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Research Article

Fine Structure Constant, Domain Walls, and Generalized Uncertainty Principle in the Universe

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We study the corrections to the fine structure constant from the generalized uncertainty principle in the spacetime of a domain wall. We also calculate the corrections to the standard formula to the energy of the electron in the hydrogen atom to the ground state, in the case of spacetime of a domain wall and generalized uncertainty principle. The results generalize the cases known in literature.

1. Introduction

In the last years, there has been an interest in cosmology with a space-time variation of the constants of nature. In 1920, in order to explain the relativistic splits of the atomic spectral lines, Arnold Sommerfeld introduced the fine structure constant

$$\alpha_0 = \frac{e^2}{4\pi\epsilon_0\hbar\, c}, \tag{1.1}$$

where c is the speed of light in vacuum, $\hbar = h/2\pi$ is the reduced Planck constant, e is the electron charge magnitude, and ϵ_0 is the permittivity of free space, all quantities were measured in the laboratories on Earth. The numerical value of the constant is $\alpha_0 \sim 1/137.035999710$ [1] that can be determined without any reference to a specific system of the units, and α gives the strength of the electromagnetic interaction. In the recent years, possible variations of the fine structure constant have been observed; these observations suggest that about 10^{10} years ago α was smaller than today. On the other hand time variation of fundamental constants has been an intriguing field of theoretical research since it was proposed by Dirac in 1937 [2–5] where in large numbers of hypotheses he conjectured that the fundamental constants are functions of the epoch. The physical motivation to search a time or a space dependence on fundamental constants originates because the effort to unify the fundamental constant implies variations of the coupling constants [6]. Let us introduce $\alpha(z)$, that is, the value that might be dependent on the time. The variations of α can be measured by the so-called "time shift density parameter"

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha(z) - \alpha_0}{\alpha_0}, \tag{1.2}$$

with α_0 value of α today.

From an experimental point of view, there are two ways to test the validity of the "constant" hypothesis of α: local and astronomical methods. The former connected with local geophysical data, the natural reactor 1.8×10^9 years ago ($z \sim 0.16$) in Oklo [7–9], these data give [10] $|\dot{\alpha}/\alpha| = (0.4 \pm 0.5) \times 10^{-17} yr^{-1}$ (or $|\Delta\alpha/\alpha| \leq 2 \times 10^{-8}$), that is, one of the most stringent constrains on the variation of α over cosmological time scales. The latter methods consider deep-space astronomical observations; they mainly consider the analysis of spectra from high red-shift quasar absorption system. Evidence of time variation of α is derived from these data [11–17]. It is important to say that these data, coming from the Keck telescope in the Northern hemisphere, give a range of the red-shift $0.2 < z < 4.2$ [18]: $\Delta\alpha/\alpha = (-0.543 \pm 0.116) \times 10^{-5}$. If we assume a linear increase of α with time, we have a drift rate $d \ \ln \alpha/dt = (6.40 \pm 1.35) \times 10^{-16}$ per year. In any case, $\Delta\alpha/\alpha$ may be more complex [19, 20], and a linear extrapolation may not be valid when we consider a cosmic time scale. However, independent analysis of the same phenomena with VLT telescope, in Chile, does not find any variation of α [21–23]; in fact, we find $\Delta\alpha/\alpha = (-0.06 \pm 0.06) \times 10^{-5}$. There is an intensive debate in literature about possible reasons for disagreement, for example, a possible reason may be that the Keck telescope is in the Northern hemisphere and VLT telescope is in the Southern hemisphere. Recently [24, 25], a reanalysis of [21–23] varying α by means of the multiple heavy element transition on the Southern hemisphere has been reported, obtaining $\Delta\alpha/\alpha = (-0.64 \pm 0.36) \times 10^{-5}$. On the other hand, this search may be connected to astronomical observations for variations in the fundamental constants in quasar absorption spectra and in laboratory [26].

The experimental physics has reached very high precision, therefore, in order to search corrections very fine to our theories in the description of the nature, it is necessary to introduce logical systems more and more sophisticated. In this context, to search corrections to the fine structure constant, it is only possible if we study very complex fields of knowledge. The conceptual utilization of the GUP may be useful for calculating the corrections to the fine structure constant. The paper follows this line in which we want to build a bridge between corrections to the alpha and GUP. On the other hand if we consider a cosmological ambit these corrections may have important consequences, if we also consider a topological defect has a domain wall on large scale in the universe. For these reasons, it is important to employ GUP and α evolution.

The search for a quantum theory of the gravitation is one of the most intriguing problems in physics. The generalized uncertainty principle is a consequence of incorporating a minimal length from a theory of quantum gravity. When we consider a quantum gravity theory, we need a fundamental distance scale of the order of the Planck length l_p. These reasonings induce the possibility to have corrections to the Heinsenberg principle in order to have a more general uncertainty principle (GUP). Thus, the Heinsenberg principle

$$\Delta x \Delta p \gtrsim \hbar \tag{1.3}$$

has to be replaced by

$$\Delta x \Delta p \gtrsim \hbar + \beta l_p^2 \frac{\Delta p^2}{\hbar}. \tag{1.4}$$

Here, Δx and Δp are the position and momentum uncertainty for a quantum particle, β is a positive dimensionless coefficient that may depend on the position x and momentum p, usually assumed to be of order one, and $l_p = (G\hbar/c^3)^{1/2} \sim 1.66 \times 10^{-33}$ cm is the Planck length. It is important to stress that $l_p^2\hbar$ may be replaced with the Newtonian constant G; therefore, the second term in (1.4) is a consequence of gravity. The physical reason considers that the quantum mechanics limits the accuracy of the position and momentum of the particle by the well-known rule $\Delta x \geq \hbar/\Delta p$; moreover, if we consider general relativity, the energy cannot be localized in a region smaller than the one defined by its gravitational radius, $\Delta x \geq l_{Pl}^2 \Delta p$. If we combine the results, there is a minimum observable length $\Delta x \geq \max(1/\Delta p; l_p^2 \Delta p) \geq l_p$. This final result is the generalized uncertainty principle, that can be summarized as in (1.4).

Generally speaking, the GUP is obtained when the Heinsenberg uncertainty principle is considered combining both quantum theory and gravity, and it may be obtained from different fields and frameworks as strings [27–34], black holes [35], and gravitation [36, 37], where the gravitational interaction between the photon and the particle modifies the Heinsenberg principle, adding an additional term in (1.4) proportional to the square of the Planck length l_p. From a physical point of view very, interesting consequences can be found in [38–48].

The initial stages of the primordial universe according to the standard model of the particles physics are often described as the era of the phase transition. In the recent years, the cosmological consequence of primordial phase transitions has been the subject of many studies in the early universe. When we have a cosmological phase transition, topological defects necessarily can be formed [49–51]; they are domain walls, cosmic strings, or monopoles. These phenomena are expected to be produced at a phase transition in various area of physics, for example, also in condensed matter physics several examples have been observed, while until today in particle physics, astrophysics, and cosmology it is not the case; on the other hand they could have very important cosmological consequences. Generally people study cosmic strings because they present interesting properties and there are not any bad cosmological consequences, instead domain walls scenarios have attracted less attention since there is the so-called Zeldovich bound [52], in which a linear scaling regime would dominate the energy density of the universe violating the observed isotropy and homogeneity. A domain wall network was proposed to explain dark matter and dark energy [53–67].

The connection between topological defects and variation of the fundamental constants is an intriguing field of work. The corrections to the fine structure constant have been calculated in the spacetime of a cosmic strings [68–70]. A recent paper [71] has studied the correlation of time variation of the fine structure constant in the spacetime of a domain wall and in particular it has been shown that the gravitational field generated by a domain wall acts as a medium with spacetime-dependent permittivity ϵ. In this way, the fine structure constant will depend on a time-dependent function at a fixed point. A further step has been obtained with the calculation of the corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle [72–75]. In this paper, we study the corrections to the fine structure constant in the spacetime of a cosmic domain wall taking into account the generalized uncertainty principls which are calculalated.

In other terms we generalize our previous study [71]. The paper is organized as follows: in Section 2, we summarize our previous results obtained considering the time variation of the fine structure constant in the spacetime of a cosmic domain wall, in Section 3, we generalized the results taking into account the generalized uncertainty principle, in Section 4, we calculate, as application, the correction to the energy ground state of the hydrogen atom, and the results are summarized in the concluding Section 5.

2. α in the Spacetime of a Domain Wall

As it is well known, a domain wall is a topologically stable kink produced when a vacuum manifold of a spontaneously broken gauge theory is disconnected [50, 51]. A very important concept regards the surface energy density σ of a domain wall because it determines the dynamics and gravitational properties, but unfortunately σ is very large, and this implies that cosmic domain walls would have an enormous impact on the homogeneity of the universe. It is possible to have constraint on the wall tension σ from the isotropy of the cosmic microwave background; in fact, if a few walls stretch across the present horizon, we have an anisotropy fluctuation temperature of CMB $\delta T/T \sim 2\pi G\sigma H_O^{-1}$ with G Newton's constant and H_0 Hubble constant. The anisotropy $\delta T/T \leq 3 \times 10^{-5}$ arises from WMAP, therefore, it is not possible to have topologically stable cosmic walls with $\sigma \geq 1\, Mev^3$.

A cosmic domain wall in the universe modifies the electromagnetic properties of the free space and in particular if we consider the gravitational field generated by a wall, it acts as a medium with space- and time-dependent permittivity. Therefore, (1.1) implies that the fine structure constant at fixed point will be a time-dependent function. In this section, we follow the way of [71].

Let us consider the line element associated to the spacetime of a thin wall [76]

$$ds^2 = e^{-4\pi G\sigma|x|}\left(c^2 dt^2 - dx^2\right) - e^{4\pi G\sigma(ct-|x|)}\left(dy^2 + dz^2\right), \tag{2.1}$$

in which we have considered a model with infinitely static domain walls in the zy-plane. Generally speaking, in a curved spacetime, the electromagnetic field tensor $F_{\mu\nu}$ has electric and magnetic fields, respectively, defined as

$$E_i = F_{0i}, \qquad B^i = -\frac{1}{2\sqrt{\gamma}}\epsilon^{ijk}F_{jk}, \tag{2.2}$$

with $\gamma = \det\|\gamma_{ij}\|$ determinant of the spatial metric and ϵ^{ijk} Levi-Civita symbol. If we consider a charged particle q, the charge density at rest in $\mathbf{x} = \mathbf{x_0}$ is

$$\rho = \frac{q}{\sqrt{\gamma}}\delta(\mathbf{x} - \mathbf{x_0}). \tag{2.3}$$

We write the divergence and curl operators in curved spacetime as

$$\operatorname{div}\, \mathbf{v} = \frac{\partial_i\left(\sqrt{\gamma} v^i\right)}{\sqrt{\gamma}}, \tag{2.4}$$

$$(\operatorname{curl}\, \mathbf{v})^i = \frac{\epsilon^{ijk}\left(\partial_j v_k - \partial_k v_j\right)}{2\sqrt{\gamma}}, \tag{2.5}$$

respectively; therefore, Maxwell's equation in three dimensions is

$$\operatorname{div}\, \mathbf{B} = 0, \qquad \operatorname{curl}\, \mathbf{E} = -\frac{1}{\sqrt{\gamma}} \frac{\partial\left(\sqrt{\gamma}\mathbf{B}\right)}{\partial t}, \tag{2.6}$$

$$\operatorname{div}\, \mathbf{D} = 4\pi\rho, \qquad \operatorname{curl}\, \mathbf{H} = \frac{1}{\sqrt{\gamma}} \frac{\partial\left(\sqrt{\gamma}\mathbf{D}\right)}{\partial t}, \tag{2.7}$$

where

$$\mathbf{D} = \frac{\mathbf{E}}{\sqrt{g_{00}}}, \qquad \mathbf{H} = \sqrt{g_{00}}\mathbf{B}. \tag{2.8}$$

If we indicate with ∇ the three-dimensional nabla operator in Euclidean space, we can rewrite the first equation of (2.7) as

$$\nabla \cdot (\epsilon \mathbf{E}) = 4\pi q \delta(\mathbf{x} - \mathbf{x_0}), \tag{2.9}$$

where $\epsilon = \sqrt{\gamma}/\sqrt{g_{00}}$. The solution of Poisson equation, (2.9), is $\epsilon\mathbf{E} = q/4\pi\epsilon r^3$ that gives for the electric field the expression

$$\mathbf{E} = \frac{q}{4\pi\epsilon r^3}\mathbf{r}. \tag{2.10}$$

It is interesting to note that if we consider the metric (2.1), a domain wall produces a gravitational field that acts as a medium with a permittivity ϵ that has the expression

$$\epsilon = \epsilon_0 e^{4\pi G\sigma(ct - |x|)}. \tag{2.11}$$

Therefore, a cosmic domain wall in the universe modifies the electromagnetic properties of the free space, and taking into account (2.11), we can say that in the free space the constant α is given by (1.1), and in the spacetime of a domain wall it is

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar\, c}, \tag{2.12}$$

that is to say, the fine structure constant in the spacetime of a domain wall is spacetime dependent.

3. α in the Spacetime of a Domain Wall from the Generalized Uncertainty Principle

Now, we calculate the corrections to the fine structure constant in the spacetime of a domain wall taking into account the generalized uncertainty principle. If we take into account the gravitational interactions, the Heinsenberg principle must be revised with the generalized uncertainty principle, that is to say, $\Delta x \Delta p \geq \hbar$ becomes $\Delta x \Delta p \gtrsim \hbar + \beta l_{Pl}^2 (\Delta p/\hbar)^2$; this is suggestd to introduce a kind of "effective" Planck constant, h_{eff}, due to the generalized uncertainty principle, defined as

$$\hbar_{\text{eff}} = \hbar \left[1 + \beta l_{Pl}^2 \left(\frac{\Delta p}{\hbar} \right)^2 \right], \tag{3.1}$$

in order to write $\Delta x \Delta p \geq \hbar_{\text{eff}}$. Therefore, the constant will be

$$\alpha_{\text{eff}} = \frac{e^2}{4\pi \epsilon \hbar_{\text{eff}}}, \tag{3.2}$$

with ϵ given by (2.11). In this way, the GUP is able to introduce "itself" in the expression and change the structure of α.

In order to obtain α_{eff}, let us consider (1.4) that we solve as a second-order equation for the momentum uncertainty in terms of the distance uncertainty, then we have

$$\frac{\Delta p}{\hbar} = \frac{\Delta x}{2\beta l_{Pl}^2} \left[1 - \sqrt{1 - \frac{4\beta l_{Pl}^2}{(\Delta x)^2}} \right] \tag{3.3}$$

(we do not consider the sign + in the parenthesis because it is nonphysical; in fact, if we impose correct classical limit $l_{pl} \to 0$, we only have minus sign).

We obtain Δx considering Bohr's radius in the spacetime of a domain wall. In the absence of a domain wall, a Bohr's atom has the radius ($n = 1$) $r_0 = 4\pi \epsilon_0 \hbar^2 / m e^2$, with m mass of the electron, but in, presence of a domain wall and the GUP, it becomes

$$\tilde{r}_0 = \frac{4\pi \epsilon \hbar^2}{m e^2} \equiv \Delta x. \tag{3.4}$$

In other terms, Bohr's radius in a spacetime of a domain wall, $\tilde{r}_0$, is connected with r_0 classical Bohr's radius by the relation

$$\tilde{r}_0 = r_0 e^{4\pi G \sigma (ct - |x|)}. \tag{3.5}$$

Now, introducing (3.3) in (3.1), we obtain h_{eff} as a function of Δx. This h_{eff} introduced in (3.2), finally gives the fine structure constant in the spacetime of a domain wall with the generalized uncertainty principle:

$$\alpha_{\text{eff}} = \frac{e^2}{4\pi\epsilon c\hbar}\left[1 + \frac{(\Delta x)^2}{4\beta l_{Pl}^2}\left(1 - \sqrt{1 - \frac{4\beta l_{Pl}^2}{(\Delta x)^2}}\right)^2\right]^{-1}. \tag{3.6}$$

We discuss (3.6) starting from the case without the spacetime of a domain wall, in other terms, α with the generalized uncertainty principle. There are several studies [77–79] that consider noncommutativity spacetime and quantum gravitational effects in the calculation of the fine structure constant with Δx given by (3.4). If we only consider the GUP effect on the fine structure constant, we have

$$\alpha_{\text{gup}} \simeq \alpha_0\left[1 - 3.6 \times 10^{-50}\right], \tag{3.7}$$

but in presence of the cosmic domain wall, it is possible to render explicit the expression of α,

$$\alpha_{\text{eff}} = \alpha_0 e^{-4\pi G\sigma(ct-|x|)}\left[1 + \frac{r_0^2}{4l_{Pl}^2}e^{8\pi G\sigma(ct-|x|)}\left(1 - \sqrt{1 - \frac{4l_{Pl}^2}{r_0^2}e^{-8\pi G\sigma(ct-|x|)}}\right)^2\right]^{-1}. \tag{3.8}$$

4. Corrections to the Energy Ground State of Hydrogen Atom

It is interesting to calculate the corrections to the energy ground state E_0 of the hydrogen atom in presence of a domain wall and considering the GUP. Classically, the hydrogen atom decays, and it is just the Heinsenberg uncertainty principle that assures the stability. The energy of the electron in the hydrogen atom is

$$E_{\text{gup}}^{dw} \sim \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon\tilde{r}_0}. \tag{4.1}$$

The GUP gives

$$\Delta p \geq \frac{\hbar}{\Delta x} + \frac{l_{Pl}^2(\Delta p)^2}{\Delta x\hbar}. \tag{4.2}$$

Now, let us iterate (4.2), neglecting the terms $O(l_{Pl}^2)$ and squaring both members, we have

$$p^2 \geq (\Delta p)^2 \geq \frac{\hbar^2}{(\Delta x)^2} + 2\frac{\hbar^2 l_{Pl}^2}{(\Delta x)^4}. \tag{4.3}$$

Therefore, (4.1) for the energy becomes

$$E_{\mathrm{gup}}^{dw} = \frac{\hbar^2}{2m\tilde{r}_0^2} - \frac{e^2}{4\pi\epsilon\tilde{r}_0} + \frac{\hbar^2 l_{Pl}^2}{m\tilde{r}_0^4}. \tag{4.4}$$

From a physical point of view, (4.4) is very interesting. If we "switch off" the domain wall contribution, the first two terms on the second member are the energy of the ground state of the electron in the hydrogen atom, $E_0 = -me^2/8\pi^2\epsilon_0^2\hbar^2 = 13.6\,\mathrm{eV}$. The third term is the correction to the ground state energy due to the generalized uncertainty principle, that is to say,

$$\Delta E_{\mathrm{gup}} = \frac{m^3 l_{Pl}^2 e^8}{(4\pi\epsilon_0)^4\hbar^6} \sim 10^{-48}\,\mathrm{eV}. \tag{4.5}$$

This corrective term, due to the GUP, is very little to be experimentally tested actually. If now we "switch on" the domain wall contribution, we have

$$E = -\frac{me^4}{8\pi^2\epsilon_0\hbar^2}e^{-8\pi G\sigma(ct-|x|)} + \frac{m^3e^8l_{Pl}^2}{(4\pi\epsilon)^4\hbar^6}e^{-16\pi G\sigma(ct-|x|)}. \tag{4.6}$$

In other terms, when we consider the domain wall, the classical and the GUP contributions to the energy are exponentially modulated; therefore, an integrate effect, starting from the early universe, may be relevant into the amplification to the correction to the energy of the electron in a hydrogen atom from an experimental point of view.

5. Conclusion

In conclusion, if we consider that the gravitational interactions may modify the Heinsenberg principle with the so-called generalized uncertainty principle and if we also consider that the fine structure constant may be different in different epochs, it is possible to study the right expression of the fine structure constant in the spacetime of a domain wall, taking into account the generalized uncertainty principle. In this paper, we have examined the effects of these two contributions on α. We have found the most general expression given by (3.8). The modification of α involves two aspects, the domain wall's contribution influences the value of ϵ_0 that becomes ϵ given by (2.11), while the GUP's contribution acts in order to modify the Planck constant $\hbar$ into $\hbar_{\mathrm{eff}}$ given by (3.1). α is very near to α_0 as we can see in (3.7); this means that the GUP does not change the numerical value in an appreciable way. The domain wall's contribution consists in exponentially modulating the α_0 value, and from a numerical point of view, if we set $ct - |x| = H_0^{-1}$, we does not change the value of α. On the other hand it is possible to think of it as a kind of "integrate effect" in the spacetime; in this way, it is possible to have a different evolution of α in the spacetime. These arguments are also very interesting because recently a sample of 153 measurements from the ESO very large telescope indicate that α appears on average to be larger than in the past [80]. Moreover, manifestations of a spatial variation in α must be independently confirmed by means of terrestrial measurements as laboratory, meteorite data, and nuclear reactor [81] and by means of a new test connected with big bang nucleosynthesis [82]. For completeness, we have also studied the corrections to the energy of the hydrogen atom if we add both the actions: GUP and domain wall. Also in this case, the corrections are still too small for the actual experiments. Future investigations are in progress by the author.

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