ERRATUM TO “HYPERFINITE AND STANDARD UNIFICATIONS FOR PHYSICAL THEORIES”

ROBERT A. HERRMANN

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This corrects the major theorem on product consequence operators.

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In [1], Definition 5.2, and Theorem 5.3 and its proof are stated incorrectly. The following is the correct definition, theorem, and proof.

Definition 5.2. Suppose one has a nonempty finite set $\mathcal{C} = \{C_1, \dots, C_m\}$ of general consequence operators, each defined on a nonempty language L_i , $1 \leq i \leq m$. Define the operator ΠC_m as follows: for any $X \subset L_1 \times \dots \times L_m$, using the projection pr_i , $1 \leq i \leq m$, define $\Pi C_m(X) = C_1(pr_1(X)) \times \dots \times C_m(pr_m(X))$.

THEOREM 5.3. *The operator ΠC_m defined on the subsets of $L_1 \times \dots \times L_m$ is a general consequence operator and if, at least, one member of $\mathcal{C}$ is axiomless, then ΠC_m is axiomless. If each member of $\mathcal{C}$ is finitary and axiomless, then ΠC_m is finitary.*

Proof. (a) Let $X \subset L_1 \times \dots \times L_m$. Then for each i , $1 \leq i \leq m$, $pr_i(X) \subset C_i(pr_i(X)) \subset L_i$. But, $X \subset pr_1(X) \times \dots \times pr_m(X) \subset C_1(pr_1(X)) \times \dots \times C_m(pr_m(X)) = \Pi C_m(X) \subset L_1 \times \dots \times L_m$. Suppose that $X \neq \emptyset$. Then $\emptyset \neq \Pi C_m(X) = C_1(pr_1(X)) \times \dots \times C_m(pr_m(X)) \subset L_1 \times \dots \times L_m$. Hence, $\emptyset \neq pr_i(\Pi C_m(X)) = C_i(pr_i(X))$, $1 \leq i \leq m$, implies that $C_i(pr_i(\Pi C_m(X))) = C_i(C_i(pr_i(X))) = C_i(pr_i(X))$, $1 \leq i \leq m$. Hence, $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Let $X = \emptyset$ and assume that no member of $\mathcal{C}$ is axiomless. Then each $pr_i(X) = \emptyset$. But, each $C_i(pr_i(X)) \neq \emptyset$ implies that $\Pi C_m(X) \neq \emptyset$. By the previous method, it follows, in this case, that $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Now suppose that there is some j such that C_j is axiomless. Hence, $C_j(pr_j(X)) = \emptyset$ implies that $\Pi C_m(X) = C_1(pr_1(X)) \times \dots \times C_m(pr_m(X)) = \emptyset$, which implies that $C_j(pr_j(\Pi C_m(X))) = \emptyset$. Consequently, $C_1(pr_1(\Pi C_m(X))) \times \dots \times C_m(pr_m(\Pi C_m(X))) = \emptyset$. Thus, $\Pi C_m(\Pi C_m(X)) = \emptyset$ and axiom (1) holds. Also in the case where at least one member of $\mathcal{C}$ is axiomless, then ΠC_m is axiomless.

(b) Let $X \subset Y \subset L_1 \times \dots \times L_m$. For each i , $1 \leq i \leq m$, $pr_i(X) \subset pr_i(Y)$, whether $pr_i(X)$ is the empty set or not. Hence, $C_i(pr_i(X)) \subset C_i(pr_i(Y))$. Therefore, $\Pi C_m(X) =$

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$C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset C_1(pr_1(Y)) \times \cdots \times C_m(pr_m(Y)) = \Pi C_m(Y)$ and axiom (2) holds. Thus, ΠC_m is, at least, a general consequence operator.

(c) Assume that each member of $\mathcal{C}$ is finitary and axiomless and let $x \in \Pi C_m(X)$ where, since ΠC_m is axiomless, X is nonempty. Then for each i , $pr_i(x) \in C_i(pr_i(X))$. Since each C_i is finitary and axiomless, then there is some nonempty finite $F_i \subset pr_i(X)$ such that $pr_i(x) \in C_i(F_i) \subset C_i(pr_i(X))$. Hence, nonempty and finite $F = F_1 \times \cdots \times F_m \subset pr_1(X) \times \cdots \times pr_m(X)$. Then for each i , $pr_i(F) = F_i$ implies that finite $F = F_1 \times \cdots \times F_m = pr_1(F) \times \cdots \times pr_m(F) \subset pr_1(X) \times \cdots \times pr_m(X)$. From axiom (2), $x \in \Pi C_m(F) = C_1(pr_1(F)) \times \cdots \times C_m(pr_m(F)) \subset \Pi C_m(pr_1(X) \times \cdots \times pr_m(X)) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X)$. This completes the proof. $\square$

References

- [1] R. A. Herrmann, *Hyperfinite and standard unifications for physical theories*, International Journal of Mathematics and Mathematical Sciences **28** (2001), no. 2, 93–102.

Robert A. Herrmann: Institute for Mathematics and Philosophy, 44890 Rivermont Terrace 100,
Ashburn VA 20147, USA
E-mail address: rah@usna.edu