Research Article

Characterization of Entire Sequences via Double Orlicz Space

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Let Γ denote the space of all entire sequences and $\wedge$ the space of all analytic sequences. This paper is a study of the characterization and general properties of entire sequences via double Orlicz space of Γ_M^2 of Γ^2 establishing some inclusion relations.

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1. Introduction

Throughout, w , Γ , and $\wedge$ denote the classes of all, entire, and analytic scalar-valued single sequences, respectively.

We write w^2 for the set of all complex sequences (x_{mn}) , where $m, n \in \mathbb{N}$ is the set of positive integers. Then, w^2 is a linear space under the coordinatewise addition and scalar multiplication.

Some initial works on double sequence spaces are found in Bromwich [1]. Later on, they were investigated by Hardy [2], Móricz [3], Móricz and Rhoades [4], Basarir and Sonalcan [5], Tripathy [6], Colak and Turkmenoglu [7], Turkmenoglu [8], and many others.

We need the following inequality in the sequel of the paper.

For $a, b \geq 0$ and $0 < p < 1$, we have

$$(a + b)^p \leq a^p + b^p. \quad (1.1)$$

The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (S_{mn}) is called convergent, where $S_{mn} = \sum_{i,j=1}^{m,n} x_{ij}$ ($m, n = 1, 2, 3, \dots$) (see [9]).

A sequence $x = (x_{mn})$ is said to be double analytic if $\sup_{m,n} |x_{mn}|^{1/m+n} < \infty$. The vector space of all double analytic sequences will be denoted by $\wedge^2$. A sequence $x = (x_{mn})$ is called

double entire sequence if $|x_{mn}|^{1/m+n} \rightarrow 0$ as $m, n \rightarrow \infty$. The double entire sequences will be denoted by Γ^2 . Let $\Phi = \{\text{all finite sequences}\}$.

Consider a double sequence $x = (x_{ij})$. The (m, n) th section $x^{[m, n]}$ of the sequence is defined by $x^{[m, n]} = \sum_{i, j=0}^{m, n} x_{ij} \delta_{ij}$ for all $m, n \in \mathbb{N}$,

$$\delta_{mn} = \begin{pmatrix} 0, 0, \dots, 0, 0, \dots \\ 0, 0, \dots, 0, 0, \dots \\ \vdots \\ 0, 0, \dots, 1, 0, \dots \\ 0, 0, \dots, 0, 0, \dots \end{pmatrix}, \quad (1.2)$$

with 1 in the (m, n) th position and zero otherwise. An FK-space (or a metric space) X is said to have AK property if (δ_{mn}) is a Schauder basis for X . Or equivalently, $x^{[m, n]} \rightarrow x$.

An FDK space is a double sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings $x = (x_k) \rightarrow (x_{mn})$ ($m, n \in \mathbb{N}$) are also continuous.

Orlicz [10] used the idea of Orlicz function to construct the space (L^M) . Lindenstrauss and Tzafriri [11] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p ($1 \leq p < \infty$). Subsequently, different classes of sequence spaces were defined by Parashar and Choudhary [12], Mursaleen et al. [13], Bektaş and Altın [14], Tripathy et al. [15], Rao and Subramanian [16], and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in [17].

Recalling [10] and [17], an Orlicz function is a function $M: [0, \infty) \rightarrow [0, \infty)$ which is continuous, nondecreasing, and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \rightarrow \infty$ as $x \rightarrow \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x) + M(y)$, then this function is called modulus function, defined by Nakano [18] and further discussed by Ruckle [19], Maddox [20], and many others.

Let (Ω, Σ, μ) be a finite measure space. We denote by $E(\mu)$ the space of all (equivalence classes of) Σ -measurable functions x from Ω into $[0, \infty)$. Given an Orlicz function M , we define on $E(\mu)$ a convex functional I_M by

$$I_M(x) = \int_{\Omega} M(x(t)) d\mu, \quad (1.3)$$

and an Orlicz space $L^M(\mu)$ by $L^M(\mu) = \{x \in E(\mu) : I_M(\lambda x) < +\infty \text{ for some } \lambda > 0\}$ (for detail, see [10, 17]).

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct Orlicz sequence space:

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}, \quad (1.4)$$

where $w = \{\text{all complex sequences}\}$. The space ℓ_M with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\} \quad (1.5)$$

becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p$ ($1 \leq p < \infty$), the spaces ℓ_M coincide with the classical sequence space ℓ_p .

If X is a sequence space, we give the following definitions:

- (i) X' = the continuous dual of X ;
- (ii) $X^\alpha = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn}x_{mn}| < \infty, \text{ for each } x \in X\}$;
- (iii) $X^\beta = \{a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn}x_{mn} \text{ is convergent, for each } x \in X\}$;
- (iv) $X^\gamma = \{a = (a_{mn}) : \sup_{m,n \geq 1} |\sum_{m,n=1}^{M,N} a_{mn}x_{mn}| < \infty, \text{ for each } x \in X\}$;
- (v) let X be an FK-space $\supset \Phi$, then $X^f = \{f(\delta_{mn}) : f \in X'\}$;
- (vi) $X^\wedge = \{a = (a_{mn}) : \sup_{(mn)} |a_{mn}x_{mn}|^{1/m+n} < \infty, \text{ for each } x \in X\}$;
- (vii) $X^\alpha, X^\beta, X^\gamma$ are called α -(or Köthe-Toeplitz) dual of X , β -(or generalized Köthe-Toeplitz) dual of X , γ -dual of X , and $\wedge$ -dual of X , respectively.

2. Definitions and preliminaries

Throughout the article, w^2 denote the spaces of all sequences. Γ_M^2 and $\wedge_M^2$ denote the Pringscheims of double Orlicz space of entire sequence and Pringscheims of double Orlicz space of bounded sequence, respectively

Let w^2 denote the set of all complex double sequences $x = (x_{mn})_{m,n=1}^{\infty}$ and $M : [0, \infty) \rightarrow [0, \infty)$ be an Orlicz function, or a modulus function. Given a double sequence, $x \in w^2$. Let t denote the double sequence with $t_{mn} = |x_{mn}|^{1/(m+n)}$ for all $m, n \in \mathbb{N}$. Define the sets

$$\begin{aligned} \Gamma_M^2 &= \left\{ x \in w^2 : \left(M\left(\frac{t_{mn}}{\rho}\right) \right) \rightarrow 0 \ (m, n \rightarrow \infty) \text{ for some } \rho > 0 \right\}, \\ \wedge_M^2 &= \left\{ x \in w^2 : \sup_{(m,n)} \left(M\left(\frac{t_{mn}}{\rho}\right) \right) < \infty \text{ for some } \rho > 0 \right\}. \end{aligned} \quad (2.1)$$

The space $\wedge_M^2$ is a metric space with the metric

$$\tilde{d}(x, y) = \inf \left\{ \rho > 0 : \sup_{(m,n)} \left(M\left(\frac{|x_{mn} - y_{mn}|^{1/m+n}}{\rho}\right) \right) \leq 1 \right\} \quad (2.2)$$

and the space Γ_M^2 is a metric space with the metric

$$d(x, y) = \left\{ \rho > 0 : \sup_{(m,n)} \left(M\left(\frac{|x_{mn} - y_{mn}|^{1/m+n}}{\rho}\right) \right) : m, n = 1, 2, 3, \dots \right\}. \quad (2.3)$$

3. Main results

PROPOSITION 3.1. *If M is a modulus function, then Γ_M^2 is a linear set over the set of complex numbers $\mathbb{C}$.*

Proof. It is trivial. Therefore, the proof is omitted. □

PROPOSITION 3.2. $(\Gamma_M^2)^\beta \subsetneq \wedge^2$.

Proof. Let $y = \{y_{mn}\}$ be an arbitrary point in $(\Gamma_M^2)^\beta$. If y is not in $\wedge^2$, then for each natural number p , we can find an index $m_p n_p$ such that

$$M\left(\frac{|y_{m_p n_p}|^{1/m_p + n_p}}{\rho}\right) > p, \quad (p = 1, 2, 3, \dots). \quad (3.1)$$

Define $x = \{x_{mn}\}$ by

$$\begin{aligned} M\left(\frac{x_{mn}}{\rho}\right) &= \frac{1}{p^{m+n}} \quad \text{for } (m, n) = (m_p, n_p) \text{ for some } p \in N, \\ M\left(\frac{x_{mn}}{\rho}\right) &= 0 \quad \text{otherwise.} \end{aligned} \quad (3.2)$$

Then x is in Γ_M^2 , but for infinitely mn ,

$$M\left(\frac{|y_{mn} x_{mn}|}{\rho}\right) > 1. \quad (3.3)$$

Consider the sequence $z = \{z_{mn}\}$, where $M(z_{11}/\rho) = M(x_{11}/\rho) - s$ with

$$s = \sum M\left(\frac{x_{mn}}{\rho}\right), \quad M\left(\frac{z_{mn}}{\rho}\right) = M\left(\frac{x_{mn}}{\rho}\right) \quad (m, n = 1, 2, 3, \dots). \quad (3.4)$$

Then, z is a point of Γ_M^2 . Also, $\sum M(z_{mn}/\rho) = 0$. Hence, z is in Γ_M^2 ; but, by (3.3), $\sum M(z_{mn} y_{mn}/\rho)$ does not converge:

$$\implies \sum x_{mn} y_{mn} \text{ diverges.} \quad (3.5)$$

Thus, the sequence y would not be in $(\Gamma_M^2)^\beta$. This contradiction proves that

$$(\Gamma_M^2)^\beta \subsetneq \wedge^2. \quad (3.6)$$

If we now choose $M = \text{id}$, where id is the identity and $y_{1n} = x_{1n} = 1$ and $y_{mn} = x_{mn} = 0$ ($m > 1$) for all n , then obviously $x \in \Gamma_M^2$ and $y \in \wedge^2$, but

$$\sum_{m,n=1}^{\infty} x_{mn} y_{mn} = \infty. \text{ Hence, } y \notin (\Gamma_M^2)^{\beta}. \quad (3.7)$$

From (3.6) and (3.7), we are granted $(\Gamma_M^2)^{\beta} \subsetneq \wedge^2$. This completes the proof. $\square$

PROPOSITION 3.3. Γ_M^2 has AK, where M is a modulus function.

Proof. Let $x = (x_{mn}) \in \Gamma_M^2$ and take $x^{[mn]} = \sum_{i,j=1}^{m,n} x_{ij} \delta_{ij}$ for all $m, n \in \mathbb{N}$. Hence,

$$d(x, x^{[rs]}) = \left\{ \rho : \sup_{(m,n)} \left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right) : m \geq r+1, n \geq s+1 \right\} \quad (3.8)$$

$$\longrightarrow 0 \quad \text{as } m, n \longrightarrow \infty.$$

Therefore, $x^{[rs]} \rightarrow x$ as $r, s \rightarrow \infty$ in Γ_M^2 . Thus, Γ_M^2 has AK. This completes the proof. $\square$

PROPOSITION 3.4. Γ_M^2 is solid.

Proof. Let $|x_{mn}| \leq |y_{mn}|$ and let $y = (y_{mn}) \in \Gamma_M^2$. $(M(|x_{mn}|^{1/m+n}/\rho)) \leq (M(|y_{mn}|^{1/m+n}/\rho))$, because M is nondecreasing. But $(M(|y_{mn}|^{1/m+n}/\rho)) \in \Gamma^2$ because $y \in \Gamma_M^2$. That is, $(M(|y_{mn}|^{1/m+n}/\rho)) \rightarrow 0$ as $m, n \rightarrow \infty$ and $(M(|x_{mn}|^{1/m+n}/\rho)) \rightarrow 0$ as $m, n \rightarrow \infty$. Therefore, $x = \{x_{mn}\} \in \Gamma_M^2$. This completes the proof. $\square$

PROPOSITION 3.5. $(\Gamma_M^2)^{\wedge} \subsetneq \wedge^2$.

Proof. Let $y \in \wedge$ -dual of Γ_M^2 . Then, $(M(|x_{mn} y_{mn}|/\rho)) \leq M^{m+n}$ for some constant $M > 0$ and for all $x \in \Gamma_M^2$. Therefore, $(M(|y_{mn}|/\rho)) \leq M^{m+n}$ for all m, n by taking $x = (\delta_{mn})$. This implies that $y \in \wedge^2$. Thus,

$$(\Gamma_M^2)^{\wedge} \subset \wedge^2. \quad (3.9)$$

We now choose $M = \text{id}$ and define the double sequences (y_{mn}) and (x_{mn}) by $y_{mn} = 1$ for all m and n , and by $x_{m1} = 2^{(m+1)^2}$ and $x_{mn} = 0$ ($n \geq 2$) for all $m = 1, 2, \dots$. Obviously, $y \in \wedge^2$ and since $x_{mn} = 0$ for all $m, n \geq 0$, (x_{mn}) converges to zero in the Pringsheim sense. Hence, $x \in \Gamma_M^2$. But,

$$|a_{m1} x_{m1}|^{1/(m+1)} = 2^{m+1} \longrightarrow \infty \quad \text{as } m \longrightarrow \infty, \text{ hence } x \notin (\Gamma_M^2)^{\wedge}. \quad (3.10)$$

From (3.9) and (3.10), we are granted $(\Gamma_M^2)^{\wedge} \subsetneq \wedge^2$. This completes the proof. $\square$

PROPOSITION 3.6. *The dual space of (Γ_M^2) is $\wedge^2$. In other words, $(\Gamma_M^2)^* = \wedge^2$.*

Proof. We recall that

$$\delta_{mn} = \begin{pmatrix} 0, 0, \dots, 0, 0, \dots \\ 0, 0, \dots, 0, 0, \dots \\ \vdots \\ 0, 0, \dots, 1, 0, \dots \\ 0, 0, \dots, 0, 0, \dots \end{pmatrix} \quad (3.11)$$

has 1 in the (m, n) th position and zero otherwise, with

$$\begin{aligned} x &= \delta_{mn}, \left\{ M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right) \right\} \\ &= \begin{pmatrix} M((0)^{1/2}/\rho), M((0)^{1/3}/\rho), M((0)^{1/2}/\rho), \dots \\ M((0)^{1/3}/\rho), M((0)^{1/4}/\rho), M((0)^{1/5}/\rho), \dots \\ \vdots \\ M((0)/\rho), M((0)/\rho), \dots, M((1)^{1/m+n}/\rho), M((0)/\rho), \dots \\ M((0)/\rho), M((0)/\rho), \dots, M((0)/\rho), M((0)/\rho), \dots \\ \vdots \end{pmatrix} \end{aligned} \quad (3.12)$$

which is a double null sequence.

Hence, $\delta_{mn} \in \Gamma_M^2 \cdot f(x) = \sum_{m,n=1}^{\infty} x_{mn} y_{mn}$ with $x \in \Gamma_M^2$ and $f \in (\Gamma_M^2)^*$, where $(\Gamma_M^2)^*$ is the dual space of Γ_M^2 . Take $x = (x_{mn}) = \delta_{mn} \in \Gamma_M^2$. Then,

$$|y_{mn}| \leq \|f\| d(\delta_{mn}, 0) < \infty \quad \forall m, n. \quad (3.13)$$

Thus, (y_{mn}) is a double bounded sequence, and hence a double analytic sequence. In other words, $y \in \wedge^2$. Therefore, $(\Gamma_M^2)^* = \wedge^2$. This completes the proof. $\square$

PROPOSITION 3.7. $(\wedge_M^2)^\beta \subsetneq \Gamma_M^2$.

Proof. Let $(x_{mn}) \in (\wedge_M^2)^\beta$,

$$\Rightarrow \sum_{m,n=1}^{\infty} x_{mn} y_{mn} \text{ converges } \forall y \in \wedge_M^2. \quad (3.14)$$

Let us assume that $(x_{mn}) \notin \Gamma_M^2$. Then, there exist a sequence positive integers $(m_p + n_p)$ strictly increasing such that

$$\left(M\left(\frac{|x_{(m_p+n_p)}|}{\rho}\right) \right) > \frac{1}{2^{(m_p+n_p)}}, \quad (p = 1, 2, 3, \dots). \quad (3.15)$$

Let

$$\begin{aligned} y_{(m_p, n_p)} &= 2^{(m_p + n_p)} \quad (\text{for } p = 1, 2, 3, \dots), \\ y_{m, n} &= 0, \quad \text{otherwise.} \end{aligned} \quad (3.16)$$

Then, $(y_{mn}) \in \wedge_M^2$.

However,

$$\sum_{m, n=1}^{\infty} \left(M \left(\frac{|x_{mn} y_{mn}|}{\rho} \right) \right) = \sum_{p=1}^{\infty} \left(M \left(\frac{|x_{(m_p, n_p)} y_{(m_p, n_p)}|}{\rho} \right) \right) > 1 + 1 + 1 + \dots \quad (3.17)$$

We know that the infinite series $1 + 1 + 1 + \dots$ diverges. Hence, $\sum_{m, n=1}^{\infty} (M(|x_{mn} y_{mn}|/\rho))$ diverges. This contradicts (3.14). Hence, $(x_{mn}) \in \Gamma_M^2$. Therefore,

$$(\wedge_M^2)^\beta \subset \Gamma_M^2. \quad (3.18)$$

If we now choose $M = \text{id}$, where id is the identity and $y_{1n} = x_{1n} = 1$ and $y_{mn} = x_{mn} = 0$ ($m > 1$) for all n , then obviously $x \in \Gamma_M^2$ and $y \in \wedge_M^2$, but

$$\sum_{m, n=1}^{\infty} x_{mn} y_{mn} = \infty. \text{ Hence, } y \notin (\wedge_M^2)^\beta. \quad (3.19)$$

From (3.18) and (3.19), we are granted $(\wedge_M^2)^\beta \subsetneq \Gamma_M^2$. This completes the proof. $\square$

Definition 3.8. Let $p = (p_{mn})$ be a double sequence of positive real numbers. Then,

$$\Gamma_M^2(p) = \left\{ x = (x_{mn}) : \left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right)^{p_{mn}} \longrightarrow 0 \text{ (} m, n \longrightarrow \infty \text{) for some } \rho > 0 \right\}. \quad (3.20)$$

Suppose that p_{mn} is a constant for all m, n , then $\Gamma_M^2(p) = \Gamma_M^2$.

PROPOSITION 3.9. Let $0 \leq p_{mn} \leq q_{mn}$ and let $\{q_{mn}/p_{mn}\}$ be bounded. Then, $\Gamma_M^2(q) \subset \Gamma_M^2(p)$.

Proof. Let

$$x \in \Gamma_M^2(q), \quad (3.21)$$

then

$$\left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right)^{q_{mn}} \longrightarrow 0 \text{ as } m, n \longrightarrow \infty. \quad (3.22)$$

Let $t_{mn} = (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$, and let $\lambda_{mn} = p_{mn}/q_{mn}$. Since $p_{mn} \leq q_{mn}$, we have $0 \leq \lambda_{mn} \leq 1$. Let $0 < \lambda < \lambda_{mn}$, then

$$\begin{aligned} u_{mn} &= \begin{cases} t_{mn} & (t_{mn} \geq 1), \\ 0 & (t_{mn} < 1), \end{cases} \\ v_{mn} &= \begin{cases} 0 & (t_{mn} \geq 1), \\ t_{mn} & (t_{mn} < 1), \end{cases} \\ t_{mn} &= u_{mn} + v_{mn}, \quad t_{mn}^{\lambda_{mn}} = u_{mn}^{\lambda_{mn}} + v_{mn}^{\lambda_{mn}}. \end{aligned} \quad (3.23)$$

Now, it follows that

$$u_{mn}^{\lambda_{mn}} \leq u_{mn} \leq t_{mn}, \quad v_{mn}^{\lambda_{mn}} \leq v_{mn}. \quad (3.24)$$

Since $t_{mn}^{\lambda_{mn}} = u_{mn}^{\lambda_{mn}} + v_{mn}^{\lambda_{mn}}$, we have $t_{mn}^{\lambda_{mn}} \leq t_{mn} + v_{mn}^{\lambda_{mn}}$. Thus, $(M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}})^{\lambda_{mn}} \leq (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$ and

$$\left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right)^{q_{mn}} \right)^{p_{mn}/q_{mn}} \leq \left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right)^{q_{mn}}, \quad (3.25)$$

which yields $(M(|x_{mn}|^{1/m+n}/\rho))^{p_{mn}} \leq (M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}}$. However, $(M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}} \rightarrow 0$ (by (3.22)). Thus, $(M(|x_{mn}|^{1/m+n}/\rho))^{p_{mn}} \rightarrow 0$ as $m, n \rightarrow \infty$.

Hence,

$$x \in \Gamma_M^2(p). \quad (3.26)$$

From (3.21) and (3.26), we are granted

$$\Gamma_M^2(q) \subset \Gamma_M^2(p). \quad (3.27)$$

This completes the proof. $\square$

PROPOSITION 3.10. (a) If $0 < \inf p_{mn} \leq p_{mn} \leq 1$, then $\Gamma_M^2(p) \subset \Gamma_M^2$.

(b) If $1 \leq p_{mn} \leq \sup p_{mn} < \infty$, then $\Gamma_M^2 \subset \Gamma_M^2(p)$.

Proof. The above statements are special cases of Proposition 3.9. Therefore, it can be proved by similar arguments. $\square$

PROPOSITION 3.11. If $0 < p_{mn} \leq q_{mn} < \infty$ for each m, n , then $\Gamma_M^2(p) \subseteq \Gamma_M^2(q)$.

Proof. Let $x \in \Gamma_M^2(p)$, then

$$\left(M \left(\frac{|x_{mn}|^{1/m+n}}{\rho} \right) \right)^{p_{mn}} \rightarrow 0 \quad \text{as } m, n \rightarrow \infty. \quad (3.28)$$

This implies that $(M(|x_{mn}|^{1/m+n}/\rho)) \leq 1$ for sufficiently large m, n . Since M is nondecreasing, we get

$$\left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{q_{mn}} \leq \left(M\left(\frac{|x_{mn}|^{1/m+n}}{\rho}\right)\right)^{p_{mn}}, \quad (3.29)$$

then $(M(|x_{mn}|^{1/m+n}/\rho))^{q_{mn}} \rightarrow 0$ as $m, n \rightarrow \infty$ (by using (3.28)). Let $x \in \Gamma_M^2(q)$. Hence, $\Gamma_M^2(p) \subseteq \Gamma_M^2(q)$. This completes the proof. $\square$

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