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Research Article

Fuzzy n-Fold BCI-Positive Implicative Ideals in BCI-Algebras

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We consider the notion of fuzzy n-fold positive implicative ideals in BCI-algebras. We analyse many properties of fuzzy n-fold positive implicative ideals. We also establish an extension properties for fuzzy n-fold positive implicative ideals in BCI-algberas. This work generalizes the corresponding results in the crisp case.

1. Introduction

Liu and Zhang [1], Wei and Jun [2] independently introduced BCI-positive implicative ideals and used these to completely describe positive implicative BCI-algebras (namely, weakly positive implicative BCI-algebras). Tebu et al. [3] introduced the notion of n-fold positive implicative ideals in BCI-algebra. Fuzzy ideals are useful tool to obtain results on BCI-algebras. The sets of provable formulas in the corresponding inference systems from the point of view of uncertain information can be described by fuzzy ideals of those algebraic semantics. So in this paper, we discuss the notion of fuzzy n-fold positive implicative ideals in BCI-algebras. We show that every fuzzy n-fold positive implicative ideal is a fuzzy ideal, and give a condition for a fuzzy ideal to be a fuzzy n-fold positive implicative ideal. Using the level set, we provide a characterization of fuzzy n-fold positive implicative ideals. Finally, we establish an extension property for fuzzy n-fold positive implicative ideal. This paper generalizes the corresponding results in BCK/BCI-algebras [4].

2. Preliminaries

We recollect some definitions and results which will be used in the following, and we shall not cite them every time they are used.

Definition 2.1 (see [5]). An algebra $X = \langle X; \star, 0\rangle$ of type $\langle 2, 0\rangle$, is said to be a BCI-algebra if it satisfies the following conditions for all $x, y, z \in X$:

(i) BCI1-$((x \star y) \star (x \star z)) \star (z \star y) = 0$;

(ii) BCI2-$(x \star (x \star y)) \star y = 0$;

(iii) BCI3-$x \star x = 0$;

(iv) BCI4-$x \star y = 0$ and $y \star x = 0$ imply $x = y$.

If a BCI-algebra X satisfies the condition $0 \star x = 0$ for all $x \in X$; then X is called a BCK-algebra. Hence, BCK-algebra form a subclass of BCI-algebra.

Proposition 2.2 (see [5]). *On every BCI-algebra X, there is a natural order called the BCI-ordering defined by $x \leq y$ if and only if $x \star y = 0$. Under this order, the following axioms hold for all $x, y, z \in X$.*

(a) $x \star 0 = x$;

(b) *if* $x \leq y$, *then* $x \star z \leq y \star z$ *and* $z \star y \leq z \star x$;

(c) $(x \star y) \star z = (x \star z) \star y$;

(d) $x \star (x \star (x \star y)) = x \star y$;

(e) $(x \star z) \star (y \star z) \leq x \star y$;

(f) $0 \star (x \star y) = (0 \star x) \star (0 \star y)$;

(g) $0 \star (x \star y) = 0 \star (0 \star (y \star x))$;

(h) $0 \star x = 0 \star (y \star (y \star x))$;

(i) $(0 \star (x \star y)) \star (y \star x) = 0$;

(j) *if* $x \leq y$, *then* $0 \star x = 0 \star y$;

(k) *if* $x \leq 0 \star y$, *then* $x = 0 \star y$.

Let n be a positive integer. Throughout this paper we appoint that $X := (X, \star, 0)$ denotes a BCI-algebra; $x \star y^n := (\cdots((x \star y) \star y) \star \cdots) \star y$, in which y occurs n times; $x \star y^0 := x$ and $x \star \prod_{i=1}^{n} y_i$ denotes $(\cdots((x \star y_1) \star y_2) \star \cdots) \star y_n$, where $x, y, y_i \in X$.

Definition 2.3 (see [5]). A nonempty subset I of a BCI-algebra is called an ideal of X if it satisfies:

(I_1) $0 \in I$,

(I_2) $x \star y \in I$ and $y \in I$ imply $x \in I$.

Definition 2.4 (see [5]). A nonempty subset I of a BCI-algebra is called a BCI-positive implicative ideal of X if it satisfies (I_1) and

(I_3) $(x \star y^2) \star (z \star y) \in I$ and $z \in I$ imply $x \star y \in I$; for all $x, y, z \in X$.

We recall [6, 7] that a fuzzy set of a set X is a function $\mu : X \to [0;1]$. For a fuzzy set μ in X and $t \in [0;1]$, define μ_t to be the set

$$\mu_t = \{x \in X \mid \mu(x) \geq t\}. \tag{2.1}$$

Definition 2.5 (see [8]). (a) Let X be a BCI-algebra. A fuzzy set in X is said to be a fuzzy ideal of X if:

(i) $\mu(0) \geq \mu(x)$ for all $x \in X$;

(ii) $\mu(x) \geq \mathrm{Min}\{\mu(x \star y), \mu(y)\}$; for all $x, y \in X$.

Note that every ideal μ is order reversing, that is, if $x \leq y$ then $\mu(x) \geq \mu(y)$.

(b) A fuzzy set μ in X is called a fuzzy positive implicative ideal of X if it satisfies:

(i) $\mu(0) \geq \mu(x)$ for all $x \in X$.

(ii) $\mu(x \star y) \geq \mathrm{Min}\{\mu((x \star y^2) \star (z \star y)), \mu(z)\}$; for all $x, y, z \in X$.

Definition 2.6. Let X be a BCK-algebra and n be a positive integer. Then the fuzzy set μ of X is called a fuzzy n-fold positive implicative ideal in BCK-algebra if it satisfies the following conditions:

(i) $\mu(0) \geq \mu(x)$, for all $x \in X$;

(ii) $\mu(x \star y^n) \geq \mathrm{Min}\{\mu((x \star y^{n+1}) \star z); \mu(z)\}$; for all $x, y, z \in X$.

Definition 2.7 (see [9]). Let X be a BCK-algebra and n be a positive integer. Then X is called an n-fold positive implicative BCK-algebra if $x \star y^{n+1} = x \star y^n$; for all $x, y \in X$.

Definition 2.8 (see [3]). Let X be a BCI-algebra and n be a positive integer. Then X is called an n-fold positive implicative BCI-algebra if $(x \star y^{n+1}) \star (0 \star y) = x \star y^n$; for all $x, y \in X$.

Definition 2.9 (see [3]). A nonempty subset I of a BCI-algebra is called an n-fold BCI-positive implicative ideal of X if it satisfies (I_1) and

$(I_5)(x \star y^{n+1}) \star (z \star y) \in I$ and $z \in I$ imply $x \star y^n \in I$; for all $x, y, z \in X$.

3. Fuzzy n-Fold Positive Implicative Ideals in BCI-Algebras

In this section, we introduce the notion of fuzzy n-fold positive implicative ideal in a BCI-algebra and study some important properties.

Definition 3.1. A fuzzy set μ of a BCI-algebra X is called a fuzzy n-fold positive implicative ideal of X if it satisfies the following conditions:

(i) $\mu(0) \geq \mu(x)$ for all $x \in X$;

(ii) $\mathrm{Min}\{\mu((x \star y^{n+1}) \star (z \star y)); \mu(z)\} \leq \mu(x \star y^n)$; for all $x, y, z \in X$.

Remark 3.2. (a) Notice that fuzzy 1-fold positive implicative ideal in BCI-algebra is a fuzzy positive implicative ideal in BCI-algebra.

(b) The notion of fuzzy n-fold positive implicative ideal in a BCI-algebras generalizes the notion of fuzzy n-fold positive implicative ideal in BCK-algebras. This is because if X is a BCK-algebra, for every $x \in X$, $0 \star x = 0$.

(c) Every fuzzy n-fold positive implicative ideal in BCI-algebra is a fuzzy ideal.

The following example shows that the converse of Remark 3.2 (c) may not be true.

Example 3.3. Consider a BCI-algebra $X = \{0, 1, 2, 3\}$ with Cayley table as follows:

$$\begin{array}{ccccc} \star & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 3 & 2 \\ 1 & 1 & 0 & 3 & 2 \\ 2 & 2 & 2 & 0 & 3 \\ 3 & 3 & 3 & 2 & 0. \end{array} \tag{3.1}$$

We define $\mu : X \to [0; 1]$ in X given by $\mu(0) = \mu(1) = 1$, $\mu(2) = \mu(3) = 1/2$. μ is a fuzzy ideal, but not a fuzzy 2-fold positive implicative ideal of X. As $\mu((2 \star 2^3) \star (3 \star 2)) = 1$, but $\mu(2 \star 2^2) = 1/2$.

Remark 3.4. In an n-fold positive implicative BCI-algebra, every fuzzy ideal is a fuzzy n-fold positive implicative.

Theorem 3.5. *Let μ be a fuzzy ideal of a BCI-algebra X. Then the following conditions are equivalent:*

(i) $\mu((x \star y^{n+1}) \star (z \star y)) \leq \mu((x \star y^n) \star z)$, *for all* $x, y, z \in X$;

(ii) μ *is a fuzzy n-fold positive implicative ideal;*

(iii) $\mu((x \star y^{n+1}) \star (0 \star y)) \leq \mu(x \star y^n)$, *for all* $x, y \in X$.

Proof. (i)⇒(ii). Let $x, y, z \in X$. We will prove that $\mu(x \star y^n) \geq \mathrm{Min}\{\mu((x \star y^{n+1}) \star (z \star y)); \mu(z)\}$

Since μ is a fuzzy ideal, we have $\mu(x \star y^n) \geq \mathrm{Min}\{\mu((x \star y^n) \star z; \mu(z)\} \geq \mathrm{Min}\{\mu((x \star y^{n+1}) \star (z \star y)); \mu(z)\}$.

(ii)⇒(iii). It is easy to obtain by setting $z = 0$.

(iii)⇒(i). Let $x, y, z \in X$ with $v = ((x \star y^{n+1}) \star (z \star y))$ and $u = x \star z$, it suffices to show that $\mu(u \star y^{n+1}) \geq \mu(v)$.

$$\begin{aligned} \mu\Big(\Big(\Big(u \star y^{n+1}\Big) \star (0 \star y)\Big) \star v\Big) &= \mu\Big(\Big(\Big(\Big(\Big(x \star y^{n+1}\Big) \star (0 \star y)\Big) \star \Big(\Big(x \star y^{n+1}\Big) \star (z \star y)\Big)\Big) \star z\Big)\Big) \\ &\geq \mu(((z \star y) \star (0 \star y)) \star z). \\ &\geq \mu((z \star 0) \star z) = \mu(0). \end{aligned} \tag{3.2}$$

Thus, $\mu[(((u \star y^{n+1}) \star (0 \star y)) \star v)] = \mu(0)$, it follows $\mu((u \star y^n) = \mathrm{Min}\{\mu[((u \star y^{n+1}) \star (0 \star y)) \star v]; \mu(v)\}$. Hence, $\mu((x \star z) \star y^n) \geq \mu(v)$. □

Theorem 3.6. *Let X be a BCI-algebra and μ be a fuzzy set in X.*

If μ is a fuzzy n-fold positive implicative ideal of X, then $\mu((x \star y^n) \star (0 \star (0 \star y))) \leq \mu(x \star y^{n+1})$.

Proof. Suppose that μ is a fuzzy n-fold positive implicative ideal of X. putting $s = x \star y$, then $\mu((x \star y^{n+1}) \star (0 \star y)) \leq \mu(x \star y^n)$, for all $x, y \in X$.

$$\begin{aligned}
&\mu\Big(\Big(\Big(\Big(s \star y^{n+1}\Big) \star (0 \star y)\Big)\Big) \star ((x \star y^n) \star (0 \star (0 \star y)))\Big) \\
&\quad = \mu(((((x \star y^n) \star (0 \star y)) \star y) \star y) \star ((x \star y^n) \star (0 \star (0 \star y)))) \\
&\quad = \mu\Big(\Big(\Big(\Big(\Big(x \star y^{n+1}\Big) \star (0 \star y)\Big) \star ((x \star y^n) \star (0 \star (0 \star y)))\Big) \star y\Big) \star y\Big) \\
&\quad \geq \mu((((0 \star (0 \star y)) \star (0 \star y)) \star y) \star y) \\
&\quad = \mu((((0 \star (0 \star y)) \star y) \star (0 \star y)) \star y) \\
&\quad = \mu((0 \star (0 \star y)) \star y) \\
&\quad = \mu(0).
\end{aligned} \tag{3.3}$$

We have $\mu(((x \star y) \star y^{n+1}) \star (0 \star y)) = \mu((s \star y^{n+1}) \star (0 \star y)) = \mu(0)$, since μ is a fuzzy n-fold positive implicative ideal, we obtain $\mu(x \star y^{n+1}) = \mu((x \star y) \star y^n) = \mu(u \star y^n) \geq \mu((x \star y^n) \star (0 \star (0 \star y)))$. The proof is complete. □

However the following example proves that the converse of Theorem 3.6 is not true.

Example 3.7. Consider a BCI-algebra $X = \{0, 1, a, b, c\}$ with the following Cayley table:

$$\begin{array}{cccccc}
\star & 0 & 1 & a & b & c \\
0 & 0 & 0 & a & a & a \\
1 & 1 & 0 & a & a & a \\
a & a & a & 0 & 0 & 0 \\
b & b & a & 1 & 0 & a \\
c & c & a & 1 & 1 & 0.
\end{array} \tag{3.4}$$

We define μ: $X \longrightarrow [0;1]$ in X by $\mu(0) = 1$ and for all $x \neq 0$ $\mu(x) = 0,3$. It is clear that for all $b, c \in X$ $\mu((c \star b^2) \star (0 \star c)) \leq \mu(c \star b)$; but μ is not a fuzzy 1-fold positive implicative ideal because $\mu((c \star b^2) \star (0 \star c)) = 1$, but $\mu(c \star b) = 0,3$.

Lemma 3.8. *Let μ be a fuzzy ideal in X. If μ is a fuzzy n-fold positive implicative ideal, then μ is a fuzzy $(n+1)$-fold positive implicative ideal.*

Proof. let μ be a fuzzy n-fold positive implicative ideal of X, let $x, y \in X$. Then $\mu((x \star y^{n+1}) \star (0 \star y)) \leq \mu(x \star y^n)$. Therefore, $\mu(((x \star y) \star y^{n+1}) \star (0 \star y)) \leq \mu((x \star y) \star y^n)$. Hence, μ is a fuzzy $(n+1)$-fold positive implicative ideal. □

Using this lemma and a simple induction argument, we obtain the following proposition.

Proposition 3.9. *Let μ be a fuzzy ideal of a BCI-algebra X and $k \in \mathbb{N} \setminus \{0\}$. If μ is a fuzzy n-fold positive implicative ideal, then μ is a fuzzy $(n+k)$-fold positive implicative ideal.*

However there exists fuzzy $(n+k)$-fold positive implicative ideals which are not fuzzy n-fold positive implicative ideals.

Example 3.10. Consider the set $X_2 = \{0, 1, 2, \pi\}$, and define the operation $\star$ on X_2 by:

$$\begin{aligned} &k \star l = \max(0, k-l) \quad \text{if } 0 \le k, l \le n, \\ &k \star \pi = \pi \star k = \pi \quad \text{if } k = 0, 1, 2, \\ &\pi \star \pi = 0. \end{aligned} \tag{3.5}$$

Consider the fuzzy ideal μ define by $\mu(0) = 0,4$ and for all $x \neq 0$, $\mu(x) = 0$. Then μ is a fuzzy 2-fold positive implicative ideal, but not a fuzzy positive implicative ideal of X_2 because $\mu((2 \star 1^2) \star (0 \star 1)) = 0,4$ and $\mu(2 \star 1) = 0$.

Now, we established some transfer principle for fuzzy n-fold positive implicative ideals in BCI-algebras.

Proposition 3.11. *Let μ be a fuzzy ideal and I be an ideal of a BCI-algebra X.*

Then I is an n-fold positive implicative ideal if and only if it's characteristic function μ_I is a fuzzy n-fold positive implicative ideal.

Proof. Assume that I is an n-fold positive implicative ideal. Let $x, y \in X$. If $x \star y^n \in I$, then $\mu(x \star y^n) \ge \mu((x \star y^{n+1}) \star (0 \star y))$. If $x \star y^n \notin I$, then $(x \star y^{n+1}) \star (0 \star y) \notin I$ because I is an n-fold positive implicative ideal. So, we also have $\mu(x \star y^n) \ge \mu((x \star y^n) \star (0 \star y))$.

Conversely, suppose that μ_I is a fuzzy n-fold positive implicative ideal. Let $x, y \in X$ such that $(x \star y^{n+1}) \star (0 \star y) \in I$. Then $\mu_I((x \star y^{n+1}) \star (0 \star y)) = 1$; since μ_I is a fuzzy n-fold positive implicative ideal, $\mu(x \star y^n) \ge \mu_I((x \star y^{n+1}) \star (0 \star y)) = 1$. We conclude that $x \star y^n \in I$. □

Theorem 3.12. *Let μ be a fuzzy ideal in a BCI-algebra X. Then μ is a fuzzy n-fold positive implicative if and only if the t-cut set $\mu_t := \{x \in X / \mu(x) \ge t\}$ either is empty or is an n-fold positive implicative ideal.*

Proof. Suppose that μ is a fuzzy n-fold positive implicative ideal of X and $\mu_t \neq \emptyset$, for all $t \in [0; 1]$. We will show that μ_t is an n-fold positive implicative ideal. Since μ_t is nonempty, there exists $x_0 \in X$ such that $x_0 \in \mu_t$; since $\mu(0) \ge \mu(x)$, for all $x \in X$, we have: $\mu(0) \ge \mu(x_0) \ge t$; thus $0 \in \mu_t$.

Let $x, y, z \in X$; such that $(x \star y^{n+1}) \star (0 \star y) \in \mu_t$. Then $\mu(((x \star y^{n+1}) \star (0 \star y)) \ge t$, since μ is fuzzy n-fold positive implicative ideal, we have $\mu(x \star y^n) \ge \mu(((x \star y^{n+1}) \star (0 \star y))) \ge t$. Hence $x \star y^n \in \mu_t$ and μ_t is an n-fold BCI-positive implicative ideal.

Conversely, assume that $\mu_t \neq \emptyset$ for all $t \in [0; 1]$ and μ_t is an n-fold BCI-positive implicative ideal. We will show that μ is a fuzzy n-fold positive implicative ideal.

It is easy to see that $\mu(0) \ge \mu(x)$, for all $x \in X$.

Now assume that there exist $a, b \in X$ such that $\mu(a \star b^n) < \mu((a \star b^{n+1}) \star (0 \star b))$.

setting $t_0 = (1/2)[\mu(a \star b^n) + \mu((a \star b^{n+1}) \star (0 \star b))]$,then $\mu(a \star b^n) < t_0 < \mu((a \star b^{n+1}) \star (0 \star b))$; it follows that $(a \star b^{n+1}) \star (0 \star b) \in \mu_{t_0}$, but $a \star b^n \notin \mu_{t_0}$. This is a contradiction. Hence μ is a fuzzy n-fold positive implicative ideal. □

Theorem 3.13. *If μ is an n-fold fuzzy positive implicative ideal of a BCI-algebra X, then the set $X_\mu = \{x \in X, \mu(x) = \mu(0)\}$ is an n-fold positive implicative ideal of X.*

Proof. Let μ be an n-fold fuzzy positive implicative ideal of X. Clearly $0 \in X_\mu$.

Let $x, y, z \in X$ such that $(x * y^{n+1}) * (0 * y) \in X_\mu$. Then $\mu(x * y^n) \geq \mu((x * y^{n+1}) * (0 * y)) = \mu(0)$. It follows by $\mu(0) \geq \mu(x)$ for all $x \in X$, that $\mu(x * y^n) = \mu(0)$ so that $x * y^n \in X_\mu$. Hence X_μ is an n-fold positive implicative ideal of X. □

Theorem 3.14 (extension property for fuzzy n-fold positive implicative ideals in BCI-algebra). *Let μ, ν two fuzzy ideals of a BCI-algebra X such that $\mu(0) = \nu(0)$ and $\mu \subset \nu$, that is, $\mu(x) \leq \nu(x)$ for all $x \in X$. If μ is a fuzzy n-fold positive implicative ideal then ν is also fuzzy n-fold positive implicative ideal.*

Proof. Using Theorem 3.5, it is sufficient to show that ν satisfies the inequality $\nu(x \star y^n) \geq \nu((x \star y^{n+1}) \star (0 \star y))$ for all $x, y \in X$.

Let $x, y \in X$; setting $r = (x \star y^{n+1}) \star (0 \star y)$, we have

$$\begin{aligned}
\nu(0) = \mu(0) = \mu\left(\left(\left(x \star y^{n+1}\right) \star (0 \star y)\right) \star r\right) &= \mu\left(\left(\left(x \star y^{n+1}\right) \star r\right) \star (0 \star y)\right) \\
&= \mu\left(\left((x \star r) \star y^{n+1}\right) \star (0 \star y)\right) \\
&\leq \mu\left((x \star y^n) \star r\right) \\
&\leq \nu\left((x \star y^n) \star r\right).
\end{aligned} \tag{3.6}$$

Since ν is a fuzzy ideal, $\nu(x \star y^n) \geq \mathrm{Min}\{\nu((x \star r) \star y^n); \nu(r)\} \geq \mathrm{Min}\{\nu(0); \nu(r)\} = \nu(r)$. □

Hence by Theorem 3.5(*iii*), ν is a fuzzy n-fold positive implicative ideal of X.

Definition 3.15 (see [10]). Let X, Y be two BCI-algebras.

A map $f : X \to Y$ is called a BCI-homomorphism if: $f(x \star y) = f(x) \star f(y)$ for all $x, y \in X$.

Definition 3.16. Let X and Y be two BCI-algebras, μ a fuzzy subset of X, ν a fuzzy subset of Y and $f : X \longrightarrow Y$ a BCI-homomorphism.

The image of μ under f denoted by $f(\mu)$ is a fuzzy set of Y defined by: For all $y \in Y$, $f(\mu)(y) = \mathrm{Sup}\{\mu(x) : x \in f^{-1}(Y)\}$ if $f^{-1}(Y) \neq \emptyset$ and $f(\mu)(y) = 0$ if $f^{-1}(Y) = \emptyset$.

The preimage of ν under f denoted by $f^{-1}(\nu)$ is a fuzzy set of X defined by: For all $x \in X, f^{-1}(\nu)(x) = \nu(f(x))$. Let $f : X \longrightarrow Y$ be an onto BCI-homomorphism, It is easy to prove that the preimage of a fuzzy n-fold positive implicative ideal ν under f is also a fuzzy n-fold positive implicative ideal.

Definition 3.17. A fuzzy subset μ of X has a sup property if for any nonempty subset A of X, there exists $a_0 \in A$ such that $\mu(a_0) = \mathrm{Sup}\{\mu(a) : a \in A\}$. Using this fact, we can prove the following result:

Proposition 3.18. *Let $f : X \longrightarrow Y$ be an onto BCI-homomorphism, the image $f(\mu)$ of a fuzzy n-fold positive implicative ideal μ with a sup property is also a fuzzy n-fold positive implicative ideal.*

4. Conclusion

To investigate the structure of an algebraic system, it is clear that ideals with special properties play an important role. The present paper introduced and studied the notion of fuzzy n-fold positive implicative ideal in BCI-algebra. The extension property of fuzzy n-fold positive implicative ideals was established. The main purpose of future work is to investigate the fuzzy foldness of other types of ideals in BCI-algebras and the relation diagram between them similar to the one in [8, 11, 12]. We are also doing some investigations on logic whose algebraic semantics is n-fold positive implicative BCI-algebras.

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