Erratum

Lower Bounds for Some Factorable Matrices

B. E. Rhoades and Pali Sen

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The purpose of this erratum is to correct both the mathematical and typographical errors made in 2006.

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The typos are as follows.

(i) Page 2, line 17, \( \ell_0 \) should read \( \ell_p \).
(ii) Page 3, line 5, \( j = r + 2 \) should read \( j = r + 1 \).
(iii) Page 3, line 15, \( \Delta y_{rp} \) should read \( \Delta y_p \).
(iv) Page 3, line 16 should read

\[
g(r) - g(r + 1) = (r + 2)a_{r+1}^p \Delta y_r^p + (r + 2)\Delta y_r^p \sum_{j=r+2}^{\infty} a_j^p. \tag{1.1}
\]

(v) Page 4, line 7, \( q < t < 1 \) should read \( r > 0 \).
(vi) Page 4, line 11, \( v(r) \) should read \( u(r) \).
(vii) Page 6, line 17, \( (t^{r+1}/(r + 2))^p \) should read \( (t^{r+1}/(r + 2))^p \times \).
(viii) Page 6, line 18, \( = \) should read \( \times \).
(ix) Page 7, line 6, \( -1 \rfloor \) should read \( -1 \rfloor \).
(x) Page 7, line 21, \( (n + 1)^{s-1} \) should read \( (n + 1)^{1-s} \).
(xi) Page 8, line 7, \( q(p - 1) \) should read \( p(p - 1) \).
(xii) Page 8, line 14, \( (r + 1)^{s-1} - (r + 2)^{s-1} \) should read \( (r + 1)^{p-1} - (r + 2)^{p-1} \).
(xiii) Page 8, line 16, \( (j + 1)^{(p-1)s} \) should read \( (j + 1)^{(s-1)p} \).
(xiv) Page 10, line 12, \( \geq Pr(r + 1) \) should read \( \geq 1/(r + 1) \).
(xv) Page 10, line 14, \( (r + 1)^{P_p} \) should read \( (r + 1)^{P_p} \).
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(xvi) Page 10, line 14, \( P_r^{p} \) should read \( P_{r+1}^{p} \).

(xvii) Page 10, lines 15, 16, \( P_r^{p} \) should read \( P_{r+1}^{p} \).

(xviii) Page 10, line 17, \( p_{r+1}/P_r \) should read \( p_{r+1}/P_r \).

The mathematical errors occur showing that \( \lim h(r) = 0 \) in Theorems 6 and 7. In Theorem 6, from the formula on line 2 of page 4,

\[
\lim h(r) = \lim_r \frac{a_{r+1}^p \Delta y_r^p}{\Delta^2 y_r^p}
= \lim_r \frac{[(r + 1)^p - (r + 2)^p]}{(r + 2)^p [((r + 1)^p - 2(r + 2)^p + (r + 3)^p]}
= \lim_r \frac{(1/(r + 2)^p) [1 - ((r + 2)/(r + 1))^p]}{(1 - 2 ((r + 2)/(r + 1))^p + ((r + 3)/(r + 2))^p)}
= ((-s/(r + 2)^{s+1})[1 - ((r + 2)/(r + 1))^p] - (-p/(r + 2)^{s+1})(r + 2)/(r + 1)\) \( p^{-1} (1/(r + 1)^2) \) + \( p((r + 3)/(r + 1))^p (2/(r + 1)^2) \)

\[
= \lim_r ((-s(r + 1)^2/2p(r + 2)^{s+1})[1 - ((r + 2)/(r + 1))^p] + (1/(r + 2)^s) ((r + 2)/(r + 1)^p_{-1}) /((((r + 2)/(r + 1))^p_{-1} - ((r + 3)/(r + 1))^p_{-1})
= \lim_r \frac{(-s(r + 1)/2p(r + 2)^{s+1}) [(r + 1)^p - (r + 2)^p] + ((r + 2)^p_{-1} - (r + 3)^p_{-1})}{(r + 2)^p_{-1} - (r + 3)^p_{-1}}
= \lim_r \frac{(-s(r + 1)/2p(r + 2)^{s+1}) [(r + 1)/(r + 2)]^p - 1 + 1/(r + 2)^s}{1 - ((r + 3)/(r + 2))^p_{-1}}
= \lim_r ((-s(r + 2)^2/2p)((r + 2 - s(r + 1))/(r + 2)^{s+1})((r + 1)/(r + 2))^p - 1) - (s/2p(r + 2)^{s-1}((r + 1)/(r + 2))^p - (s/(r + 2)^{s-1}) /((p - 1)((r + 3)/(r + 2))^p_{-2} = \lim_r A, \quad (1.2)

where

\[
A = -s(r + 2 - s(r + 1)) \left( \frac{(r + 1)^p}{r + 2} - 1 \right). \quad (1.3)
\]
If $s \geq 2$, then, clearly $\lim rA = 0$. Suppose that $1 < s < 2$, denoting the expressions involved:

$$\lim_{r} A = \lim_{r} \frac{-s/2p(p-1)}{(r+2)s^{-1}p^{-1}/(r+2-s(r+1))} = 0.$$ \hspace{1cm} (1.4)

Thus $g$ is monotone decreasing in $r$. The balance of the proof of [1, Theorem 6] is correct, and $L^p = f(\infty)$.

In Theorem 7, \[(1.5)\]

$$h(r) = \frac{(r+1)^p - (r+2)^p}{(r+1)^p[(r+1)^p - 2(r+2)^p + (r+3)^p]}.$$ which is the same $h$ as in Theorem 6, with $s$ replaced by $p$. Therefore, $\lim_{r} h(r) = 0$. In the proof of Theorem 7, $g(0) \leq 0$, so $L^p = f(0)$. 

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B. E. Rhoades: Department of Mathematics, Indiana University, Bloomington, IN 47405-7106, USA
Email address: rhoades@indiana.edu

Pali Sen: Department of Mathematics and Statistics, University of North Florida, Jacksonville, FL 32224, USA
Email address: psen@unf.edu