Research Article

Coupled Fixed Point Results in Complete Partial Metric Spaces

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Received 23 March 2012; Revised 18 September 2012; Accepted 25 September 2012

Academic Editor: A. Zayed

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We establish some coupled fixed point theorems for a mapping satisfying some contraction conditions in complete partial metric spaces. Our consequences extend the results of H. Aydi (2011).

1. Introduction and Mathematical Preliminaries

The notion of a partial metric space (PMS) was introduced in 1992 by Matthews [1, 2]. Matthews proved a fixed point theorem on this spaces, analogous to the Banach's fixed point theorem. Recently, many authors have focused on partial metric spaces and their topological properties (see e.g. [3–9]).

The definition of a partial metric space is given by Matthews (see [1, 2]) as follows:

Definition 1.1. Let X be a nonempty set and let $p : X \times X \rightarrow R^+$ satisfies

$$(P1) \quad x = y \Leftrightarrow p(x, x) = p(y, y) = p(x, y), \text{ for all } x, y \in X,$$

$$(P2) \quad p(x, x) \leq p(x, y), \text{ for all } x, y \in X,$$

$$(P3) \quad p(x, y) = p(y, x), \text{ for all } x, y \in X,$$

$$(P4) \quad p(x, y) \leq p(x, z) + p(z, y) - p(z, z), \text{ for all } x, y, z \in X.$$

Then the pair (X, p) is called a partial metric space and p is called a partial metric on X .

The function $d_p : X \times X \rightarrow R^+$ defined by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y) \quad (1.1)$$

satisfies the conditions of a metric on X ; therefore it is a (usual) metric on X .

Remark 1.2. if $x = y$, $p(x, y)$ may not be 0.

- (1) A famous example of partial metric spaces is the pair (R^+, p) , where $p(x, y) = \max\{x, y\}$ for all $x, y \in R^+$. In this case, d_p is the Euclidian metric $d_p(x, y) = |x - y|$.
- (2) Each partial metric p on X generates a T_0 topology τ_p on X which has a base of open p -balls $B_p(x, \varepsilon)$, where $x \in X$ and $\varepsilon > 0$ ($B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$).

The following concepts has been defined as follows on a partial metric space.

Definition 1.3 (see e.g., [1, 2]). (i) A sequence $\{x_n\}$ in a PMS (X, p) converges to $x \in X$ if and only if $p(x, x) = \lim_{n \rightarrow \infty} p(x, x_n)$.

(ii) A sequence $\{x_n\}$ in a PMS (X, p) is called Cauchy if and only if $\lim_{n, m \rightarrow \infty} p(x_n, x_m)$ exists (and is finite).

(iii) A PMS (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p , to a point $x \in X$ such that $p(x, x) = \lim_{n, m \rightarrow \infty} p(x_n, x_m)$.

The concept of coupled fixed point have been introduced in [10] by Bhaskar and Lakshmikantham as follows.

Definition 1.4 (see [10]). An element $(x, y) \in X \times X$ is called a coupled fixed point of mapping $F : X \times X \rightarrow X$ if $x = F(x, y)$ and $y = F(y, x)$.

Aydi in [11] has obtained some coupled fixed point results for mappings satisfying different contractive conditions on complete partial metric spaces. Some of these results are the following cases.

Theorem 1.5 (see [11, Theorem 2.1]). Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \rightarrow X$ satisfies the following contractive condition:

$$p(F(x, y), F(u, v)) \leq kp(x, u) + lp(y, v), \quad (1.2)$$

for all $x, y, u, v \in X$, where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Theorem 1.6 (see [11, Theorem 2.4]). Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \rightarrow X$ satisfies the following contractive condition:

$$p(F(x, y), F(u, v)) \leq kp(F(x, y), x) + lp(F(u, v), u), \quad (1.3)$$

for all $x, y, u, v \in X$, where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Theorem 1.7 (see [11, Theorem 2.5]). *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \rightarrow X$ satisfies the following contractive condition:*

$$p(F(x, y), F(u, v)) \leq kp(F(x, y), u) + lp(F(u, v), x), \quad (1.4)$$

for all $x, y, u, v \in X$, where k, l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

For a survey of fixed point theory, its applications, and related results in partial metric spaces we refer the reader to [4, 5, 12–20] and the references mentioned therein. Also, many researchers have obtained coupled fixed point results for mappings under various contractive conditions in the framework of partial metric spaces (see, e.g., [21, 22]).

In this paper we establish some coupled fixed point results of contractive mappings in the framework of complete partial metric spaces. Our results extend and generalize the results of Aydi [11].

2. Main Results

We recall three easy lemmas which have an essential role in the proof of the main result. These results can be derived easily (see, e.g., [1, 2, 6]).

Lemma 2.1. (1) *A sequence $\{x_n\}$ is a Cauchy sequence in the PMS (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p) .*

(2) *A PMS (X, p) is complete if and only if the metric space (X, d_p) is complete. Moreover,*

$$\lim_{n \rightarrow \infty} d_p(x, x_n) = 0 \iff p(x, x) = \lim_{n \rightarrow \infty} p(x, x_n) = \lim_{n, m \rightarrow \infty} p(x_n, x_m). \quad (2.1)$$

Lemma 2.2 (see [3]). *Assume that $x_n \rightarrow z$ as $n \rightarrow \infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then, $\lim_{n \rightarrow \infty} p(x_n, y) = p(z, y)$, for every $y \in X$.*

Lemma 2.3 (see, e.g., [3, 4]). *Let (X, p) be a complete PMS. Then,*

- (a) *if $p(x, y) = 0$ then, $x = y$,*
- (b) *if $x \neq y$, then $p(x, y) > 0$.*

Throughout this paper, we assume that all of the constants are nonnegative. Our main result is the following. The method of the proof can be found in [11].

Theorem 2.4. *Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that*

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \alpha_1 p(x, u) + \alpha_2 p(y, v) \\ &\quad + \alpha_3 p(F(x, y), x) + \alpha_4 p(F(y, x), y) \\ &\quad + \alpha_5 p(F(x, y), u) + \alpha_6 p(F(y, x), v) + \alpha_7 p(F(u, v), x) \\ &\quad + \alpha_8 p(F(v, u), y) + \alpha_9 p(F(u, v), u) + \alpha_{10} p(F(v, u), v), \end{aligned} \quad (2.2)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $\sum_{i=1}^{10} \alpha_i < 1$. Then, F has a unique coupled fixed point in X .

Proof. Let $x_0, y_0 \in X$ be arbitrary. Define $x_1, y_1 \in X$ such that $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$ and in this way, we construct the sequences $\{x_n\}$ and $\{y_n\}$ as $x_n = F(x_{n-1}, y_{n-1})$ and $y_n = F(y_{n-1}, x_{n-1})$, for all $n \geq 0$.

We will complete the proof in three steps.

Step I. Let $\delta_n = p(x_{n-1}, x_n) + p(y_{n-1}, y_n)$. We will show that $\lim_{n \rightarrow \infty} \delta_n = 0$.

Using (2.2) we obtain that

$$\begin{aligned}
 p(x_n, x_{n+1}) &= p(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\
 &\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(F(x_{n-1}, y_{n-1}), x_{n-1}) \\
 &\quad + \alpha_4 p(F(y_{n-1}, x_{n-1}), y_{n-1}) \\
 &\quad + \alpha_5 p(F(x_{n-1}, y_{n-1}), x_n) + \alpha_6 p(F(y_{n-1}, x_{n-1}), y_n) + \alpha_7 p(F(x_n, y_n), x_{n-1}) \\
 &\quad + \alpha_8 p(F(y_n, x_n), y_{n-1}) + \alpha_9 p(F(x_n, y_n), x_n) + \alpha_{10} p(F(y_n, x_n), y_n) \\
 &= \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(x_n, x_{n-1}) + \alpha_4 p(y_n, y_{n-1}) + \alpha_5 p(x_n, x_n) \\
 &\quad + \alpha_6 p(y_n, y_n) + \alpha_7 p(x_{n+1}, x_{n-1}) + \alpha_8 p(y_{n+1}, y_{n-1}) + \alpha_9 p(x_{n+1}, x_n) \\
 &\quad + \alpha_{10} p(y_{n+1}, y_n) \\
 &\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(x_n, x_{n-1}) + \alpha_4 p(y_n, y_{n-1}) \\
 &\quad + (\alpha_5 - \alpha_7) p(x_n, x_n) + (\alpha_6 - \alpha_8) p(y_n, y_n) \\
 &\quad + \alpha_7 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] + \alpha_8 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})] \\
 &\quad + \alpha_9 p(x_{n+1}, x_n) + \alpha_{10} p(y_{n+1}, y_n).
 \end{aligned} \tag{2.3}$$

Analogously, starting from $p(x_{n+1}, x_n) = p(F(x_n, y_n), F(x_{n-1}, y_{n-1}))$, we have

$$\begin{aligned}
 p(x_{n+1}, x_n) &= p(F(x_n, y_n), F(x_{n-1}, y_{n-1})) \\
 &\leq \alpha_1 p(x_n, x_{n-1}) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(F(x_n, y_n), x_n) + \alpha_4 p(F(y_n, x_n), y_n) \\
 &\quad + \alpha_5 p(F(x_n, y_n), x_{n-1}) + \alpha_6 p(F(y_n, x_n), y_{n-1}) + \alpha_7 p(F(x_{n-1}, y_{n-1}), x_n) \\
 &\quad + \alpha_8 p(F(y_{n-1}, x_{n-1}), y_n) + \alpha_9 p(F(x_{n-1}, y_{n-1}), x_{n-1}) + \alpha_{10} p(F(y_{n-1}, x_{n-1}), y_{n-1}) \\
 &= \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(x_{n+1}, x_n) + \alpha_4 p(y_{n+1}, y_n) + \alpha_5 p(x_{n+1}, x_{n-1}) \\
 &\quad + \alpha_6 p(y_{n+1}, y_{n-1}) + \alpha_7 p(x_n, x_n) + \alpha_8 p(y_n, y_n) + \alpha_9 p(x_n, x_{n-1}) + \alpha_{10} p(y_n, y_{n-1}) \\
 &\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(x_{n+1}, x_n) + \alpha_4 p(y_{n+1}, y_n) \\
 &\quad + \alpha_5 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] + \alpha_6 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})]
 \end{aligned}$$

$$\begin{aligned}
& + (\alpha_7 - \alpha_5)p(x_n, x_n) + (\alpha_8 - \alpha_6)p(y_n, y_n) \\
& + \alpha_9 p(x_n, x_{n-1}) + \alpha_{10} p(y_n, y_{n-1}).
\end{aligned} \tag{2.4}$$

In a similar way, we have

$$\begin{aligned}
p(y_n, y_{n+1}) &= p(F(y_{n-1}, x_{n-1}), F(y_n, x_n)) \\
&\leq \alpha_1 p(y_{n-1}, y_n) + \alpha_2 p(x_{n-1}, x_n) + \alpha_3 p(y_n, y_{n-1}) + \alpha_4 p(x_n, x_{n-1}) \\
&\quad + (\alpha_5 - \alpha_7)p(y_n, y_n) + (\alpha_6 - \alpha_8)p(x_n, x_n) \\
&\quad + \alpha_7 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})] + \alpha_8 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] \\
&\quad + \alpha_9 p(y_{n+1}, y_n) + \alpha_{10} p(x_{n+1}, x_n).
\end{aligned} \tag{2.5}$$

Analogously, starting from $p(y_{n+1}, y_n) = p(F(y_n, x_n), F(y_{n-1}, x_{n-1}))$, we have

$$\begin{aligned}
p(y_{n+1}, y_n) &= p(F(y_n, x_n), F(y_{n-1}, x_{n-1})) \\
&\leq \alpha_1 p(y_{n-1}, y_n) + \alpha_2 p(x_{n-1}, x_n) + \alpha_3 p(y_{n+1}, y_n) + \alpha_4 p(x_{n+1}, x_n) \\
&\quad + \alpha_5 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})] + \alpha_6 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] \\
&\quad + (\alpha_7 - \alpha_5)p(y_n, y_n) + (\alpha_8 - \alpha_6)p(x_n, x_n) \\
&\quad + \alpha_9 p(y_n, y_{n-1}) + \alpha_{10} p(x_n, x_{n-1}).
\end{aligned} \tag{2.6}$$

Adding (2.3), (2.4), (2.5), and (2.6) we obtain that

$$\begin{aligned}
2\delta_{n+1} &\leq [2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}]\delta_n \\
&\quad + [\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}]\delta_{n+1},
\end{aligned} \tag{2.7}$$

or, equivalently,

$$\delta_{n+1} \leq \lambda \delta_n, \tag{2.8}$$

where, $\lambda = [2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}] / (2 - [\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}])$.

Repeating the above mentioned process, we have

$$\delta_{n+1} \leq \lambda \delta_n \leq \lambda^2 \delta_{n-1} \leq \dots \leq \lambda^{n+1} \delta_0, \tag{2.9}$$

where, from our assumption about coefficients α_i , $\lambda \in [0, 1)$; hence,

$$\lim_{n \rightarrow \infty} \delta_n = 0. \tag{2.10}$$

Step II. $\{x_n\}$ and $\{y_n\}$ are Cauchy.

If $\delta_0 = 0$ then, $p(x_0, x_1) + p(y_0, y_1) = 0$. Hence, we get $x_0 = x_1 = F(x_0, y_0)$ and $y_0 = y_1 = F(y_0, x_0)$; that is, (x_0, y_0) is a coupled fixed point of F . Now, let $\delta_0 > 0$. For each $m \geq n$, we have

$$\begin{aligned}
 p(x_m, x_n) + p(y_m, y_n) &\leq p(x_m, x_{m-1}) + p(y_m, y_{m-1}) \\
 &\quad + p(x_{m-1}, x_{m-2}) + p(y_{m-1}, y_{m-2}) \\
 &\quad + \cdots \\
 &\quad + p(x_{n+1}, x_n) + p(y_{n+1}, y_n) \\
 &= \delta_m + \delta_{m-1} + \cdots + \delta_{n+1} \\
 &\leq [\lambda^m + \lambda^{m-1} + \cdots + \lambda^{n+1}] \delta_0 \leq \frac{\lambda^{n+1}}{1-\lambda} \delta_0.
 \end{aligned} \tag{2.11}$$

So, we have $\lim_{n,m \rightarrow \infty} p(x_n, x_m) + p(y_n, y_m) = 0$. This proves that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in (X, p) and hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in the metric space (X, d_p) . From Lemma 2.1, (X, d_p) is complete, so $\{x_n\}$ and $\{y_n\}$ converge to some $x, y \in X$, respectively; that is, $\lim_{n \rightarrow \infty} d_p(x_n, x) = 0$ and $\lim_{n \rightarrow \infty} d_p(y_n, y) = 0$. Therefore, from Lemma 2.1 and (2.10), we have

$$p(x, x) = \lim_{n \rightarrow \infty} p(x_n, x) = \lim_{n,m \rightarrow \infty} p(x_n, x_m) = 0, \tag{2.12}$$

$$p(y, y) = \lim_{n \rightarrow \infty} p(y_n, y) = \lim_{n,m \rightarrow \infty} p(y_n, y_m) = 0. \tag{2.13}$$

Step III. We will show that F has a unique coupled fixed point. From the above step,

$$\lim_{n \rightarrow \infty} p(F(x_n, y_n), x) = \lim_{n \rightarrow \infty} p(F(y_n, x_n), y) = 0. \tag{2.14}$$

Next, we will prove that $x = F(x, y)$ and $y = F(y, x)$.

We have

$$p(x, F(x, y)) \leq p(x, x_{n+1}) + p(x_{n+1}, F(x, y)) - p(x_{n+1}, x_{n+1}). \tag{2.15}$$

Taking the limit as $n \rightarrow \infty$ in the above inequality, as $x_{n+1} = F(x_n, y_n)$ and using triangle inequality and (2.12), we have

$$\begin{aligned}
 p(x, F(x, y)) &\leq \lim_{n \rightarrow \infty} p(x, x_{n+1}) + \lim_{n \rightarrow \infty} p(F(x_n, y_n), F(x, y)) \\
 &= \lim_{n \rightarrow \infty} p(F(x_n, y_n), F(x, y)).
 \end{aligned} \tag{2.16}$$

But, for all $n \geq 0$, from (2.2),

$$\begin{aligned}
 p(F(x_n, y_n), F(x, y)) &\leq \alpha_1 p(x_n, x) + \alpha_2 p(y_n, y) \\
 &\quad + \alpha_3 p(F(x_n, y_n), x_n) + \alpha_4 p(F(y_n, x_n), y_n) \\
 &\quad + \alpha_5 p(F(x_n, y_n), x) + \alpha_6 p(F(y_n, x_n), y) + \alpha_7 p(F(x, y), x_n) \\
 &\quad + \alpha_8 p(F(y, x), y_n) + \alpha_9 p(F(x, y), x) + \alpha_{10} p(F(y, x), y) \\
 &= \alpha_1 p(x_n, x) + \alpha_2 p(y_n, y) + \alpha_3 p(x_{n+1}, x) \\
 &\quad + \alpha_4 p(y_{n+1}, y) + \alpha_5 p(x_{n+1}, x) + \alpha_6 p(y_{n+1}, y) + \alpha_7 p(F(x, y), x_n) \\
 &\quad + \alpha_8 p(F(y, x), y_n) + \alpha_9 p(F(x, y), x) + \alpha_{10} p(F(y, x), y).
 \end{aligned} \tag{2.17}$$

In the above inequality, if $n \rightarrow \infty$, using (2.12) and Lemma 2.2 we have

$$\lim_{n \rightarrow \infty} p(F(x_n, y_n), F(x, y)) \leq (\alpha_7 + \alpha_9) p(F(x, y), x) + (\alpha_8 + \alpha_{10}) p(F(y, x), y). \tag{2.18}$$

Analogously,

$$p(y, F(y, x)) \leq p(y, y_{n+1}) + p(y_{n+1}, F(y, x)) - p(y_{n+1}, y_{n+1}). \tag{2.19}$$

Taking the limit as $n \rightarrow \infty$ in the above inequality, since $y_{n+1} = F(y_n, x_n)$ and using triangle inequality and (2.13), we have

$$\begin{aligned}
 p(y, F(y, x)) &\leq \lim_{n \rightarrow \infty} p(y, y_{n+1}) + \lim_{n \rightarrow \infty} p(F(y_n, x_n), F(y, x)) \\
 &= \lim_{n \rightarrow \infty} p(F(y_n, x_n), F(y, x)).
 \end{aligned} \tag{2.20}$$

Similar to (2.17), we have

$$\lim_{n \rightarrow \infty} p(F(y_n, x_n), F(y, x)) \leq (\alpha_7 + \alpha_9) p(F(y, x), y) + (\alpha_8 + \alpha_{10}) p(F(x, y), x). \tag{2.21}$$

Adding (2.18) and (2.21) and using (2.15) and (2.19), we obtain that

$$\begin{aligned}
 p(x, F(x, y)) + p(y, F(y, x)) \\
 \leq (\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}) [p(x, F(x, y)) + p(y, F(y, x))].
 \end{aligned} \tag{2.22}$$

Therefore, $p(x, F(x, y)) + p(y, F(y, x)) = 0$; that is, $F(x, y) = x$ and $F(y, x) = y$. $\square$

Remark 2.5. (1) If in the above theorem, we assume that $\alpha_i = 0$, for all $3 \leq i \leq 10$, then we obtain the result of Aydi in [11] which is noted here in Theorem 1.5.

(2) If in the above theorem, $\alpha_i = 0$, for all $1 \leq i \leq 10$, unless $i = 3, 9$, then we obtain the result of Aydi in [11] which is mentioned here in Theorem 1.6.

(3) If in the above theorem, we assume that $\alpha_i = 0$, for all $3 \leq i \leq 10$, except that $i \neq 5, 7$, then we obtain the result of Aydi in [11] (Theorem 1.7).

Many results can be deduced from the above theorem as follows.

Corollary 2.6. *Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that*

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \alpha_1 p(F(x, y), x) + \alpha_2 p(F(y, x), y) \\ &\quad + \alpha_3 p(F(u, v), u) + \alpha_4 p(F(v, u), v), \end{aligned} \quad (2.23)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $\sum_{i=1}^4 \alpha_i < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.7. *Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that*

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \alpha_1 p(F(x, y), u) + \alpha_2 p(F(y, x), v) \\ &\quad + \alpha_3 p(F(u, v), x) + \alpha_4 p(F(v, u), y), \end{aligned} \quad (2.24)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $\sum_{i=1}^4 \alpha_i < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.8. *Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that*

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \alpha_1 p(F(x, y), x) + \alpha_2 p(F(y, x), y) \\ &\quad + \alpha_3 p(F(u, v), x) + \alpha_4 p(F(v, u), y), \end{aligned} \quad (2.25)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.9. *Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that*

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \alpha_1 p(F(x, y), u) + \alpha_2 p(F(y, x), v) \\ &\quad + \alpha_3 p(F(u, v), u) + \alpha_4 p(F(v, u), v), \end{aligned} \quad (2.26)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $\sum_{i=1}^4 \alpha_i < 1$. Then, F has a unique coupled fixed point in X .

Also, we have the following results, when the constants in the above corollaries are equal.

Corollary 2.10. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(x, u) + p(y, v)] \\ &\quad + \frac{l}{2} [p(F(x, y), x) + p(F(y, x), y)] \\ &\quad + \frac{r}{2} [p(F(x, y), u) + p(F(y, x), v)] + \frac{s}{2} [p(F(u, v), x) + p(F(v, u), y)] \\ &\quad + \frac{t}{2} [p(F(u, v), u) + p(F(v, u), v)], \end{aligned} \quad (2.27)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l + r + s + t < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.11. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(F(x, y), x) + p(F(y, x), y)] \\ &\quad + \frac{l}{2} [p(F(u, v), u) + p(F(v, u), v)], \end{aligned} \quad (2.28)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.12. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(F(x, y), u) + p(F(y, x), v)] \\ &\quad + \frac{l}{2} [p(F(u, v), x) + p(F(v, u), y)], \end{aligned} \quad (2.29)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.13. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(F(x, y), x) + p(F(y, x), y)] \\ &\quad + \frac{l}{2} [p(F(u, v), x) + p(F(v, u), y)], \end{aligned} \quad (2.30)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.14. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(x, u) + p(y, v)] \\ &+ \frac{l}{4} [p(F(x, y), x) + p(F(y, x), y) + p(F(u, v), u) + p(F(v, u), v)] \\ &+ \frac{r}{4} [p(F(x, y), u) + p(F(y, x), v) + p(F(u, v), x) + p(F(v, u), y)], \end{aligned} \quad (2.31)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l + r < 1$. Then, F has a unique coupled fixed point in X .

Corollary 2.15. Let (X, p) be a complete partial metric space and $F : X \times X \rightarrow X$ be a mapping such that

$$\begin{aligned} p(F(x, y), F(u, v)) &\leq \frac{k}{2} [p(F(x, y), u) + p(F(y, x), v)] \\ &+ \frac{l}{2} [p(F(u, v), u) + p(F(v, u), v)], \end{aligned} \quad (2.32)$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l < 1$. Then F has a unique coupled fixed point in X .

Example 2.16. Let $X = [0, \infty)$ and p on X be given as $p(a, b) = \max\{a, b\}$. Obviously, the partial metric space (X, p) is complete (see, e.g., Example 2.3 of [11]).

Define $F : X \times X \rightarrow X$ as $F(x, y) = (x + y)/30$, for all $x, y \in X$.

Now, we have

$$\begin{aligned} p(F(x, y), F(u, v)) &= \frac{1}{30} \max\{x + y, u + v\} \\ &\leq \frac{1}{27} [\max\{x, u\} + \max\{y, v\}] \\ &\leq \frac{1}{27} [\max\{x, u\} + \max\{y, v\}] \\ &\quad + \frac{1}{27} \left[\max\left\{\frac{x + y}{30}, x\right\} + \max\left\{\frac{y + x}{30}, y\right\} \right] \\ &\quad + \frac{1}{27} \left[\max\left\{\frac{x + y}{30}, u\right\} + \max\left\{\frac{y + x}{30}, v\right\} \right] \\ &\quad + \frac{1}{27} \left[\max\left\{\frac{u + v}{30}, x\right\} + \max\left\{\frac{v + u}{30}, y\right\} \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{27} \left[\max \left\{ \frac{u+v}{30}, u \right\} + \max \left\{ \frac{v+u}{30}, v \right\} \right] \\
& = \alpha_1 p(x, u) + \alpha_2 p(y, v) + \alpha_3 p(F(x, y), x) + \alpha_4 p(F(y, x), y) \\
& \quad + \alpha_5 p(F(x, y), u) + \alpha_6 p(F(y, x), v) + \alpha_7 p(F(u, v), x) \\
& \quad + \alpha_8 p(F(v, u), y) + \alpha_9 p(F(u, v), u) + \alpha_{10} p(F(v, u), v).
\end{aligned} \tag{2.33}$$

Thus, (2.2) is satisfied with $\alpha_i = 1/27$. Obviously, all the conditions of Theorem 2.4 are satisfied. Moreover, $(0, 0)$ is the unique coupled fixed point of F .

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