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Research Article

On Starlike and Convex Functions with Respect to k-Symmetric Points

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We introduce new subclasses $S_k^{\sigma,s}(\lambda,\delta,\phi)$ and $K_k^{\sigma,s}(\lambda,\delta,\phi)$ of analytic functions with respect to k-symmetric points defined by differential operator. Some interesting properties for these classes are obtained.

1. Introduction

Let A denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$.

Also let $\wp$ be the class of analytic functions p with $p(0) = 1$, which are convex and univalent in U and satisfy the following inequality:

$$\Re\{p(z)\} > 0, \quad z \in U. \tag{1.2}$$

A function $f \in A$ is said to be starlike with respect to symmetrical points in U if it satisfies

$$\Re\left\{\frac{zf'(z)}{f(z) - f(-z)}\right\} > 0, \quad z \in U. \tag{1.3}$$

This class was introduced and studied by Sakaguchi in 1959 [1]. Some related classes are studied by Shanmugam et al. [2].

In 1979, Chand and Singh [3] defined the class of starlike functions with respect to k-symmetric points of order α $(0 \le \alpha < 1)$. Related classes are also studied by Das and Singh [4].

Recall that the function F is subordinate to G if there exists a function ω, analytic in U, with $\omega(0) = 0$ and $|\omega(z)| < 1$, such that $F(z) = G(w(z))$, $z \in U$. We denote this subordination by $F(z) \prec G(z)$. If $G(z)$ is univalent in U, then the subordination is equivalent to $F(0) = G(0)$ and $F(U) \subset G(U)$.

A function $f \in A$ is in the class $S_k(\phi)$ satisfying

$$\frac{zf'(z)}{f_k(z)} \prec \phi(z), \quad z \in U, \tag{1.4}$$

where $\phi \in \wp$, k is a fixed positive integer, and $f_k(z)$ is given by the following:

$$\begin{aligned} f_k(z) &= \frac{1}{k}\sum_{\nu=0}^{k-1} \varepsilon^{-\nu} f(\varepsilon^{\nu} z) \\ &= z + \sum_{\iota=2}^{\infty} a_{k(\iota-1)+1} z^{k(\iota-1)+1}, \quad \left(\varepsilon = \exp\left(\frac{2\pi i}{k}\right), z \in U\right). \end{aligned} \tag{1.5}$$

The classes $S_k(\phi)$ of starlike functions with respect to k-symmetric points and $K_k(\phi)$ of convex functions with respect to k-symmetric points were considered recently by Wang et al. [5]. Moreover, the special case

$$\phi(z) = \frac{1+\beta z}{1-\alpha\beta z}, \quad 0 \le \alpha \le 1,\ 0 < \beta \le 1 \tag{1.6}$$

imposes the class $S_k(\alpha, \beta)$, which was studied by Gao and Zhou [6], and the class $S_1(\phi) = S^*(\phi)$ was studied by Ma and Minda [7].

Let two functions given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ and $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$ be analytic in U. Then the Hadamard product (or convolution) $f * g$ of the two functions f, g is defined by

$$f(z) * g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \tag{1.7}$$

and for several function $f_1(z), \ldots, f_m(z) \in A$,

$$f_1(z) * \cdots * f_m(z) = z + \sum_{n=2}^{\infty} (a_{1n} \cdots a_{mn}) z^n, \quad z \in U. \tag{1.8}$$

The theory of differential operators plays important roles in geometric function theory. Perhaps, the earliest study appeared in the year 1900, and since then, many mathematicians have worked extensively in this direction. For recent work see, for example, [8–12].

We now define differential operator as follows:

$$D_{\lambda,\delta}^{\sigma,s} f(z) = z + \sum_{n=2}^{\infty} n^s (C(\delta,n)[1+\lambda(n-1)])^{\sigma} a_n z^n, \tag{1.9}$$

where $\lambda \geq 0$, $C(\delta,n) = (\delta+1)_{n-1}/(n-1)!$, for $\delta,\sigma,s \in N_0 = \{0,1,2,\ldots\}$, and $(x)_n$ is the Pochhammer symbol defined by

$$(x)_n = \frac{\Gamma(x+n)}{\Gamma(x)} = \begin{cases} 1, & n = 0, \\ x(x+1)\cdots(x+n-1), & n = \{1,2,3,\ldots\}. \end{cases} \tag{1.10}$$

Here $D_{\lambda,\delta}^{\sigma,s} f(z)$ can also be written in terms of convolution as

$$\begin{gathered} \varphi(z) = \left[\frac{\lambda z}{(1-z)^2} - \frac{\lambda z}{1-z} + \frac{z}{1-z}\right] * \frac{z}{(1-z)^{\delta+1}}, \quad z \in U, \\ D_{\lambda,\delta}^{\sigma,s} f(z) = \underbrace{\varphi(z) * \cdots * \varphi(z)}_{\sigma\text{-times}} * \sum_{n=1}^{\infty} n^s z^n * f(z) = \underbrace{D_\delta * \cdots * D_\delta}_{\sigma\text{-times}} * D_\lambda^{\sigma,s} f(z), \end{gathered} \tag{1.11}$$

where $D_\delta = z + \sum_{n=2}^{\infty} C(\delta,n) z^n$ and $D_\lambda^{\sigma,s} = z + \sum_{n=2}^{\infty} n^s [1+\lambda(n-1)]^{\sigma} z^n$.

Note that $D_{\lambda,\delta}^{0,1} f(z) = D_{1,0}^{1,0} f(z) = z\, f'(z)$ and $D_{\lambda,\delta}^{0,0} f(z) = f(z)$. When $\sigma = 0$, we get the Sŭlŭgean differential operator [9], when $\lambda = s = 0$, $\sigma = 1$ we obtain the Ruscheweyh operator [8], when $s = 0$, $\sigma = 1$, we obtain the Al-Shaqsi and Darus [11], and when $\delta = s = 0$, we obtain the Al-Oboudi differential operator [10].

In this paper, we introduce new subclasses of analytic functions with respect to k-symmetric points defined by differential operator. Some interesting properties of $S_k^{\sigma,s}(\lambda,\delta,\phi)$ and $K_k^{\sigma,s}(\lambda,\delta,\phi)$ are obtained.

Applying the operator $D_{\lambda,\delta}^{\sigma,s} f(z)$

$$D_{\lambda,\delta}^{\sigma,s} f_k(z) = \frac{1}{k} \sum_{\nu=0}^{k-1} \varepsilon^{-\nu} D_{\lambda,\delta}^{\sigma,s} f(\varepsilon^{\nu} z), \quad \varepsilon^k = 1, \tag{1.12}$$

where k is a fixed positive integer, we now define classes of analytic functions containing the differential operator.

Definition 1.1. Let $S_k^{\sigma,s}(\lambda,\delta,\phi)$ denote the class of functions in A satisfying the condition

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s} f_k(z)} \prec \phi(z), \tag{1.13}$$

where $\phi \in \wp$.

Definition 1.2. Let $K_k^{\sigma,s}(\lambda,\delta,\phi)$ denote the class of functions in A satisfying the condition

$$\frac{\left(z\left(D_{\lambda,\delta}^{\sigma,s}f(z)\right)'\right)'}{\left(D_{\lambda,\delta}^{\sigma,s}f_k(z)\right)'} \prec \phi(z), \tag{1.14}$$

where $\phi \in \wp$.

In order to prove our results, we need the following lemmas.

Lemma 1.3 (see [13]). *Let $c > -1$, and let $I_c : A \to A$ be the integral operator defined by $F = I_c(f)$, where*

$$F(z) = \frac{c+1}{z^c}\int_0^z t^{c-1}f(t)dt. \tag{1.15}$$

Let ϕ be a convex function, with $\phi(0) = 1$ and $\Re\{\phi(z) + c\} > 0$ in U. If $f \in A$ and $zf'(z)/f(z) \prec \phi(z)$, then $zF'(z)/F(z) \prec q(z) \prec \phi(z)$, where q is univalent and satisfies the differential equation

$$q(z) + \frac{zq'(z)}{q(z)+c} = \phi(z). \tag{1.16}$$

Lemma 1.4 (see [14]). *Let κ, υ be complex numbers. Let ϕ be convex univalent in U with $\phi(0) = 1$ and $\Re[\kappa\phi + \upsilon] > 0$, $z \in U$, and let $q(z) \in A$ with $q(0) = 1$ and $q(z) \prec \phi(z)$. If $p(z) = 1 + p_1 z + p_2 z^2 + \cdots \in \wp$ with $p(0) = 1$, then*

$$p(z) + \frac{zp'(z)}{\kappa q(z) + \upsilon} \prec \phi(z) \Longrightarrow p(z) \prec \phi(z). \tag{1.17}$$

Lemma 1.5 (see [15]). *Let f and g, respectively, be in the classes convex function and starlike function. Then, for every function $H \in A$, one has*

$$\frac{(f(z) * g(z)H(z))}{f(z) * g(z)} \in \overline{\mathrm{co}}(H(U)), \quad z \in U, \tag{1.18}$$

where $\overline{\mathrm{co}}(H(U))$ denotes the closed convex hull of $H(U)$.

2. Main Results

Theorem 2.1. *Let $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$. Then f_k defined by* (1.5) *is in $S_1^{\sigma,s}(\lambda,\delta,\phi) = S^{\sigma,s}(\lambda,\delta,\phi)$.*

Proof. Let $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$, then by Definition 1.1 we have

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s}f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s}f_k(z)} \prec \phi(z). \tag{2.1}$$

Substituting z by $\varepsilon^{\nu}z$, where $\varepsilon^{k}=1$ $(\nu=0,1,\ldots,k-1)$ in (2.1), respectively, we have

$$\frac{\varepsilon^{\nu}z\left(D_{\lambda,\delta}^{\sigma,s}f(\varepsilon^{\nu}z)\right)'}{D_{\lambda,\delta}^{\sigma,s}f_k(\varepsilon^{\nu}z)}\prec\phi(z). \tag{2.2}$$

According to the definition of f_k and $\varepsilon^{k}=1$, we know $f_k(\varepsilon^{\nu}z)=\varepsilon^{\nu}f_k(z)$ for any $\nu=0,1,\ldots,k-1$, and summing up, we can get

$$\frac{1}{k}\sum_{\nu=0}^{k-1}\varepsilon^{-\nu}\left[\frac{z\left(D_{\lambda,\delta}^{\sigma,s}f(\ \varepsilon^{\nu}z\ )\right)'}{D_{\lambda,\delta}^{\sigma,s}f_k(z)}\right]=\frac{z\left[(1/k)\sum_{\nu=0}^{k-1}\varepsilon^{-\nu}D_{\lambda,\delta}^{\sigma,s}f(\varepsilon^{\nu}z)\right]'}{D_{\lambda,\delta}^{\sigma,s}f_k(z)}=\frac{z\left(D_{\lambda,\delta}^{\sigma,s}f_k(z)\right)'}{D_{\lambda,\delta}^{\sigma,s}f_k(z)}. \tag{2.3}$$

Hence there exist ζ_{ν} in U such that

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s}f_k(z)\right)'}{D_{\lambda,\delta}^{\sigma,s}f_k(z)}=\frac{1}{k}\sum_{\nu=0}^{k-1}\phi(\zeta_{\nu})=\phi(\zeta_0), \tag{2.4}$$

for $\zeta_0\in U$ since $\phi(U)$ is convex. Thus $f_k\in S^{\sigma,s}(\lambda,\delta,\phi)$. □

Theorem 2.2. *Let $f\in A$ and $\phi\in\wp$. Then*

$$f\in K_k^{\sigma,s}(\lambda,\delta,\phi)\Longleftrightarrow zf'\in S_k^{\sigma,s}(\lambda,\delta,\phi). \tag{2.5}$$

Proof. Let

$$g(z)=z+\sum_{n=2}^{\infty}n^{s}(C(\delta,n)[1+\lambda(n-1)])^{\sigma}z^{n}, \tag{2.6}$$

and the operator $D_{\lambda,\delta}^{\sigma,s}f$ can be written as $D_{\lambda,\delta}^{\sigma,s}f=g*f$.

Then from the definition of the differential operator $D_{\lambda,\delta}^{\sigma,s}$, we can verify

$$\frac{\left(z\left(D_{\lambda,\delta}^{\sigma,s}f(z)\right)'\right)'}{\left(D_{\lambda,\delta}^{\sigma,s}f_k(z)\right)'}=\frac{\left(z(g*f)'\right)'(z)}{(g*f)_k'(z)}=\frac{z(g*zf')'(z)}{(g*zf')_k(z)}=\frac{z\left(D_{\lambda,\delta}^{\sigma,s}zf'(z)\right)'}{D_{\lambda,\delta}^{\sigma,s}zf_k'(z)}. \tag{2.7}$$

Thus $f\in K_k^{\sigma,s}(\lambda,\delta,\phi)$ if and only if $zf'\in S_k^{\sigma,s}(\lambda,\delta,\phi)$. □

By using Theorems 2.2 and 2.1, we get the following.

Corollary 2.3. *Let $f\in K_k^{\sigma,s}(\lambda,\delta,\phi)$. Then f_k defined by* (1.5) *is in $K_1^{\sigma,s}(\lambda,\delta,\phi)=K^{\sigma,s}(\lambda,\delta,\phi)$.*

Proof. Let $f \in K_k^{\sigma,s}(\lambda,\delta,\phi)$. Then Theorem 2.2 shows that $zf' \in S_k^{\sigma,s}(\lambda,\delta,\phi)$. We deduce from Theorem 2.1 that $(zf')_k \in S^{\sigma,s}(\lambda,\delta,\phi)$. From $(zf')_k = zf_k'$, Theorem 2.2 now shows that $f_k \in K_1^{\sigma,s}(\lambda,\delta,\phi) = K^{\sigma,s}(\lambda,\delta,\phi)$. □

Theorem 2.4. *Let $\phi \in \wp$, $\lambda > 0$ with $\Re[\phi(z) + (1/\lambda) - 1] > 0$. If $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$, then*

$$\frac{z\left(D_{\lambda,\delta}^{\sigma-1,s}(D_\delta f_k(z))\right)'}{D_{\lambda,\delta}^{\sigma-1,s}(D_\delta f_k(z))} \prec q(z) \prec \phi(z), \tag{2.8}$$

where $D_{\lambda,\delta}^{\sigma-1,s}(D_\delta f_k(z)) = D_{\lambda,\delta}^{\sigma-1,s} * D_\delta f_k(z)$ and q is the univalent solution of the differential equation

$$q(z) + \frac{zq'(z)}{q(z) + (1/\lambda) - 1} = \phi(z). \tag{2.9}$$

Proof. Let $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$. Then in view of Theorem 2.1, $f_k \in S^{\sigma,s}(\lambda,\delta,\phi)$, that is,

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s} f_k(z)\right)'}{D_{\lambda,\delta}^{\sigma,s} f_k(z)} \prec \phi(z). \tag{2.10}$$

From the definition of $D_{\lambda,\delta}^{\sigma,s}$, we see that

$$D_{\lambda,\delta}^{s,\sigma} f_k(z) = (1-\lambda)\left(D_{\lambda,\delta}^{s,\sigma-1} * D_\delta f_k(z)\right) + \lambda z\left(D_{\lambda,\delta}^{s,\sigma-1} * D_\delta f_k(z)\right)' \tag{2.11}$$

which implies that

$$D_{\lambda,\delta}^{s,\sigma-1} * D_\delta f_k(z) = \frac{1}{\lambda z^{(1/\lambda)-1}} \int_0^z t^{(1/\lambda)-2} D_{\lambda,\delta}^{s,\sigma} f_k(t)\,dt. \tag{2.12}$$

Using (2.10) and (2.12), we see that Lemma 1.3 can be applied to get (2.8), where $c = (1/\lambda) - 1 > -1$ and $\Re\{\phi\} > 0$ with $\Re[\phi(z) + (1/\lambda) - 1] > 0$ and q satisfies (2.9). We thus complete the proof of Theorem 2.4. □

Theorem 2.5. *Let $\phi \in \wp$ and $s \in N_0$. Then*

$$S_k^{\sigma,s+1}(\lambda,\delta,\phi) \subset S_k^{\sigma,s}(\lambda,\delta,\phi). \tag{2.13}$$

Proof. Let $f \in S_k^{\sigma,s+1}(\lambda,\delta,\phi)$. Then

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s+1} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} \prec \phi(z). \tag{2.14}$$

Set

$$p(z) = \frac{z\left(D_{\lambda,\delta}^{\sigma,s} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s} f_k(z)}, \tag{2.15}$$

where p is analytic function with $p(0) = 1$. By using the equation

$$z\left(D_{\lambda,\delta}^{\sigma,s} f(z)\right)' = D_{\lambda,\delta}^{\sigma,s+1} f(z), \tag{2.16}$$

we get

$$p(z) = \frac{D_{\lambda,\delta}^{\sigma,s+1} f(z)}{D_{\lambda,\delta}^{\sigma,s} f_k(z)} \tag{2.17}$$

and then differentiating, we get

$$z\left(D_{\lambda,\delta}^{\sigma,s+1} f(z)\right)' = zD_{\lambda,\delta}^{\sigma,s} f_k(z) p'(z) + z\left(D_{\lambda,\delta}^{\sigma,s} f_k(z)\right)' p(z), \tag{2.18}$$

Hence

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s+1} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} = \frac{D_{\lambda,\delta}^{\sigma,s} f_k(z)}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} zp'(z) + \frac{z\left(D_{\lambda,\delta}^{\sigma,s} f_k(z)\right)'}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} p(z). \tag{2.19}$$

Applying (2.16) for the function f_k we obtain

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s+1} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} = \frac{D_{\lambda,\delta}^{\sigma,s} f_k(z)}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} zp'(z) + p(z). \tag{2.20}$$

Using (2.20) with $q(z) = (D_{\lambda,\delta}^{\sigma,s+1} f_k(z))/(D_{\lambda,\delta}^{\sigma,s} f_k(z))$, we obtain

$$\frac{z\left(D_{\lambda,\delta}^{\sigma,s+1} f(z)\right)'}{D_{\lambda,\delta}^{\sigma,s+1} f_k(z)} = \frac{zp'(z)}{q(z)} + p(z). \tag{2.21}$$

Since $f \in S_k^{\sigma,s+1}(\lambda, \delta, \phi)$, then by using (2.14) in (2.21) we get the following.

$$\frac{zp'(z)}{q(z)} + p(z) \prec \phi(z). \tag{2.22}$$

We can see that $q(z) \prec \phi(z)$, hence applying Lemma 1.4 we obtain the required result. □

By using Theorems 2.2 and 2.5, we get the following.

Corollary 2.6. *Let $\phi \in \wp$ and $s \in N_0$. Then*

$$K_k^{\sigma,s+1}(\lambda,\delta,\phi) \subset K_k^{\sigma,s}(\lambda,\delta,\phi). \tag{2.23}$$

Now we prove that the class $S_k^{\sigma,s}(\lambda,\delta,\phi)$, $\phi \in \wp$, is closed under convolution with convex functions.

Theorem 2.7. *Let $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$, $\phi \in \wp$, and φ is a convex function with real coefficients in U. Then $f * \varphi \in S_k^{\sigma,s}(\lambda,\delta,\phi)$.*

Proof. Let $f \in S_k^{\sigma,s}(\lambda,\delta,\phi)$, then Theorem 2.1 asserts that $D_{\lambda,\delta}^{\sigma,s} f_k(z) \in S^*(\phi)$, where $\Re\{\phi\} > 0$. Applying Lemma 1.5 and the convolution properties we get

$$\begin{aligned}
\frac{z\left(D_{\lambda,\delta}^{\sigma,s}(f*\varphi)(z)\right)'}{D_{\lambda,\delta}^{\sigma,s}(f_k*\varphi)(z)} &= \frac{z\left(D_{\lambda,\delta}^{\sigma,s} f(z) * \varphi(z)\right)'}{\varphi(z) * D_{\lambda,\delta}^{\sigma,s} f_k(z)} \\
&= \frac{\varphi(z) * \left(z\left(D_{\lambda,\delta}^{\sigma,s} f(z)\right)' / f D_{\lambda,\delta}^{\sigma,s} f_k(z)\right) D_{\lambda,\delta}^{\sigma,s} f_k(z)}{\varphi(z) * D_{\lambda,\delta}^{\sigma,s} f_k(z)} \\
&\in \overline{\mathrm{co}}\left(\frac{z\left(D_{\lambda,\delta}^{\sigma,s} f\right)'}{D_{\lambda,\delta}^{\sigma,s} f_k}(U)\right) \subseteq \phi(U).
\end{aligned} \tag{2.24}$$

□

Corollary 2.8. *Let $f \in K_k^{\sigma,s}(\lambda,\delta,\phi)$, $\phi \in \wp$, and φ is a convex function with real coefficients in U. Then $f * \varphi \in K_k^{\sigma,s}(\lambda,\delta,\phi)$.*

Proof. Let $f \in K_k^{\sigma,s}(\lambda,\delta,\phi)$, $\phi \in \wp$. Then Theorem 2.2 shows that $zf' \in S_k^{\sigma,s}(\lambda,\delta,\phi)$. The result of Theorem 2.7 yields $(zf') * \varphi = z(f*\varphi)' \in S_k^{\sigma,s}(\lambda,\delta,\phi)$, and thus $f * \varphi \in K_k^{\sigma,s}(\lambda,\delta,\phi)$. □

Some other works related to other differential operators with respect to symmetric points for different types of problems can be seen in ([16–21]).

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References

[1] K. Sakaguchi, "On a certain univalent mapping," *Journal of the Mathematical Society of Japan*, vol. 11, pp. 72–75, 1959.

[2] T. N. Shanmugam, C. Ramachandran, and V. Ravichandran, "Fekete-Szegő problem for subclasses of starlike functions with respect to symmetric points," *Bulletin of the Korean Mathematical Society*, vol. 43, no. 3, pp. 589–598, 2006.

[3] R. Chand and P. Singh, "On certain schlicht mappings," *Indian Journal of Pure and Applied Mathematics*, vol. 10, no. 9, pp. 1167–1174, 1979.
[4] R. N. Das and P. Singh, "On subclasses of schlicht mapping," *Indian Journal of Pure and Applied Mathematics*, vol. 8, no. 8, pp. 864–872, 1977.
[5] Z.-G. Wang, C.-Y. Gao, and S.-M. Yuan, "On certain subclasses of close-to-convex and quasi-convex functions with respect to k-symmetric points," *Journal of Mathematical Analysis and Applications*, vol. 322, no. 1, pp. 97–106, 2006.
[6] C. Gao and S. Zhou, "A new subclass of close-to-convex functions," *Soochow Journal of Mathematics*, vol. 31, no. 1, pp. 41–49, 2005.
[7] W. C. Ma and D. Minda, "A unified treatment of some special classes of univalent functions," in *Proceedings of the Conference on Complex Analysis*, pp. 157–169.
[8] S. Ruscheweyh, "New criteria for univalent functions," *Proceedings of the American Mathematical Society*, vol. 49, pp. 109–115, 1975.
[9] G. Sălăgean, "Subclasses of univalent functions," in *Complex Analysis—Fifth Romanian-Finnish Seminar*, vol. 1013 of *Lecture Notes in Mathematics*, pp. 362–372, Springer, Berlin, Germany, 1983.
[10] F. M. Al-Oboudi, "On univalent functions defined by a generalized Sălăgean operator," *International Journal of Mathematics and Mathematical Sciences*, no. 25-28, pp. 1429–1436, 2004.
[11] K. Al Shaqsi and M. Darus, "On subclass of close-to-convex functions," *International Journal of Contemporary Mathematical Sciences*, vol. 2, no. 13-16, pp. 745–757, 2007.
[12] M. Darus and R. W. Ibrahim, "On subclasses for generalized operators of complex order," *Far East Journal of Mathematical Sciences*, vol. 33, no. 3, pp. 299–308, 2009.
[13] S. S. Miller and P. T. Mocanu, *Differential Subordinations Theory and Applications*, vol. 225, Marcel Dekker, New York, NY, USA, 2000.
[14] K. S. Padmanabhan and R. Parvatham, "Some applications of differential subordination," *Bulletin of the Australian Mathematical Society*, vol. 32, no. 3, pp. 321–330, 1985.
[15] St. Ruscheweyh and T. Sheil-Small, "Hadamard products of Schlicht functions and the Pólya-Schoenberg conjecture," *Commentarii Mathematici Helvetici*, vol. 48, pp. 119–135, 1973.
[16] M. Harayzeh Al-Abbadi and M. Darus, "Differential subordination defined by new generalised derivative operator for analytic functions," *International Journal of Mathematics and Mathematical Sciences*, vol. 2010, Article ID 369078, 15 pages, 2010.
[17] A. A. Ali Abubaker and M. Darus, "On classes of analytic functions with respect to k-symmetric conjugate points involving operator," *Far East Journal of Mathematical Sciences*, vol. 45, no. 1, pp. 27–40, 2010.
[18] K. Al-Shaqsi and M. Darus, "On univalent functions with respect to k-symmetric points defined by a generalized Ruscheweyh derivatives operator," *Journal of Analysis and Applications*, vol. 7, no. 1, pp. 53–61, 2009.
[19] K. Al-Shaqsi and M. Darus, "On harmonic univalent functions with respect to k-symmetric points," *International Journal of Contemporary Mathematical Sciences*, vol. 3, no. 3, pp. 111–118, 2008.
[20] K. Al-Shaqsi and M. Darus, "On meromorphic harmonic functions with respect to k-symmetric points," *Journal of Inequalities and Applications*, vol. 2008, Article ID 259205, 11 pages, 2008.
[21] K. Al-Shaqsi and M. Darus, "Fekete-Szegő problem for univalent functions with respect to k-symmetric points," *The Australian Journal of Mathematical Analysis and Applications*, vol. 5, no. 2, article 7, pp. 1–12, 2008.