New characterizations of almost contra-precontinuity are presented. These characterizations are used to develop a new weak form of almost contra-precontinuity. This new weak form is then used to extend several results in the literature concerning almost contra-precontinuity.

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1. Introduction

Almost contra-precontinuous functions were introduced by Ekici [7] and recently have been investigated further by Noiri and Popa [13]. The purpose of this note is to develop some new characterizations of almost contra-precontinuous functions and to introduce a new weak form of almost contra-precontinuity, which we call subalmost contra-precontinuity. It is shown that subalmost contra-precontinuity implies subalmost weak continuity and is independent of subweak continuity. Subalmost contra-precontinuity is used to extend several results in the literature concerning almost contra-precontinuity. For example, we show that the graph of a subalmost contra-precontinuous function with a Hausdorff codomain is $P$-regular and that the domain of a subalmost contra-precontinuous injection with a weakly Hausdorff codomain is pre-$T_1$. These results extend the analogous results for an almost contra-precontinuous function.

2. Preliminaries

The symbols $X$ and $Y$ denote topological spaces with no separation axioms assumed unless explicitly stated. All sets are considered to be subsets of topological spaces. The closure and interior of a set $A$ are signified by $\text{Cl}(A)$ and $\text{Int}(A)$, respectively. A set $A$ is regular open if $A = \text{Int}(\text{Cl}(A))$. A set $A$ is preopen [12] (resp., semiopen [11], $\beta$-open [1]) provided that $A \subseteq \text{Int}(\text{Cl}(A))$ (resp., $A \subseteq \text{Cl}(\text{Int}(A))$, $A \subseteq \text{Cl}(\text{Int}(\text{Cl}(A)))$). A set is $\theta$-open provided that it contains a closed neighborhood of each of its points. A set $A$ is preclosed (resp., semiclosed, $\beta$-closed, regular closed, $\theta$-closed) if its complement is preopen.
for every open subset \(V\).

Theorem 3.4. Noiri and Popa proved the following characterizations of almost contra-precontinuity.

Definition 2.3. A function \(f : X \rightarrow Y\) is said to be almost contra-precontinuous [7] if

\[ f^{-1}(V) \in PC(X) \text{ for every } V \in RO(Y). \]

Definition 2.2. A function \(f : X \rightarrow Y\) is said to be subweakly continuous [14] (resp., sub-almost weakly continuous [3], subweakly \(\beta\)-continuous [4]) provided that there is an open base \(\mathcal{B}\) for the topology on \(Y\) such that for every \(V \in \mathcal{B}\), \(\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))\) (resp., \(p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))\), \(\beta\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))\)).

Definition 2.1. A function \(f : X \rightarrow Y\) is said to be almost contra-precontinuous [7] if

\[ f^{-1}(V) \in PC(X) \text{ for every } V \in RO(Y). \]

Theorem 3.1 (Noiri and Popa [13]). For a function \(f : X \rightarrow Y\), the following properties are equivalent:

(a) \(f\) is almost contra-precontinuous;
(b) \(f(p\text{Cl}(A)) \subseteq s\text{Cl}_\theta(f(A))\) for every subset \(A\) of \(X\);
(c) \(p\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(s\text{Cl}_\theta(B))\) for every subset \(B\) of \(Y\).

We extend these characterizations by showing that Theorem 3.1(c) can be stated for open sets only. The following lemmas will be useful.

Lemma 3.2. If \(V\) is an open set, then \(s\text{Cl}_\theta(V) = s\text{Cl}(V)\).

Proof. Obviously \(s\text{Cl}(V) \subseteq s\text{Cl}_\theta(V)\). Suppose that \(x \notin s\text{Cl}(V)\). Then there exists \(U \in \text{SO}(X)\) such that \(U \cap V = \emptyset\). Then, since \(V\) is open, \(\text{Cl}(U) \cap V = \emptyset\). Therefore \(x \notin s\text{Cl}_\theta(V)\). Hence \(s\text{Cl}_\theta(V) \subseteq s\text{Cl}(V)\).

Lemma 3.3 (Di Maio and Noiri [6]). If \(V\) is an open set, then \(s\text{Cl}(V) = \text{Int}(\text{Cl}(V))\).

Theorem 3.4. For a function \(f : X \rightarrow Y\), the following conditions are equivalent:

(a) \(f \) is almost contra-precontinuous;
(b) \(p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_\theta(V))\) for every open subset \(V\) of \(Y\);
(c) \(p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_\theta(V))\) for every open subset \(V\) of \(Y\);
(d) \(p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))\) for every open subset \(V\) of \(Y\);
(e) \(\text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))\) for every open subset \(V\) of \(Y\).
Proof. (a)⇒(b) follows from Theorem 3.1(c).
(b)⇒(c) follows from Lemma 3.2.
(c)⇒(d) follows form Lemma 3.3.
(d)⇒(e). Since \( p \text{Cl}(f^{-1}(V)) = f^{-1}(V) \cup \text{Cl}({\text{Int}(f^{-1}(V))}) \), it follows from (d) that \( \text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) \).
(e)⇒(a). Let \( V \in \text{RO}(Y) \). Then by (e), \( \text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) = f^{-1}(V) \).

Therefore \( f^{-1}(V) \) is preclosed, which proves that \( f \) is almost contra-precontinuous. \( \square \)

The next result is an immediate consequence of Theorems 3.1 and 3.4.

**Theorem 3.5.** Let \( f : X \to Y \) be a function and let \( \mathcal{F} \) be any collection of subsets of \( Y \) containing the open sets. Then \( f \) is almost contra-precontinuous if and only if \( p \text{Cl}(f^{-1}(S)) \subseteq f^{-1}(s\text{Cl}_0(S)) \) for every \( S \in \mathcal{F} \).

**Corollary 3.6.** For a function \( f : X \to Y \), the following properties are equivalent:
(a) \( f \) is almost contra-precontinuous;
(b) \( p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_0(V)) \) for every \( V \in \text{SO}(Y) \);
(c) \( p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_0(V)) \) for every \( V \in \text{PO}(Y) \);
(d) \( p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_0(V)) \) for every \( V \in \beta\text{O}(Y) \).

### 4. Subalmost contra-precontinuous functions

We define a function \( f : X \to Y \) to be subalmost contra-precontinuous provided that there exists an open base \( \mathcal{B} \) for the topology on \( Y \) such that \( p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_0(V)) \) for every \( V \in \mathcal{B} \). Obviously almost contra-precontinuity implies subalmost contra-precontinuity. The following example shows that the converse does not hold.

Recall that a space \( X \) is extremally disconnected (ED) if the closure of every open set is open in \( X \).

**Example 4.1.** Let \( X \) be a non-ED, \( T_1 \)-space and let \( Y = X \) have the discrete topology. The identity mapping \( f : X \to Y \) is subalmost contra-precontinuous with respect to the base for \( Y \) consisting of the singleton sets. However, \( f \) is not almost contra-precontinuous. Note that for \( y \in Y \), \( p \text{Cl}_X(f^{-1}([y])) = \{y\} \). Also, since \( X \) is non-ED, there exists an open set \( V \) of \( X \) such that \( \text{Cl}_X(V) \) is not open. Then \( f^{-1}(s\text{Cl}_Y(V)) = V \), but \( p \text{Cl}_X(f^{-1}(V)) = \text{Cl}_X(V) \).

Since \( s\text{Cl}(A) \subseteq \text{Cl}(A) \) for every set \( A \), it follows that subalmost contra-precontinuity implies subalmost weak continuity, and hence it also implies subweak \( \beta \)-continuity. The following example shows that subalmost contra-precontinuity and subalmost weak continuity are not equivalent.

**Example 4.2.** Let \( X = \{a, b, c\} \) have the topology \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\} \). The identity mapping \( f : X \to X \) is obviously subalmost weakly continuous (in fact, continuous). However, \( f \) is not subalmost contra-precontinuous because any base for \( \tau \) must contain \( \{a\} \) and \( p \text{Cl}(f^{-1}(\{a\})) \not\subseteq f^{-1}(s\text{Cl}(\{a\})) \).

Since the function in Example 4.2 is obviously subweakly continuous, we see that subweak continuity does not imply subalmost contra-precontinuity. The following example...
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completes the proof that subalmost contra-precontinuity is independent of subweak continuity.

Example 4.3. Let $X$ be an indiscrete space with at least two points and let $Y = X$ have the discrete topology. Since $p \text{Cl}(\{x\}) = \{x\}$ for every $x \in X$, the identity mapping $f : X \rightarrow Y$ is subalmost contra-precontinuous with respect to the base for $Y$ consisting of the singleton sets. However, since every singleton set of $X$ is dense, $f$ is not subweakly continuous.

The following characterizations of subalmost contra-precontinuity are analogous to those in Theorem 3.4 for almost contra-precontinuity.

Theorem 4.4. For a function $f : X \rightarrow Y$, the following conditions are equivalent:

(a) $f$ is subalmost contra-precontinuous;
(b) there exists an open base $\mathcal{B}$ for $Y$ such that $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}_0(V))$ for every $V \in \mathcal{B}$;
(c) there exists an open base $\mathcal{B}$ for $Y$ such that $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))$ for every $V \in \mathcal{B}$;
(d) there exists an open base $\mathcal{B}$ for $Y$ such that $\text{Cl}(\text{Int}(f^{-1}(V))) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))$ for every $V \in \mathcal{B}$.

Theorem 4.5. If $f : X \rightarrow Y$ is subalmost weakly continuous and satisfies the additional property that images of preclosed sets are open, then $f$ is subalmost contra-precontinuous.

Proof. Since $f$ is subalmost weakly continuous, there exists an open base $\mathcal{B}$ for $Y$ such that $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$ for every $V \in \mathcal{B}$. Since images of preclosed sets are open, we have $f(p \text{Cl}(f^{-1}(V))) \subseteq \text{Int}(\text{Cl}(V))$ or $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Int}(\text{Cl}(V)))$. Therefore by Theorem 4.4, $f$ is subalmost contra-precontinuous.

Since subweak continuity implies subalmost weak continuity, we have the following result.

Corollary 4.6. If $f : X \rightarrow Y$ is subweakly continuous and satisfies the additional property that images of preclosed sets are open, then $f$ is subalmost contra-precontinuous.

Theorem 4.7. If $f : X \rightarrow Y$ is subalmost contra-precontinuous and semicontinuous, then $f$ is subweakly continuous.

Proof. Since $f$ is subalmost contra-precontinuous, there exists an open base $\mathcal{B}$ for the topology on $Y$ such that $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V))$ for every $V \in \mathcal{B}$. Because $f$ is semicontinuous, $f^{-1}(V)$ is semiopen for every $V \in \mathcal{B}$, and hence $p \text{Cl}(f^{-1}(V)) = \text{Cl}(f^{-1}(V))$ for every $V \in \mathcal{B}$. Finally, since $s\text{Cl}(A) \subseteq \text{Cl}(A)$ for every set $A$, we have $\text{Cl}(f^{-1}(V)) = p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V)) \subseteq f^{-1}(\text{Cl}(V))$. Therefore, $f$ is subweakly continuous.

5. Graph-related properties of subalmost contra-precontinuous functions

By the graph of a function $f : X \rightarrow Y$, we mean the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $X \times Y$.

Definition 5.1. The graph of a function $f : X \rightarrow Y$, $G(f)$ is said to be $P$-regular [7] provided that for every $(x, y) \in X \times Y - G(f)$, there exist a preclosed subset $U$ of $X$ and regular open subset $V$ of $Y$ such that $(x, y) \in U \times V \subseteq X \times Y - G(f)$.
Theorem 5.2. If \( f : X \to Y \) is subalmost contra-precontinuous and \( Y \) is Hausdorff, then the graph of \( f \), \( G(f) \) is \( P \)-regular.

Proof. Let \((x, y) \in X \times Y - G(f)\). Then \( y \neq f(x) \). Let \( \mathcal{B} \) be an open base for \( Y \) such that \( p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}((\text{Int} \text{Cl}(V))) \) for every \( V \in \mathcal{B} \). Since \( Y \) is Hausdorff, there exist disjoint open sets \( V \) and \( W \) such that \( f(x) \in V \), \( y \in W \), and \( V \in \mathcal{B} \). Then, since \( \text{Int} \text{Cl}(V) \cap \text{Int} \text{Cl}(W) = \emptyset \), it follows that \((x, y) \in p\text{Cl}(f^{-1}(V)) \times \text{Int} \text{Cl}(W) \subseteq X \times Y - G(f)\), which proves that \( G(f) \) is \( P \)-regular. \(\square\)

Corollary 5.3 (Ekici [7, Theorem 17]). If \( f : X \to Y \) is almost contra-precontinuous and \( Y \) is Hausdorff, then \( G(f) \) is \( P \)-regular.

Recall that the graph function of a function \( f : X \to Y \) is the function \( g : X \to X \times Y \) given by \( g(x) = (x, f(x)) \) for every \( x \in X \).

Theorem 5.4. Let \( f : (X, \tau) \to (Y, \sigma) \) be a function and let \( \mathcal{B} \) be an open base for \( \sigma \). Let \( \mathcal{C} = \{ U \times V : U \in \tau, V \in \mathcal{B} \} \). If the graph function of \( f \), \( g : X \to X \times Y \) is subalmost contra-precontinuous with respect to \( \mathcal{C} \), then \( f \) is subalmost contra-precontinuous with respect to \( \mathcal{B} \).

Proof. If \( V \in \mathcal{B} \), then \( p\text{Cl}(f^{-1}(V)) = p\text{Cl}(g^{-1}(X \times V)) \subseteq g^{-1}(s\text{Cl}(X \times V)) = g^{-1}(X \times s\text{Cl}(V)) = f^{-1}(s\text{Cl}(V)) \). Hence \( f \) is subalmost contra-precontinuous with respect to \( \mathcal{B} \). \(\square\)

If we let \( \mathcal{B} = \sigma \) in Theorem 5.4, we obtain the following result.

Corollary 5.5. If the graph function of \( f : X \to Y \) is subalmost contra-precontinuous with respect to the usual base for the product topology for the product space \( X \times Y \), then \( f \) is almost contra-precontinuous.

Corollary 5.6 (Ekici [7, Theorem 4]). If the graph function of \( f : X \to Y \) is almost contra-precontinuous, then \( f \) is almost contra-precontinuous.

Recall that a space \( X \) is said to be zero-dimensional provided that \( X \) has a clopen base.

Theorem 5.7. If the function \( f : X \to Y \) is subalmost contra-precontinuous and \( X \) is zero-dimensional, then the graph function of \( f \), \( g : X \to X \times Y \) is subalmost contra-precontinuous.

Proof. Let \( \mathcal{B} \) be an open base for \( Y \) such that \( p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}((\text{Int} \text{Cl}(V))) \) for every \( V \in \mathcal{B} \). Then \( \mathcal{C} = \{ U \times V : U \subseteq X \text{ is clopen and } V \in \mathcal{B} \} \) is a base for \( X \times Y \). For \( U \times V \in \mathcal{C} \), we have \( p\text{Cl}(g^{-1}(U \times V)) = p\text{Cl}(U \cap f^{-1}(V)) \subseteq U \cap p\text{Cl}(f^{-1}(V)) \subseteq \text{Int} \text{Cl}(U) \cap f^{-1}(\text{Int} \text{Cl}(V)) = g^{-1}(\text{Int} \text{Cl}(U) \times \text{Int} \text{Cl}(V)) = g^{-1}(\text{Int} \text{Cl}(U \times V)) \). Therefore the graph function \( g \) is subalmost contra-precontinuous. \(\square\)

Remark 5.8. In Theorem 5.7 the requirement that \( X \) be zero-dimensional can be replaced by the assumption that \( X \) is an ED space.

6. Additional properties of subalmost contra-precontinuous functions

The following generalizations of the \( T_1 \) and Hausdorff properties will be useful.
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Definition 6.1. A space $X$ is said to be pre-$T_1$ [10] provided that for every pair of distinct points $x$ and $y$ of $X$, there exist preopen sets $U$ and $V$ containing $x$ and $y$, respectively, with $y \notin U$ and $x \notin V$.

Definition 6.2. A space $X$ is said to be weakly Hausdorff [15] if each element of $X$ is an intersection of regular closed sets.

Theorem 6.3. If $f : X \to Y$ is a subalmost contra-precontinuous injection and $Y$ is weakly Hausdorff, then $X$ is pre-$T_1$.

Proof. Let $x_1$ and $x_2$ be distinct points in $X$. Then $f(x_1) \neq f(x_2)$, and since $Y$ is weakly Hausdorff, there exists a regular closed subset $F$ of $Y$ such that $f(x_1) \in F$ and $f(x_2) \notin F$. Then $f(x_2) \in X - F$, which is regular open. Let $\mathcal{B}$ be an open base for $Y$ such that $p \text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V))$ for every $V \in \mathcal{B}$. Then let $V \in \mathcal{B}$ such that $f(x_2) \in V \subseteq Y - F$. Then $x_2 \notin X - p \text{Cl}(f^{-1}(V))$, which is preopen. Also $f(x_1) \in F$, which is regular closed and therefore also semiopen. Since $F \cap V = \emptyset$, it follows that $f(x_1) \notin s\text{Cl}(V)$, and hence $x_1 \notin f^{-1}(s\text{Cl}(V))$. Then $x_1 \in X - f^{-1}(s\text{Cl}(V)) \subseteq X - p\text{Cl}(f^{-1}(V))$. Therefore, $X - p\text{Cl}(f^{-1}(V))$ is a preopen set containing $x_1$ but not $x_2$, which proves that $X$ is pre-$T_1$.

Corollary 6.4 (Ekici [7, Theorem 11]). If $f : X \to Y$ is an almost contra-precontinuous injection and $Y$ is weakly Hausdorff, then $X$ is pre-$T_1$.

The following example shows that the restriction of a subalmost contra-precontinuous function is not necessarily subalmost contra-precontinuous.

Example 6.5. Let $X = \{a, b, c, d\}$ have the topology $\tau = \{X, \emptyset, \{a, b\}\}$ and let $Y = X$ have the discrete topology. Since the singleton subsets of $X$ are preclosed [10], the identity mapping $f : X \to Y$ is subalmost contra-precontinuous with respect to the base for $Y$ consisting of the singleton sets. However, if $A = \{a, c\}$, then $f|_A : A \to Y$ fails to be subalmost contra-precontinuous.

Next we show that the restriction of a subalmost contra-precontinuous function to a semiopen set is subalmost contra-precontinuous. The following lemma will be useful.

Lemma 6.6 (Baker [3]). If $B \subseteq A \subseteq X$ and $A$ is semiopen in $X$, then $p\text{Cl}_A(B) \subseteq p\text{Cl}(B)$.

Theorem 6.7. If $f : X \to Y$ is subalmost contra-precontinuous with respect to the open base $\mathcal{B}$ for $Y$ and $A$ is a semiopen subset of $X$, then $f|_A : A \to Y$ is subalmost contra-precontinuous with respect to $\mathcal{B}$.

Proof. Let $V \in \mathcal{B}$. Then using Lemma 6.6, we see that $p\text{Cl}_A(f|_A^{-1}(V)) \subseteq A \cap p\text{Cl}(f|_A^{-1}(V)) = A \cap p\text{Cl}(f^{-1}(V) \cap A) \subseteq A \cap p\text{Cl}(f^{-1}(V)) \cap p\text{Cl}(A) = A \cap p\text{Cl}(f^{-1}(V)) \subseteq A \cap f^{-1}(s\text{Cl}(V)) = f|_A^{-1}(s\text{Cl}(V))$. Hence, $f|_A : A \to Y$ is subalmost contra-precontinuous with respect to $\mathcal{B}$.

If we take $\mathcal{B}$ to be the topology on $Y$ in Theorem 6.7, we obtain the following result.

Corollary 6.8 (Ekici [7, Theorem 2]). If $f : X \to Y$ is almost contra-precontinuous and $A$ is a semiopen subset of $X$, then $f|_A : A \to Y$ is almost contra-precontinuous.
Theorem 6.9. If \( f : X \to Y \) is subalmost contra-precontinuous and \( A \) is an open subset of \( Y \) with \( f(X) \subseteq A \), then \( f : X \to A \) is subalmost contra-precontinuous.

Proof. Let \( \mathcal{B} \) be an open base for \( Y \) such that \( p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V)) \) for every \( V \in \mathcal{B} \). Then \( \mathcal{C} = \{V \cap A : V \in \mathcal{B}\} \) is an open base for the relative topology on \( A \). For \( V \in \mathcal{B} \), we have \( p\text{Cl}(f^{-1}(V \cap A)) = p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) = f^{-1}(\text{Int}(\text{Cl}(V)) \cap A) \).

Now we show that \( \text{Int}(\text{Cl}(V)) \cap A \subseteq \text{Int}_A(\text{Cl}_A(V \cap A)) \).

Let \( y \in \text{Cl}(V) \cap A \) and let \( W \subseteq A \) be open in the relative topology on \( A \) with \( y \in W \). Since \( A \) is open in \( Y \), we see that \( W \) is open in \( Y \). Because \( y \in \text{Cl}(V) \), \( V \cap W \neq \emptyset \). Then \( W \cap (V \cap A) = W \cap V \neq \emptyset \), and hence \( y \in \text{Cl}_A(V \cap A) \). Then \( \text{Cl}(V) \cap A \subseteq \text{Cl}_A(V \cap A) \), and therefore \( \text{Int}(\text{Cl}(V) \cap A) \subseteq \text{Int}_A(\text{Cl}_A(V \cap A)) \). Since \( \text{Int}(\text{Cl}(V)) \cap A = \text{Int}(\text{Cl}(V)) \cap A \) and \( \text{Int}(\text{Cl}_A(V \cap A)) \subseteq \text{Int}_A(\text{Cl}_A(V \cap A)) \), it follows that \( \text{Int}(\text{Cl}(V)) \cap A \subseteq \text{Int}_A(\text{Cl}_A(V \cap A)) \).

Recall that we established in the first part of the proof that \( p\text{Cl}(f^{-1}(V \cap A)) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) \). Therefore \( p\text{Cl}(f^{-1}(V \cap A)) \subseteq f^{-1}(\text{Int}_A(\text{Cl}_A(V \cap A))) \), which by Theorem 4.4 proves that \( f : X \to A \) is subalmost contra-precontinuous with respect to the base \( \mathcal{C} \).

Theorem 6.10. If \( f : X \to Y \) is subalmost contra-precontinuous, then for every \( \theta \)-open (resp., \( \theta \)-closed) subset \( W \) of \( Y \), \( f^{-1}(W) \) is a union of preclosed sets (resp., an intersection of preopen sets).

Proof. Let \( \mathcal{B} \) be an open base for \( Y \) such that \( p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V)) \) for every \( V \in \mathcal{B} \). Let \( W \) be a \( \theta \)-open set of \( Y \) and let \( x \in f^{-1}(W) \). Let \( V \in \mathcal{B} \) such that \( f(x) \in V \subseteq \text{Cl}(V) \subseteq W \). Then \( x \in p\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(s\text{Cl}(V)) \subseteq f^{-1}(\text{Cl}(V)) \subseteq f^{-1}(W) \). Since \( p\text{Cl}(f^{-1}(V)) \) is preclosed, it follows that \( f^{-1}(W) \) is a union of preclosed sets. An argument using complements will prove the remaining part of the theorem.

References

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