This corrects the major theorem on product consequence operators.

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In [1], Definition 5.2, and Theorem 5.3 and its proof are stated incorrectly. The following is the correct definition, theorem, and proof.

**Definition 5.2.** Suppose one has a nonempty finite set $\mathcal{C} = \{C_1, \ldots, C_m\}$ of general consequence operators, each defined on a nonempty language $L_i$, $1 \leq i \leq m$. Define the operator $\Pi C_m$ as follows: for any $X \subset L_1 \times \cdots \times L_m$, using the projection $pr_i$, $1 \leq i \leq m$, define $\Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X))$.

**Theorem 5.3.** The operator $\Pi C_m$ defined on the subsets of $L_1 \times \cdots \times L_m$ is a general consequence operator and if, at least, one member of $\mathcal{C}$ is axiomless, then $\Pi C_m$ is axiomless. If each member of $\mathcal{C}$ is finitary and axiomless, then $\Pi C_m$ is finitary.

**Proof.** (a) Let $X \subset L_1 \times \cdots \times L_m$. Then for each $i$, $1 \leq i \leq m$, $pr_i(X) \subset C_i(pr_i(X)) \subset L_i$. But, $X \subset pr_1(X) \times \cdots \times pr_m(X) \subset C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X) \subset L_1 \times \cdots \times L_m$. Suppose that $X \neq \emptyset$. Then $\emptyset \neq \Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset L_1 \times \cdots \times L_m$. Hence, $\emptyset \neq pr_i(\Pi C_m(X)) = C_i(pr_i(X))$, $1 \leq i \leq m$, implies that $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Let $X = \emptyset$ and assume that no member of $\mathcal{C}$ is axiomless. Then each $pr_i(X) = \emptyset$. But, each $C_i(pr_i(X)) \neq \emptyset$ implies that $\Pi C_m(X) \neq \emptyset$. By the previous method, it follows, in this case, that $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Now suppose that there is some $j$ such that $C_j$ is axiomless. Hence, $C_j(pr_j(X)) = \emptyset$ implies that $\Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \emptyset$, which implies that $C_j(pr_j(\Pi C_m(X))) = \emptyset$. Consequently, $C_1(pr_1(\Pi C_m(X))) \times \cdots \times C_m(pr_m(\Pi C_m(X))) = \emptyset$. Thus, $\Pi C_m(\Pi C_m(X)) = \emptyset$ and axiom (1) holds. Also in the case where at least one member of $\mathcal{C}$ is axiomless, then $\Pi C_m$ is axiomless.

(b) Let $X \subset Y \subset L_1 \times \cdots \times L_m$. For each $i$, $1 \leq i \leq m$, $pr_i(X) \subset pr_i(Y)$, whether $pr_i(X)$ is the empty set or not. Hence, $C_i(pr_i(X)) \subset C_i(pr_i(Y))$. Therefore, $\Pi C_m(X) = \Pi C_m(Y)$.
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\[ C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset C_1(pr_1(Y)) \times \cdots \times C_m(pr_m(Y)) = \Pi C_m(Y) \] and axiom (2) holds. Thus, \( \Pi C_m \) is, at least, a general consequence operator.

(c) Assume that each member of \( \mathcal{C} \) is finitary and axiomless and let \( x \in \Pi C_m(X) \) where, since \( \Pi C_m \) is axiomless, \( X \) is nonempty. Then for each \( i \), \( pr_i(x) \in C_i(pr_i(X)) \). Since each \( C_i \) is finitary and axiomless, then there is some nonempty finite \( F_i \subset pr_i(X) \) such that \( pr_i(x) \in C_i(F_i) \subset C_i(pr_i(X)) \). Hence, nonempty and finite \( F = F_1 \times \cdots \times F_m \subset pr_1(X) \times \cdots \times pr_m(X) \). Then for each \( i \), \( pr_i(F) = F_i \) implies that finite \( F = F_1 \times \cdots \times F_m = pr_1(F) \times \cdots \times pr_m(F) \subset pr_1(X) \times \cdots \times pr_m(X) \). From axiom (2), \( x \in \Pi C_m(F) = C_1(pr_1(F)) \times \cdots \times C_m(pr_m(F)) \subset \Pi C_m(pr_1(X) \times \cdots \times pr_m(X)) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X) \). This completes the proof. \( \square \)

References


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