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Research Article

Some Coupled Fixed Point Results on Partial Metric Spaces

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We give some coupled fixed point results for mappings satisfying different contractive conditions on complete partial metric spaces.

1. Introduction and Preliminaries

For a given partially ordered set X, Bhaskar and Lakshmikantham in [1] introduced the concept of coupled fixed point of a mapping $F : X \times X \to X$. Later in [2], Círíc and Lakshmikantham investigated some more coupled fixed point theorems in partially ordered sets. The following is the corresponding definition of a coupled fixed point.

Definition 1.1 (see [3]). An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Sabetghadam et al. [4] obtained the following.

Theorem 1.2. *Let (X, d) be a complete cone metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$d(F(x, y), F(u, v)) \leq kd(x, u) + ld(y, v), \tag{1.1}$$

where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

In this paper, we give the analogous of this result (and some others in [4]) on partial metric spaces, and we establish some coupled fixed point results.

The concept of partial metric space (X,p) was introduced by Matthews in 1994. In such spaces, the distance of a point in the self may not be zero. First, we start with some preliminaries definitions on the partial metric spaces [3, 5–13].

Definition 1.3 (see ([6–8])). A partial metric on a nonempty set X is a function $p : X\times X \longrightarrow \mathbb{R}^+$ such that for all $x,y,z \in X$:

(p1) $x = y \Longleftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p2) $p(x,x) \leq p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Remark 1.4. It is clear that if $p(x,y) = 0$, then from (p1), (p2), and (p3), $x = y$. But if $x = y$, $p(x,y)$ may not be 0.

If p is a partial metric on X, then the function $p^s : X \times X \longrightarrow R_+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y), \tag{1.2}$$

is a metric on X.

Definition 1.5 (see ([6–8])). Let (X,p) be a partial metric space. Then,

(i) a sequence $\{x_n\}$ in a partial metric space (X,p) converges to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to+\infty} p(x,x_n)$;

(ii) a sequence $\{x_n\}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to+\infty} p(x_n,x_m)$;

(iii) a partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$, that is, $p(x,x) = \lim_{n,m\to+\infty} p(x_n,x_m)$.

Lemma 1.6 (see ([6, 7, 9])). *Let (X,p) be a partial metric space;*

(a) *$\{x_n\}$ is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s),*

(b) *a partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete; furthermore, $\lim_{n\to+\infty} p^s(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to+\infty} p(x_n,x) = \lim_{n,m\to+\infty} p(x_n,x_m). \tag{1.3}$$

2. Main Results

Our first main result is the following.

Theorem 2.1. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$p(F(x,y), F(u,v)) \leq kp(x,u) + lp(y,v), \tag{2.1}$$

where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Proof. Choose $x_0, y_0 \in X$ and set $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Repeating this process, set $x_{n+1} = F(x_n, y_n)$ and $y_{n+1} = F(y_n, x_n)$. Then, by (2.1), we have

$$\begin{aligned} p(x_n, x_{n+1}) &= p(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\ &\leq kp(x_{n-1}, x_n) + lp(y_{n-1}, y_n), \end{aligned} \tag{2.2}$$

and similarly

$$\begin{aligned} p(y_n, y_{n+1}) &= p(F(y_{n-1}, x_{n-1}), F(y_n, x_n)) \\ &\leq kp(y_{n-1}, y_n) + lp(x_{n-1}, x_n). \end{aligned} \tag{2.3}$$

Therefore, by letting

$$d_n = p(x_n, x_{n+1}) + p(y_n, y_{n+1}), \tag{2.4}$$

we have

$$\begin{aligned} d_n &= p(x_n, x_{n+1}) + p(y_n, y_{n+1}) \\ &\leq kp(x_{n-1}, x_n) + lp(y_{n-1}, y_n) + kp(y_{n-1}, y_n) + lp(x_{n-1}, x_n) \\ &= (k+l)\left[p(y_{n-1}, y_n) + p(x_{n-1}, x_n)\right] \\ &= (k+l)d_{n-1}. \end{aligned} \tag{2.5}$$

Consequently, if we set $\delta = k + l$, then, for each $n \in \mathbb{N}$, we have

$$d_n \leq \delta d_{n-1} \leq \delta^2 d_{n-2} \leq \cdots \leq \delta^n d_0. \tag{2.6}$$

If $d_0 = 0$ then $p(x_0, x_1) + p(y_0, y_1) = 0$. Hence, from Remark 1.4, we get $x_0 = x_1 = F(x_0, y_0)$ and $y_0 = y_1 = F(y_0, x_0)$, meaning that (x_0, y_0) is a coupled fixed point of F. Now, let $d_0 > 0$. For each $n \geq m$, we have, in view of the condition (p4)

$$\begin{aligned} p(x_n, x_m) &\leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) - p(x_{n-1}, x_{n-1}) \\ &\quad + p(x_{n-2}, x_{n-3}) + p(x_{n-3}, x_{n-4}) - p(x_{n-3}, x_{n-3}) \\ &\quad + \cdots + p(x_{m+2}, x_{m+1}) + p(x_{m+1}, x_m) - p(x_{m+1}, x_{m+1}) \\ &\leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) + \cdots + p(x_{m+1}, x_m). \end{aligned} \tag{2.7}$$

Similarly, we have

$$p(y_n, y_m) \leq p(y_n, y_{n-1}) + p(y_{n-1}, y_{n-2}) + \cdots + p(y_{m+1}, y_m). \tag{2.8}$$

Thus,

$$\begin{aligned} p(x_n, x_m) + p(y_n, y_m) &\leq d_{n-1} + d_{n-2} + \cdots + d_m \\ &\leq \left(\delta^{n-1} + \delta^{n-2} + \cdots + \delta^m\right) d_0 \\ &\leq \frac{\delta^m}{1-\delta} d_0. \end{aligned} \tag{2.9}$$

By definition of p^s, we have $p^s(x, y) \leq 2p(x, y)$, so, for any $n \geq m$

$$p^s(x_n, x_m) + p^s(y_n, y_m) \leq 2p(x_n, x_m) + 2p(y_n, y_m) \leq 2\frac{\delta^m}{1-\delta} d_0, \tag{2.10}$$

which implies that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in (X, p^s) because of $0 \leq \delta = k + l < 1$. Since the partial metric space (X, p) is complete, hence thanks to Lemma 1.6, the metric space (X, p^s) is complete, so there exist $u^*, v^* \in X$ such that

$$\lim_{n \to +\infty} p^s(x_n, u^*) = \lim_{n \to +\infty} p^s(y_n, v^*) = 0. \tag{2.11}$$

Again, from Lemma 1.6, we get

$$\begin{aligned} p(u^*, u^*) &= \lim_{n \to +\infty} p(x_n, u^*) = \lim_{n \to +\infty} p(x_n, x_n), \\ p(v^*, v^*) &= \lim_{n \to +\infty} p(y_n, v^*) = \lim_{n \to +\infty} p(y_n, y_n). \end{aligned} \tag{2.12}$$

But, from condition (p2) and (2.6),

$$p(x_n, x_n) \leq p(x_n, x_{n+1}) \leq d_n \leq \delta^n d_0, \tag{2.13}$$

so since $\delta \in [0, 1[$, hence letting $n \to +\infty$, we get $\lim_{n \to +\infty} p(x_n, x_n) = 0$. It follows that

$$p(u^*, u^*) = \lim_{n \to +\infty} p(x_n, u^*) = \lim_{n \to +\infty} p(x_n, x_n) = 0. \tag{2.14}$$

Similarly, we get

$$p(v^*, v^*) = \lim_{n \to +\infty} p(y_n, v^*) = \lim_{n \to +\infty} p(y_n, y_n) = 0. \tag{2.15}$$

Therefore, we have, using (2.1),

$$\begin{aligned} p(F(u^*,v^*),u^*) &\le p(F(u^*,v^*),x_{n+1}) + p(x_{n+1},u^*) - p(x_{n+1},x_{n+1}), \quad \text{by (p4)} \\ &\le p\big(F(u^*,v^*),F(x_n,y_n)\big) + p(x_{n+1},u^*) \\ &\le kp(x_n,u^*) + lp(y_n,v^*) + p(x_{n+1},u^*), \end{aligned} \tag{2.16}$$

and letting $n \to +\infty$, then from (2.14) and (2.15), we obtain $p(F(u^*,v^*),u^*)) = 0$, so $F(u^*,v^*) = u^*$. Similarly, we have $F(v^*,u^*) = v^*$, meaning that (u^*,v^*) is a coupled fixed point of F.

Now, if (u',v') is another coupled fixed point of F, then

$$\begin{aligned} p(u',u^*) &= p\big(F(u',v'),F(u^*,v^*)\big) \le kp(u',u^*) + lp(v',v^*), \\ p(v',v^*) &= p\big(F(v',u'),F(v^*,u^*)\big) \le kp(v',v^*) + lp(u',u^*). \end{aligned} \tag{2.17}$$

It follows that

$$p(u',u^*) + p(v',v^*) \le (k+l)\big[p(u',u^*) + p(v',v^*)\big]. \tag{2.18}$$

In view of $k + l < 1$, this implies that $p(u',u^*) + p(v',v^*) = 0$, so $u^* = u'$ and $v^* = v'$. The proof of Theorem 2.1 is completed. □

It is worth noting that when the constants in Theorem 2.1 are equal, we have the following corollary

Corollary 2.2. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$p\big(F(x,y),F(u,v)\big) \le \frac{k}{2}\big(p(x,u) + p(y,v)\big), \tag{2.19}$$

where $0 \le k < 1$. Then, F has a unique coupled fixed point.

Example 2.3. Let $X = [0,+\infty[$ endowed with the usual partial metric p defined by $p : X \times X \to [0,+\infty[$ with $p(x,y) = \max\{x,y\}$. The partial metric space (X,p) is complete because (X,p^s) is complete. Indeed, for any $x, y \in X$,

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) = 2\max\{x,y\} - (x+y) = |x-y|, \tag{2.20}$$

Thus, (X,p^s) is the Euclidean metric space which is complete. Consider the mapping $F : X \times X \to X$ defined by $F(x,y) = (x+y)/6$. For any $x, y, u, v \in X$, we have

$$\begin{aligned} p\big(F(x,y),F(u,v)\big) &= \frac{1}{6}\max\{x+y,u+v\} \le \frac{1}{6}\big[\max\{x,u\} + \max\{y,v\}\big] \\ &= \frac{1}{6}\big[p(x,u) + p(y,v)\big]. \end{aligned} \tag{2.21}$$

which is the contractive condition (2.19) for $k = 1/3$. Therefore, by Corollary 2.2, F has a unique coupled fixed point, which is $(0,0)$. Note that if the mapping $F : X \times X \to X$ is given by $F(x,y) = (x+y)/2$, then F satisfies the contractive condition (2.19) for $k = 1$, that is,

$$\begin{aligned} p(F(x,y),F(u,v)) &= \frac{1}{2}\max\{x+y,u+v\} \le \frac{1}{2}[\max\{x,u\}+\max\{y,v\}] \\ &= \frac{1}{2}[p(x,u)+p(y,v)]. \end{aligned} \tag{2.22}$$

In this case, $(0,0)$ and $(1,1)$ are both coupled fixed points of F, and, hence, the coupled fixed point of F is not unique. This shows that the condition $k < 1$ in Corollary 2.2, and hence $k + l < 1$ in Theorem 2.1 cannot be omitted in the statement of the aforesaid results.

Theorem 2.4. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x,y,u,v \in X$*

$$p(F(x,y),F(u,v)) \le kp(F(x,y),x) + lp(F(u,v),u), \tag{2.23}$$

where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Proof. We take the same sequences $\{x_n\}$ and $\{y_n\}$ given in the proof of Theorem 2.1 by

$$x_{n+1} = F(x_n,y_n), \quad y_{n+1} = F(y_n,x_n) \quad \text{for any } n \in \mathbb{N}. \tag{2.24}$$

Applying (2.23), we get

$$p(x_n,x_{n+1}) \le \delta p(x_{n-1},x_n) \tag{2.25}$$

$$p(y_n,y_{n+1}) \le \delta p(y_{n-1},y_n) \tag{2.26}$$

where $\delta = k/(1-l)$. By definition of p^s, we have

$$p^s(x_n,x_{n+1}) \le 2p(x_n,x_{n+1}) \le 2\delta^n p(x_1,x_0)\ , \tag{2.27}$$

$$p^s(y_n,y_{n+1}) \le 2p(y_n,y_{n+1}) \le 2\delta^n p(y_1,y_0). \tag{2.28}$$

Since $k+l < 1$, hence $\delta < 1$, so the sequences $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in the metric space (X,p^s). The partial metric space (X,p) is complete, hence from Lemma 1.6, (X,p^s) is complete, so there exist $u^*, v^* \in X$ such that

$$\lim_{n\to+\infty} p^s(x_n,u^*) = \lim_{n\to+\infty} p^s(y_n,v^*) = 0. \tag{2.29}$$

From Lemma 1.6, we get

$$\begin{aligned} p(u^*,u^*) &= \lim_{n\to+\infty} p(x_n,u^*) = \lim_{n\to+\infty} p(x_n,x_n), \\ p(v^*,v^*) &= \lim_{n\to+\infty} p(y_n,v^*) = \lim_{n\to+\infty} p(y_n,y_n). \end{aligned} \tag{2.30}$$

By the condition (p2) and (2.25), we have

$$p(x_n,x_n) \le p(x_n,x_{n+1}) \le \delta^n p(x_1,x_0), \tag{2.31}$$

so $\lim_{n\to+\infty} p(x_n,x_n) = 0$. It follows that

$$p(u^*,u^*) = \lim_{n\to+\infty} p(x_n,u^*) = \lim_{n\to+\infty} p(x_n,x_n) = 0. \tag{2.32}$$

Similarly, we find

$$p(v^*,v^*) = \lim_{n\to+\infty} p(y_n,v^*) = \lim_{n\to+\infty} p(y_n,y_n) = 0. \tag{2.33}$$

Therefore, by (2.23),

$$\begin{aligned} p(F(u^*,v^*),u^*) &\le p(F(u^*,v^*),x_{n+1}) + p(x_{n+1},u^*) \\ &= p\big(F(u^*,v^*),F(x_n,y_n)\big) + p(x_{n+1},u^*) \\ &\le kp(F(u^*,v^*),u^*) + lp\big(F(x_n,y_n),x_n\big) + p(x_{n+1},u^*) \\ &= kp(F(u^*,v^*),u^*) + lp(x_{n+1},x_n) + p(x_{n+1},u^*), \end{aligned} \tag{2.34}$$

and letting $n \to +\infty$, then from (2.27)–(2.32), we obtain

$$p(F(u^*,v^*),u^*) \le kp(F(u^*,v^*),u^*). \tag{2.35}$$

From the preceding inequality, we can deduce a contradiction if we assume that $p(F(u^*,v^*),u^*) \neq 0$, because in that case we conclude that $1 \le k$ and now this inequality is, in fact, a contradiction, so $p(F(u^*,v^*),u^*) = 0$, that is, $F(u^*,v^*) = u^*$. Similarly, we have $F(v^*,u^*) = v^*$, meaning that (u^*,v^*) is a coupled fixed point of F. Now, if (u',v') is another coupled fixed point of F, then, in view of (2.23),

$$\begin{aligned} p(u',u^*) &= p\big(F(u',v'),F(u^*,v^*)\big) \\ &\le kp\big(F(u',v'),u'\big) + lp(F(u^*,v^*),u^*) \\ &= kp(u',u') + lp(u^*,u^*) \\ &\le kp(u',u^*) + lp(u',u^*) = (k+l)p(u',u^*), \quad \text{using (p2)}, \end{aligned} \tag{2.36}$$

that is, $p(u', u^*) = 0$ since $(k+l) < 1$. It follows that $u^* = u'$. Similarly, we can have $v^* = v'$, and the proof of Theorem 2.4 is completed. □

Theorem 2.5. *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$p(F(x,y), F(u,v)) \leq kp(F(x,y), u) + lp(F(u,v), x), \tag{2.37}$$

where k, l are nonnegative constants with $k + 2l < 1$. Then, F has a unique coupled fixed point.

Proof. Since, $k + 2l < 1$, hence $k + l < 1$, and as a consequence the proof of the uniqueness in this theorem is as trivial as in the other results. To prove the existence of the fixed point, choose the sequences $\{x_n\}$ and $\{y_n\}$ like in the proof of Theorem 2.1, that is

$$x_{n+1} = F(x_n, y_n), \quad y_{n+1} = F(y_n, x_n), \quad \text{for any } n \in \mathbb{N}. \tag{2.38}$$

Applying again (2.37), we have

$$\begin{aligned}
p(x_n, x_{n+1}) &= p(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\
&\leq kp(F(x_{n-1}, y_{n-1}), x_n) + lp(F(x_n, y_n), x_{n-1}) \\
&= kp(x_n, x_n) + lp(x_{n+1}, x_{n-1}) \\
&\leq kp(x_{n+1}, x_n) + lp(x_{n+1}, x_{n-1})], \quad \text{by (p2)} \\
&\leq kp(x_{n+1}, x_n) + lp(x_{n+1}, x_n) + lp(x_n, x_{n-1}) - lp(x_n, x_n), \quad \text{using (p4)} \\
&\leq (k+l)p(x_n, x_{n+1}) + lp(x_{n-1}, x_n).
\end{aligned} \tag{2.39}$$

It follows that for any $n \in \mathbb{N}^*$

$$p(x_n, x_{n+1}) \leq \frac{l}{1-l-k} p(x_{n-1}, x_n). \tag{2.40}$$

Let us take $\delta = l/(1-l-k)$. Hence, we deduce

$$p^s(x_n, x_{n+1}) \leq 2p(x_n, x_{n+1}) \leq 2\delta^n p(x_0, x_1). \tag{2.41}$$

Under the condition $0 \leq k + 2l < 1$, we get $0 \leq \delta < 1$. From this fact, we immediately obtain that $\{x_n\}$ is Cauchy in the complete metric space (X, p^s). Of course, similar arguments apply to the case of the sequence $\{y_n\}$ in order to prove that

$$p^s(y_n, y_{n+1}) \leq 2p(y_n, y_{n+1}) \leq 2\delta^n p(y_0, y_1), \tag{2.42}$$

and, thus, that the sequence $\{y_n\}$ is Cauchy in (X, p^s). Therefore, there exist $u^*, v^* \in X$ such that

$$\lim_{n\to+\infty} p^s(x_n, u^*) = \lim_{n\to+\infty} p^s(y_n, v^*) = 0. \tag{2.43}$$

Thanks to Lemma 1.6, we have

$$\begin{aligned} \lim_{n\to+\infty} p(x_n, u^*) &= \lim_{n\to+\infty} p(x_n, x_n) = p(u^*, u^*), \\ \lim_{n\to+\infty} p(y_n, v^*) &= \lim_{n\to+\infty} p(y_n, y_n) = p(v^*, v^*). \end{aligned} \tag{2.44}$$

The condition (p2) together with (2.41) yield that

$$p(x_n, x_n) \le p(x_n, x_{n+1}) \le \delta^n p(x_0, x_1), \tag{2.45}$$

hence letting $n \to +\infty$, we get $\lim_{n\to+\infty} p(x_n, x_n) = 0$. It follows that

$$p(u^*, u^*) = \lim_{n\to+\infty} p(x_n, u^*) = \lim_{n\to+\infty} p(x_n, x_n) = 0. \tag{2.46}$$

Similarly, we have

$$p(v^*, v^*) = \lim_{n\to+\infty} p(y_n, v^*) = \lim_{n\to+\infty} p(y_n, y_n) = 0. \tag{2.47}$$

Therefore, we have, using (2.37),

$$\begin{aligned} p(F(u^*, v^*), u^*) &\le p(F(u^*, v^*), x_{n+1}) + p(x_{n+1}, u^*) \\ &= p(F(u^*, v^*), F(x_n, y_n)) + p(x_{n+1}, u^*) \\ &\le kp(F(u^*, v^*), x_n) + lp(F(x_n, y_n), u^*) + p(x_{n+1}, u^*) \\ &= kp(F(u^*, v^*), x_n) + lp(x_{n+1}, u^*) + p(x_{n+1}, u^*) \\ &\le kp(F(u^*, v^*), u^*) + kp(u^*, x_n) + lp(x_{n+1}, u^*) + p(x_{n+1}, u^*), \quad \text{using (p4).} \end{aligned} \tag{2.48}$$

Letting $n \to +\infty$ yields, using (2.46),

$$p(F(u^*, v^*), u^*) \le kp(F(u^*, v^*), u^*), \tag{2.49}$$

and since $k < 1$, we have $p(F(u^*, v^*), u^*) = 0$, that is, $F(u^*, v^*) = u^*$. Similarly, thanks to (2.47), we get $F(v^*, u^*) = v^*$, and hence (u^*, v^*) is a coupled fixed point of F. □

When the constants in Theorems 2.4 and 2.5 are equal, we get the following corollaries.

Corollary 2.6. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$p(F(x,y), F(u,v)) \leq \frac{k}{2}(p(F(x,y),x) + p(F(u,v),u)), \tag{2.50}$$

where $0 \leq k < 1$. Then, F has a unique coupled fixed point.

Corollary 2.7. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$*

$$p(F(x,y), F(u,v)) \leq \frac{k}{2}(p(F(x,y),u) + p(F(u,v),x)), \tag{2.51}$$

where $0 \leq k < 2/3$. Then, F has a unique coupled fixed point.

Proof. The condition $0 \leq k < 2/3$ follows from the hypothesis on k and l given in Theorem 2.5. □

Remark 2.8. (i) Theorem 2.1 extends the Theorem 2.2 of [4] on the class of partial metric spaces.

(ii) Theorem 2.4 extends the Theorem 2.5 of [4] on the class of partial metric spaces.

Remark 2.9. Note that in Theorem 2.4, if the mapping $F : X \times X \to X$ satisfies the contractive condition (2.23) for all $x, y, u, v \in X$, then F also satisfies the following contractive condition:

$$\begin{aligned} p(F(x,y), F(u,v)) &= p(F(u,v), F(x,y)) \\ &\leq kp(F(u,v),u) + lp(F(x,y),x). \end{aligned} \tag{2.52}$$

Consequently, by adding (2.23) and (2.52), F also satisfies the following:

$$p(F(x,y), F(u,v)) \leq \frac{k+l}{2}p(F(u,v),u) + \frac{k+l}{2}p(F(x,y),x), \tag{2.53}$$

which is a contractive condition of the type (2.50) in Corollary 2.6 with equal constants. Therefore, one can also reduce the proof of general case (2.23) in Theorem 2.4 to the special case of equal constants. A similar argument is valid for the contractive conditions (2.37) in Theorem 2.5 and (2.51) in Corollary 2.7.

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