of elements of $A - M$ such that $\Psi(\lambda) = \sum_{k=0}^{\infty} y_k / \lambda^k$, whenever $|\lambda| > R$) on $\mathbb{C} \cup \{\infty\}$. For it, let again $\lambda_0 \in \mathbb{C}$. Then $\Psi(\lambda) = \pi_M(a(\lambda_0) d(\lambda)^{-1})$ for each $\lambda \in O(\lambda_0)$ and there exists a number $\delta > 0$ such that $\lambda_0 + \lambda \in O(\lambda_0)$, whenever $|\lambda| < \delta$.

Now, $\Psi(\lambda_0 + h) = \pi_M(a(\lambda_0) d(\lambda_0 + h)^{-1}) = \pi_M\left(a(\lambda_0) \sum_{k=0}^{\infty} h^k a(\lambda_0)^k\right) = \sum_{k=0}^{\infty} h^k \pi_M(a(\lambda_0)^{k+1}),$ (3.10)

if $|h| < \delta$, where $\pi_M(a(\lambda_0)^{k+1}) \in A - M$ for each $k \in \mathbb{N}$.

By Corollary 2.2, there is a number $R > 0$ such that

$$\sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}}$$ (3.11)

converges in $A$, if $|\lambda| > R$. Easy calculation shows that

$$z(\lambda) \sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}} = \sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}} z(\lambda) = e_A.$$ (3.12)

Therefore,

$$z(\lambda)^{-1} = \sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}},$$ (3.13)

whenever $|\lambda| > R$. Since $z(\lambda)^{-1} z(\lambda) - e_A \in M$ for each $\lambda$ with $|\lambda| > R$, then

$$\Psi(\lambda) = \pi_M(z(\lambda)^{-1}) = \pi_M\left(\sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}}\right) = \sum_{k=0}^{\infty} \pi_M\left(\frac{z^k}{\lambda^{k+1}}\right),$$ (3.14)

if $|\lambda| > R$, where $\pi_M(z^k) \in A - M$ for each $k \in \mathbb{N}$. Consequently, $\Psi$ is an analytic $(A - M)$-valued function on $\mathbb{C} \cup \{\infty\}$. Since $A - M$ is an exponentially galbed Hausdorff space, $\Psi$ is a constant map by Turpin’s theorem (see [19, page 56]).

We show that $\Psi(\lambda) = \theta_{A - M}$ for each $\lambda \in \mathbb{C}$. So, if $O$ is any neighbourhood of zero in $A$, then there exist in $A$ a closed neighbourhood $O'$ of zero and a neighbourhood $V$ of zero such that $O' \subset O$ and

$$\left\{\sum_{k=0}^{n} \frac{y_k}{z_k} : v_1, \ldots, v_n \in V\right\} \subset O'$$ (3.15)

for each $n \in \mathbb{N}$. Moreover, there are $\mu_z \in \mathbb{C} \setminus \{0\}$ and $\rho_V > 0$ such that

$$\left(\frac{z}{\mu_z}\right)^k \in \rho_V V$$ (3.16)
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for each \( k \in \mathbb{N} \). If now \(|\lambda| > \max\{3|\mu_z|, \rho_V\}\), then

\[
v_k(\lambda) = \frac{2^k z^k}{\lambda^{k+1}} = \frac{1}{\rho_V} \frac{\rho_V}{\lambda} \left( \frac{2\mu_z}{\lambda} \right)^k \left( \frac{z}{\mu_z} \right)^k \leq \frac{1}{\rho_V} \left[ \frac{\rho_V}{\lambda} \left( \frac{2\mu_z}{\lambda} \right)^k \right] \rho_V V \subset V \tag{3.17}
\]

for each \( k \in \mathbb{N} \). Therefore,

\[
\sum_{k=0}^{n} \frac{z^k}{\lambda^{k+1}} = \sum_{k=0}^{n} \frac{v_k(\lambda)}{2^k} \in O' \tag{3.18}
\]

for each \( n \in \mathbb{N} \). Since \( O' \) is closed, then

\[
z(\lambda)^{-1} = \sum_{k=0}^{\infty} \frac{z^k}{\lambda^{k+1}} = \lim_{n \to \infty} \sum_{k=0}^{n} \frac{v_k(\lambda)}{2^k} \in O' \subset O, \tag{3.19}
\]

whenever \(|\lambda| > \max\{3|\mu_z|, \rho_V, R\}\). Hence,

\[
\lim_{|\lambda| \to \infty} z(\lambda)^{-1} = \theta_A,
\]

\[
\lim_{|\lambda| \to \infty} \Psi(\lambda) = \lim_{|\lambda| \to \infty} \pi_M(z(\lambda)^{-1}) = \pi_M \left( \lim_{|\lambda| \to \infty} z(\lambda)^{-1} \right) = \theta_{A-M}. \tag{3.20}
\]

Thus, \( \Psi(\lambda) = \theta_{A-M} \) or \( a(\lambda) \in M \) for each \( \lambda \in \mathbb{C} \). Therefore,

\[
e_A = -(a(\lambda)z(\lambda) - e_A) + a(\lambda)z(\lambda) \in M, \tag{3.21}
\]

which is a contradiction. Consequently, every \( z \in Z(A) \) defines a \( \lambda_z \in \mathbb{C} \) such that \( z = \lambda_z e_A \). Hence, \( Z(A) \) is isomorphic to \( \mathbb{C} \).

Moreover, the isomorphism \( \rho \), defined by \( \rho(z) = \lambda_z \) for each \( z \in Z(A) \), is continuous. Indeed, if \( O \) is a neighbourhood of zero in \( \mathbb{C} \), then there exists an \( \epsilon > 0 \) such that

\[
O_\epsilon = \{ \lambda \in \mathbb{C} : |\lambda| < \epsilon \} \subset O. \tag{3.22}
\]

Let \( \lambda_0 \in O_\epsilon \setminus \{0\} \). Since \( A \) is a Hausdorff space, there exists a balanced neighbourhood \( V \) of zero in \( A \) such that \( \lambda_0 e_A \notin V \). But then we also have

\[
\lambda_0 e_A \notin V' = V \cap Z(A). \tag{3.23}
\]

If \(|\lambda_z| \geq |\lambda_0|\), then \(|\lambda_0 \lambda_z^{-1}| \leq 1\) and \( \lambda_0 e_A = (\lambda_0 \lambda_z^{-1})z \in V' \) for each \( z \in V' \), which is not possible. Hence, \( \lambda_z \in O \) for each \( z \in V' \). Thus, \( \rho \) is continuous (\( \rho^{-1} \) is continuous because \( Z(A) \) is a topological linear space in the subspace topology). Consequently, \( Z(A) \) is topologically isomorphic to \( \mathbb{C} \). \( \square \)
Remark 3.2. Using Theorem 3.1, it is possible to describe all closed maximal regular one-sided and two-sided ideals in sequentially complete exponentially galbed algebras with bounded elements.

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