Research Article

Weak Forms of Continuity and Associated Properties

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Received 17 July 2008; Accepted 17 September 2008

Recommended by Dalibor Froncek

We introduce slightly p -continuous mapping and almost p -open mapping and investigate the relationships between these mappings and related types of mappings, and also study some properties of these mappings.

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1. Introduction and preliminaries

A subset A of a space X is called regular open if $A = \text{Int } \overline{A}$, and regular closed if $X \setminus A$ is regular open, or equivalently, if $A = \overline{\text{Int } A}$. It is well known that a subset A of a space X is regular open if and only if $A = \text{Int } F$, where F is closed and A is regular closed if and only if $A = \overline{U}$, where U is open. A is called semi-open [1] (resp., preopen [2], semi-preopen [3], b -open [4]) if $A \subset \overline{\text{Int } A}$ (resp., $A \subset \text{Int } \overline{A}$, $A \subset \overline{\text{Int } A}$, $A \subset \overline{\text{Int } A} \cup \text{Int } \overline{A}$). It is known that a set A is semi-open if and only if $U \subset A \subset \overline{U}$ for some open set U , and that A is preopen (resp., semi-preopen) if and only if $A = U \cap D$, where U is open (resp., semi-open) and D is dense. The concept of a preopen set was introduced in [5], where the term locally dense was used and the concept of a semi-preopen set was introduced in [6] under the name β -open. It was pointed out in [3] that A is semi-preopen if and only if $P \subset A \subset \overline{P}$ for some preopen set P . Clearly, every regular closed set is semi-open, every open set is both semi-open and preopen, semi-open sets as well as preopen sets are b -open and b -open sets are semi-preopen. It is also known that the closure of every semi-preopen set is regular closed and that the arbitrary union of semi-open (resp., preopen, semi-preopen, b -open) sets is semi-open (resp., preopen, semi-preopen, b -open). A is called semi-closed (resp., preclosed, semi-preclosed, b -closed) if $X \setminus A$ is semi-open (resp., preopen, semi-preopen, b -open). It is well known that a subset A is regular closed if and only if A is both closed and semi-open if and only if A is both closed and semi-preopen.

A mapping f from a space X into a space Y is called **regular open** [7] if it maps regular open subsets onto regular open sets, **almost open** [8] if $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is open in Y , **slightly continuous** [7] if $f(\overline{U}) \subset \overline{f(U)}$ whenever U is open in X , **semi-continuous** [1] if the inverse image of each open set is semi-open, **β -continuous** [6] if the inverse image of each open set is β -open, **weakly continuous** [9] if for each $x \in X$ and for each open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset \overline{V}$, **weakly θ -irresolute** [10] if the inverse image of each regular closed set is semi-open, **rc-continuous** [11] if the inverse image of each regular closed set is regular closed, and **wrc-continuous** [12] if the inverse image of each regular closed set is semi-preopen. We will use the term **semi-precontinuous** to indicate β -continuous. Clearly, every semi-continuous mapping is semi-precontinuous, every rc-continuous mapping is weakly θ -irresolute, and every weakly θ -irresolute mapping is wrc-continuous. In [7], it is shown that the properties semi-continuous and slightly continuous are independent of each other.

A space X is called a **weak P -space** [13] if for each countable family $\{U_n : n \in \mathbb{N}\}$ of open subsets of X , $\overline{\bigcup U_n} = \bigcup \overline{U_n}$. Clearly, X is a weak P -space if and only if the countable union of regular closed subsets of X is regular closed (closed).

A space X is called **rc-Lindelöf** [14] if every regular closed cover of X has a countable subcover, and called **almost rc-Lindelöf** [15] if every regular closed cover of X has a countable subfamily whose union is dense in X .

A subset A of a space X is called an **S -set** in X [16] if every cover of A by regular closed subsets of X has a finite subcover, and called an **rc-Lindelöf set** in X (resp., an **almost rc-Lindelöf set** in X) [17] if every cover of A by regular closed subsets of X admits a countable subfamily that covers A (resp., the closure of the union of whose members contains A). Obviously, every S -set is an rc-Lindelöf set and every rc-Lindelöf set is an almost rc-Lindelöf set. It is also clear that a subset A of a weak P -space X is rc-Lindelöf in X if and only if it is almost rc-Lindelöf in X .

Throughout this paper, $\mathbb{N}$ (resp., $\mathbb{Q}$, $\mathbb{R}$) denotes the set of natural (resp., rational, real) numbers. For the concepts not defined here, we refer the reader to [18].

2. Slightly p -continuous mappings

This section is mainly devoted to study several properties of slightly p -continuous mappings. Now, we begin with the following lemma which was pointed out in [19] without proof. We will, however, state and prove it for its special importance in the material of our paper.

Lemma 2.1. (i) *Let $f : X \rightarrow Y$ be a semi-continuous and almost open mapping. Then f is weakly θ -irresolute.*

(ii) *Let $f : X \rightarrow Y$ be a semi-precontinuous and almost open mapping. Then f is wrc-continuous.*

Proof. (i) Let U be an open subset of Y . Since f is almost open, then $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$. Since f is semi-continuous, then $f^{-1}(U)$ is semi-open, hence there exists an open subset V of X such that $V \subset f^{-1}(U) \subset \overline{V}$, therefore, $V \subset f^{-1}(U) \subset f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)} \subset \overline{V}$. Thus $f^{-1}(\overline{U})$ is semi-open, and f is weakly θ -irresolute.

(ii) Let U be an open subset of Y . Since f is almost open, then $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$. Since f is semi-precontinuous, then $f^{-1}(U)$ is semi-preopen, hence there exists a preopen subset V of X such that $V \subset f^{-1}(U) \subset \overline{V}$, therefore, $V \subset f^{-1}(U) \subset f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)} \subset \overline{V}$. Thus $f^{-1}(\overline{U})$ is semi-preopen, and f is wrc-continuous. $\square$

Corollary 2.2 (see [12]). *Let $f : X \rightarrow Y$ be a semi-continuous and almost open mapping. Then f is wrc-continuous.*

Proposition 2.3. *For a mapping $f : X \rightarrow Y$, the following are equivalent:*

- (i) f is slightly continuous;
- (ii) $f(\overline{U}) \subset \overline{f(U)}$ whenever U is semi-open in X .

Proof. Since every open set is semi-open, it suffices to show that (i) $\rightarrow$ (ii). Let U be a semi-open subset of X . Then there exists an open subset V of X such that $V \subset U \subset \overline{V}$. Thus by (i), $f(\overline{U}) = f(\overline{V}) \subset \overline{f(V)} \subset \overline{f(U)}$. $\square$

Proposition 2.4. *Let $f : X \rightarrow Y$ be a slightly continuous mapping. Then the following are equivalent:*

- (i) f is weakly θ -irresolute;
- (ii) f is rc-continuous.

Proof. Since every regular closed set is semi-open, it suffices to show that (i) $\rightarrow$ (ii). Let A be a regular closed subset of Y . By (i), $f^{-1}(A)$ is semi-open, but f is slightly continuous, so by Proposition 2.3, $f(\overline{f^{-1}(A)}) \subset \overline{f(f^{-1}(A))} \subset \overline{A} = A$. Thus $\overline{f^{-1}(A)} \subset f^{-1}f(\overline{f^{-1}(A)}) \subset f^{-1}(A)$, that is, $f^{-1}(A)$ is closed, but $f^{-1}(A)$ is semi-open, so $f^{-1}(A)$ is regular closed. Hence f is rc-continuous. $\square$

Corollary 2.5. *Let $f : X \rightarrow Y$ be a slightly continuous, semi-continuous, and almost open mapping. Then f is rc-continuous.*

Proof. Follows from Lemma 2.1(i) and Proposition 2.4. $\square$

Proposition 2.6. *Let $f : X \rightarrow Y$ be a slightly continuous and semi-continuous mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y .*

Proof. Let U be an open subset of Y . Since f is semi-continuous, it follows that $f^{-1}(U)$ is semi-open, but f is slightly continuous, so it follows from Proposition 2.3 that $f(\overline{f^{-1}(U)}) \subset \overline{f(f^{-1}(U))} \subset \overline{U}$. Thus $\overline{f^{-1}(U)} \subset f^{-1}f(\overline{f^{-1}(U)}) \subset f^{-1}(\overline{U})$. $\square$

The following corollary is a slight improvement of Corollary 2.5. This is because the closure of every semi-open set is regular closed.

Corollary 2.7. *Let $f : X \rightarrow Y$ be a slightly continuous, semi-continuous, and almost open mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y .*

Proposition 2.8. *Let $f : X \rightarrow Y$ be an rc-continuous mapping. If A is rc-Lindelöf in X , then $f(A)$ is rc-Lindelöf in Y .*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of $f(A)$ by regular closed subsets of Y . Then $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a cover of A by regular closed subsets of X (as f is rc-continuous). Since A is rc-Lindelöf in X , it follows that there exist $\alpha_1, \alpha_2, \dots \in \Lambda$ such that $A \subset \bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})$, thus it follows that $f(A) \subset \bigcup_{i=1}^{\infty} f(f^{-1}(U_{\alpha_i})) \subset \bigcup_{i=1}^{\infty} U_{\alpha_i}$. Hence $f(A)$ is rc-Lindelöf in Y . $\square$

Corollary 2.9 (see [19]). *Let $f : X \rightarrow Y$ be a slightly continuous and weakly θ -irresolute mapping. If A is rc-Lindelöf in X , then $f(A)$ is rc-Lindelöf in Y .*

Proof. Follows from Propositions 2.4 and 2.8. □

Proposition 2.10 (see [20]). *Let $f : X \rightarrow Y$ be a weakly continuous and almost open mapping. Then f is rc-continuous.*

Corollary 2.11. *Let $f : X \rightarrow Y$ be a weakly continuous and almost open mapping. If A is rc-Lindelöf in X , then $f(A)$ is rc-Lindelöf in Y .*

Proof. Follows from Propositions 2.10 and 2.8. □

Now, we prove the following known result using a slight modification on the previous proof.

Proposition 2.12 (see [7]). *Let $f : X \rightarrow Y$ be a slightly continuous and weakly θ -irresolute mapping. If A is almost rc-Lindelöf in X , then $f(A)$ is almost rc-Lindelöf in Y .*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of $f(A)$ by regular closed subsets of Y . Since f is slightly continuous and weakly θ -irresolute, it follows from Proposition 2.4 that f is rc-continuous and thus $\{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a cover of A by regular closed subsets of X . Since A is almost rc-Lindelöf in X , it follows that there exist $\alpha_1, \alpha_2, \dots \in \Lambda$ such that $A \subset \overline{\bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})}$. Now, $f^{-1}(U_{\alpha_i})$ is regular closed and thus semi-open, but the arbitrary union of semi-open sets is semi-open, so $\bigcup_{i=1}^{\infty} f^{-1}(U_{\alpha_i})$ is semi-open. Since f is slightly continuous, it follows from Proposition 2.3 that $f(A) \subset \overline{\bigcup_{i=1}^{\infty} f(f^{-1}(U_{\alpha_i}))} \subset \overline{\bigcup_{i=1}^{\infty} U_{\alpha_i}}$. Hence $f(A)$ is almost rc-Lindelöf in Y . □

Definition 2.13. A mapping f from a space X into a space Y is said to be slightly p -continuous if $f(\overline{U}) \subset \overline{f(U)}$ whenever U is preopen in X .

Proposition 2.14. *For a mapping $f : X \rightarrow Y$, the following are equivalent:*

- (i) f is slightly p -continuous;
- (ii) $f(\overline{U}) \subset \overline{f(U)}$ whenever U is semi-preopen in X ;
- (iii) $f(\overline{U}) \subset \overline{f(U)}$ whenever U is b -open in X .

Proof. (i) $\rightarrow$ (ii): Let U be a semi-preopen subset of X . Then there exists a preopen subset V of X such that $V \subset U \subset \overline{V}$. Thus by (i), $f(\overline{U}) = f(\overline{V}) \subset \overline{f(V)} \subset \overline{f(U)}$.

(ii) $\rightarrow$ (iii) $\rightarrow$ (i): follow since every preopen set is b -open and every b -open set is semi-preopen. □

Definition 2.15. A mapping $f : X \rightarrow Y$ is called brc-continuous if $f^{-1}(A)$ is b -open for every regular closed subset A of Y .

Clearly, every weakly θ -irresolute mapping is brc-continuous and every brc-continuous mapping is wrb-continuous; the converses are, however, not true as the following two examples tell.

Example 2.16. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$, and $\tau^* = \{X, \phi, \{a\}, \{b, c\}\}$. Then the identity mapping from (X, τ) onto (X, τ^*) is brc-continuous but not weakly θ -irresolute (observe that the regular closed subsets of (X, τ^*) are the members of τ^* , each of which is preopen and thus b -open in (X, τ)). However, $\{a\}$ is not semi-open in (X, τ) .

Example 2.17. Let τ_u be the usual topology on the set of real numbers R and $\tau = \{R, \phi, A, R \setminus A\}$, where $A = [0, 1) \cap Q$. Then the identity mapping from (R, τ_u) onto (R, τ) is wrc-continuous but not brc-continuous (observe that the regular closed subsets of (R, τ) are the members of τ , each of which is semi-preopen in (R, τ_u)). However, A is not b -open in (R, τ_u) .

Proposition 2.18. *Let $f : X \rightarrow Y$ be a slightly p -continuous mapping. Then the following are equivalent:*

- (i) f is weakly θ -irresolute;
- (ii) f is rc-continuous;
- (iii) f is wrc-continuous;
- (iv) f is brc-continuous.

Proof. (ii) $\rightarrow$ (i) $\rightarrow$ (iv) $\rightarrow$ (iii): follow since every regular closed set is semi-open, every semi-open set is b -open and every b -open set is semi-preopen.

(iii) $\rightarrow$ (ii): let A be a regular closed subset of Y . By (iii), $f^{-1}(A)$ is semi-preopen, but f is slightly p -continuous, so by Proposition 2.14, $f(\overline{f^{-1}(A)}) \subset \overline{f(f^{-1}(A))} \subset \overline{A} = A$. Thus $\overline{f^{-1}(A)} \subset f^{-1}f(\overline{f^{-1}(A)}) \subset f^{-1}(A)$, that is, $f^{-1}(A)$ is closed, but $f^{-1}(A)$ is semi-preopen, so $f^{-1}(A)$ is regular closed. Hence f is rc-continuous. $\square$

Corollary 2.19. *Let $f : X \rightarrow Y$ be a slightly p -continuous, semi-precontinuous, and almost open mapping. Then f is rc-continuous.*

Proof. Follows from Lemma 2.1(ii) and Proposition 2.18. $\square$

Proposition 2.20. *Let $f : X \rightarrow Y$ be a slightly p -continuous and semi-precontinuous mapping. Then $\overline{f^{-1}(U)} \subset f^{-1}(\overline{U})$ for every open subset U of Y .*

Proof. Let U be an open subset of Y . Since f is semi-precontinuous, it follows that $f^{-1}(U)$ is semi-preopen, but f is slightly p -continuous, so it follows from Proposition 2.14 that $f(\overline{f^{-1}(U)}) \subset \overline{f(f^{-1}(U))} \subset \overline{U}$. Thus $\overline{f^{-1}(U)} \subset f^{-1}f(\overline{f^{-1}(U)}) \subset f^{-1}(\overline{U})$. $\square$

Observing that the closure of every semi-preopen set is regular closed, the following corollary seems a slight improvement of Corollary 2.19.

Corollary 2.21. *Let $f : X \rightarrow Y$ be a slightly p -continuous, semi-precontinuous, and almost open mapping. Then $\overline{f^{-1}(U)} = f^{-1}(\overline{U})$ for every open subset U of Y .*

Obviously, every continuous mapping is both semi-continuous and slightly p -continuous and every slightly p -continuous mapping is slightly continuous, the converses are, however, not true as the following two examples tell.

Example 2.22. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$, and $\tau^* = \{X, \phi, \{a\}, \{a, b\}\}$. Then the identity mapping from (X, τ) onto (X, τ^*) is slightly continuous and weakly θ -irresolute (observe that the regular closed subsets of (X, τ^*) are X and ϕ). However, it is not slightly

p -continuous (consider the preopen subset $\{b, c\}$ of (X, τ)). We observe also that this is an example of a mapping that is both slightly continuous and semi-precontinuous but neither slightly p -continuous nor semi-continuous (observe that $\{a\}, \{a, b\}$ are both dense and thus preopen in (X, τ) . However, $\{a\}$ is not semi-open in (X, τ)). This example also shows that the converses of Propositions 2.6 and 2.20 are not true.

Example 2.23. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$, and $\tau^* = \{X, \phi, \{a, b\}\}$. Then the identity mapping from (X, τ) onto (X, τ^*) is slightly p -continuous (observe that the nonempty preopen subsets of (X, τ) are the supersets of $\{a\}$); it is, moreover, semi-continuous and almost open. However, it is not continuous.

Corollary 2.24. *Let $f : X \rightarrow Y$ be a slightly p -continuous and wrc-continuous mapping. If A is rc-Lindelöf (resp., almost rc-Lindelöf) in X , then $f(A)$ is rc-Lindelöf (resp., almost rc-Lindelöf) in Y .*

Proof. We observe from Proposition 2.18 that a mapping that is both slightly p -continuous and wrc-continuous is both slightly continuous and weakly θ -irresolute (the converse is not true as Example 2.22 tells). Thus the result follows from Corollary 2.9 and Proposition 2.12. $\square$

Corollary 2.25. *Let $f : X \rightarrow Y$ be a slightly p -continuous, semi-precontinuous, and almost open mapping. If A is rc-Lindelöf (resp., almost rc-Lindelöf) in X , then $f(A)$ is rc-Lindelöf (resp., almost rc-Lindelöf) in Y .*

Proof. Follows from Lemma 2.1(ii) and Corollary 2.24. $\square$

Remark 2.26. Since every dense set is preopen, one easily observes that if f is a slightly p -continuous mapping from a space X onto a space Y , then f maps dense subsets of X onto dense subsets of Y .

Recall that a space X is called submaximal (resp., strongly irresolvable) if every dense subset of X is open (resp., semi-open), or equivalently if, every preopen subset of X is open (resp., semi-open).

The following proposition is a direct consequence of Proposition 2.3.

Proposition 2.27. *Let $f : X \rightarrow Y$ be a mapping from a strongly irresolvable space X into a space Y . Then the following are equivalent:*

- (i) f is slightly p -continuous;
- (ii) f is slightly continuous.

3. Almost p -open mappings

Definition 3.1. A mapping f from a space X into a space Y is said to be semi-regular open (resp., semi- p -regular open) if it maps regular open subsets onto semi-closed (resp., semi-preclosed) subsets.

Remark 3.2. Since every regular open set is semi-closed and every semi-closed set is semi-preclosed, it is obvious that every regular open mapping is semi-regular open and every semi-regular open mapping is semi- p -regular open. The converses are, however, not true as the following examples show.

Example 3.3. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}, \{b\}\}$, and $\tau^* = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then the identity mapping f from (X, τ) onto (X, τ^*) is semi-regular open (observe that the regular open subsets of (X, τ) are the members of τ , each of which is semi-closed in (X, τ^*)); it is, however, not regular open since $\{a, c\}$ is not regular open in (X, τ^*) .

Example 3.4. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, c\}, \{b\}\}$, and $\tau^* = \{X, \phi, \{a, b\}, \{c\}\}$. Then the identity mapping f from (X, τ) onto (X, τ^*) is semi- p -regular open (observe that $\{a, c\}$ and $\{b\}$ are preopen and thus semi-preopen in (X, τ^*)); it is, however, not semi-regular open since $\{a, c\}$ is not semi-closed in (X, τ^*) .

Definition 3.5. A mapping f from a space X into a space Y is said to be almost p -open if $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is preopen in Y .

Proposition 3.6. For a mapping $f : X \rightarrow Y$, the following are equivalent:

- (i) f is almost open;
- (ii) $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is semi-open in Y .

Proof. Since every open set is semi-open, it suffices to show that (i) $\rightarrow$ (ii). Let U be a semi-open subset of Y . Then there exists an open subset V of Y such that $V \subset U \subset \overline{V}$. Thus by (i), $f^{-1}(\overline{U}) = f^{-1}(\overline{V}) \subset \overline{f^{-1}(V)} \subset \overline{f^{-1}(U)}$. $\square$

Proposition 3.7. For a mapping $f : X \rightarrow Y$, the following are equivalent:

- (i) f is almost p -open;
- (ii) $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is semi-preopen in Y ;
- (iii) $f^{-1}(\overline{U}) \subset \overline{f^{-1}(U)}$ whenever U is b -open in Y .

Proof. (i) $\rightarrow$ (ii): Let U be a semi-preopen subset of Y . Then there exists a preopen subset V of Y such that $V \subset U \subset \overline{V}$. Thus by (i), $f^{-1}(\overline{U}) = f^{-1}(\overline{V}) \subset \overline{f^{-1}(V)} \subset \overline{f^{-1}(U)}$.

(ii) $\rightarrow$ (iii) $\rightarrow$ (i): follow since every preopen set is b -open and every b -open set is semi-preopen. $\square$

Remark 3.8. Since every open set is preopen, it is obvious that every almost p -open mapping is almost open. However, the converse is not true as the following example tells.

Example 3.9. Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$, and $\tau^* = \{X, \phi, \{a, b\}, \{c\}\}$. Then the identity mapping f from (X, τ) onto (X, τ^*) is almost open and even regular open (observe that the regular open subsets of (X, τ) are X and ϕ); it is, however, not almost p -open since $\{b, c\}$ is dense and thus preopen in (X, τ^*) but not dense in (X, τ) .

Proposition 3.10. For an almost p -open mapping $f : X \rightarrow Y$, the following are equivalent:

- (i) f is semi- p -regular open;
- (ii) semi-regular open;
- (iii) regular open.

Proof. (i) $\rightarrow$ (iii): Let A be a regular open subset of X . By assumption, $f(A)$ is semi-preclosed, that is, $Y \setminus f(A)$ is semi-preopen. By Proposition 3.7, $f^{-1}(\overline{Y \setminus f(A)}) \subset \overline{f^{-1}(Y \setminus f(A))} \subset \overline{X \setminus A} = X \setminus A$. Thus $f^{-1}(Y \setminus f(A)) \cap A = \emptyset$ and, therefore, $(Y \setminus f(A)) \cap f(A) = \emptyset$, that is, $f(A) \subset \text{Int } f(A)$, that is, $f(A)$ is open, but $f(A)$ is semi-preclosed, so $f(A)$ is regular open.

(iii) $\rightarrow$ (ii) $\rightarrow$ (i): follow since every regular open mapping is semi-regular open and every semi-regular open mapping is semi- p -regular open. $\square$

Proposition 3.11. *For an almost open mapping $f : X \rightarrow Y$, the following are equivalent:*

- (i) *semi-regular open;*
- (ii) *regular open.*

Proof. Since every regular open mapping is semi-regular open, it suffices to show that (i) $\rightarrow$ (ii). Let A be a regular open subset of X . By assumption, $f(A)$ is semi-closed, that is, $Y \setminus f(A)$ is semi-open. By Proposition 3.6, $f^{-1}(\overline{Y \setminus f(A)}) \subset \overline{f^{-1}(Y \setminus f(A))} \subset \overline{X \setminus A} = X \setminus A$. Thus $f^{-1}(Y \setminus f(A)) \cap A = \emptyset$ and, therefore, $(Y \setminus f(A)) \cap f(A) = \emptyset$, that is, $f(A) \subset \text{Int } f(A)$, that is, $f(A)$ is open, but $f(A)$ is semi-closed, so $f(A)$ is regular open. $\square$

Proposition 3.12 (see [19]). *Let f be an almost open and regular open mapping from a space X onto a space Y . Then the following hold.*

- (i) *If for each $y \in Y$, $f^{-1}(y)$ is an S -set in X , then $f^{-1}(A)$ is almost rc -Lindelöf in X whenever A is almost rc -Lindelöf in Y .*
- (ii) *If for each $y \in Y$, $f^{-1}(y)$ is rc -Lindelöf in X , then $f^{-1}(A)$ is rc -Lindelöf in X whenever A is almost rc -Lindelöf in Y provided that X is a weak P -space.*

Corollary 3.13. *Let f be an almost p -open and semi- p -regular open mapping from a space X onto a space Y . Then the following hold.*

- (i) *If for each $y \in Y$, $f^{-1}(y)$ is an S -set in X , then $f^{-1}(A)$ is almost rc -Lindelöf in X whenever A is almost rc -Lindelöf in Y .*
- (ii) *If for each $y \in Y$, $f^{-1}(y)$ is rc -Lindelöf in X , then $f^{-1}(A)$ is rc -Lindelöf in X whenever A is almost rc -Lindelöf in Y provided that X is a weak P -space.*

Proof. We observe from Proposition 3.10 that a mapping that is both almost p -open and semi- p -regular open is both almost open and regular open (the converse is not true as Example 3.9 tells). Thus the result follows from Proposition 3.12. $\square$

Remark 3.14. Since every dense set is preopen, one easily observes that if f is an almost p -open mapping from a space X into a space Y , then the inverse image of a dense subset of Y is a dense subset of X .

The following proposition is a direct consequence of Proposition 3.6.

Proposition 3.15. *Let $f : X \rightarrow Y$ be a mapping from a space X into a strongly irresolvable space Y . Then the following are equivalent:*

- (i) *f is almost p -open;*
- (ii) *f is almost open.*

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