We give a counterexample to the article “On the fixed points of affine nonexpansive mappings” (2001).

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Let $K$ be a nonempty, closed convex subset of a real Banach space $E$. A mapping $T : K \to K$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in K$. $T$ is said to be affine if for each $x, y \in K$ and $0 < \lambda < 1$, $T(\lambda x + (1 - \lambda)y) = \lambda Tx + (1 - \lambda)Ty$.

In the main theorem of the above-referenced paper [2, Theorem 2.4], the author proves that when $K$ is a nonempty, closed convex and bounded subset of $E$ and $T : K \to K$ is a nonexpansive and affine mapping, then it has a fixed point in $K$.

Here, we give an example to show that the mentioned theorem above is not correct.

1. Counterexample

We consider $c_0$, the real Banach space of all sequences $(x_1, x_2, \ldots, x_n, \ldots)$ such that $\lim_{n \to \infty} x_n = 0$, equipped by the maximum norm (i.e., $\|(x_1, x_2, \ldots, x_n, \ldots)\| := \max_n |x_n|$).

Define $T : B_1 \to B_1$ by $T(x_1, x_2, \ldots) := (1, x_1, x_2, \ldots)$ for each $x = (x_1, x_2, \ldots) \in B_1$, where $B_1$ is the closed unit ball in $c_0$. It is easy to show that $\|Tx - Ty\| = \|x - y\|$, for every $x, y$ in $B_1$ and also that $T$ is affine. Therefore, the conditions of the main theorem of [2] hold. However, $T$ does not have a fixed point.

It is worth mentioning that if we impose weak compactness on $K$, then the theorem will be true. For details and some other related results, it is convenient to see [1, 3] and most importantly [4].

References


2 A counterexample


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