Research Article

On Symmetric Left Bi-Derivations in BCI-Algebras

G. Muhiuddin, 1 Abdullah M. Al-roqi, 1 Young Bae Jun, 2 and Yilmaz Ceven 3

1 Department of Mathematics, University of Tabuk, Tabuk 71491, Saudi Arabia
2 Department of Mathematics Education (and RINS), Gyeongsang National University, Jinju 660-701, Republic of Korea
3 Department of Mathematics, Faculty of Science, Suleyman Demirel University, 32260 Isparta, Turkey

Correspondence should be addressed to G. Muhiuddin; chishtygm@gmail.com

Received 15 February 2013; Accepted 30 May 2013

Copyright © 2013 G. Muhiuddin et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The notion of symmetric left bi-derivation of a BCI-algebra X is introduced, and related properties are investigated. Some results on componentwise regular and d-regular symmetric left bi-derivations are obtained. Finally, characterizations of a p-semisimple BCI-algebra are explored, and it is proved that, in a p-semisimple BCI-algebra, F is a symmetric left bi-derivation if and only if it is a symmetric bi-derivation.

1. Introduction

BCI-algebras and BCI-algebras are two classes of nonclassical logic algebras which were introduced by Imai and Iséki in 1966 [1, 2]. They are algebraic formulation of BCK-system and BCI-system in combinatory logic. Later on, the notion of BCI-algebras has been extensively investigated by many researchers (see [3–6], and references therein). The notion of a BCI-algebra generalizes the notion of a BCK-algebra in the sense that every BCK-algebra is a BCI-algebra but not vice versa (see [7]). Hence, most of the algebras related to the t-norm-based logic such as MTL [8], BL, hoop, MV [9] (i.e lattice implication algebra), and Boolean algebras are extensions of BCK-algebras (i.e. they are subclasses of BCK-algebras) which have a lot of applications in computer science (see [10]). This shows that BCK-/BCI-algebras are considerably general structures.

Throughout our discussion, X will denote a BCI-algebra unless otherwise mentioned. In the year 2004, Jun and Xin [11] applied the notion of derivation in ring and near-ring theory to BCI-algebras, and as a result they introduced a new concept, called a (regular) derivation, in BCI-algebras. Using this concept as defined they investigated some of its properties. Using the notion of a regular derivation, they also established characterizations of a p-semisimple BCI-algebra. For a self-map d of a BCI-algebra, they defined a d-invariant ideal and gave conditions for an ideal to be d-invariant. According to Jun and Xin, a self map d : X &rarr; X is called a left-right derivation (briefly (l, r)-derivation) of X if d(x * y) = d(x) * y &amp; x * d(y) holds for all x, y ∈ X. Similarly, a self map d : X &rarr; X is called a right-left derivation (briefly (r, l)-derivation) of X if d(x * y) = x * d(y) &amp; d(x) * y holds for all x, y ∈ X. Moreover, if d is both (l, r)- and (r, l)-derivation, it is a derivation on X. After the work of Jun and Xin [11], many research articles have appeared on the derivations of BCI-algebras and a greater interest has been devoted to the study of derivations in BCI-algebras on various aspects (see [12–17]).

Inspired by the notions of σ-derivation [18], left derivation [19], and symmetric bi-derivations [20, 21] in rings and near-rings theory, many authors have applied these notions in a similar way to the theory of BCI-algebras (see [12, 13, 17]). For instance in 2005 [17], Zhan and Liu have given the notion of f-derivation of BCI-algebras as follows: a self map d_f : X &rarr; X is said to be a left-right f-derivation or (l, r)-f-derivation of X if it satisfies the identity d_f(x * y) = d_f(x) * f(y) &amp; f(x) * d_f(y) for all x, y ∈ X. Similarly, a self map d_f : X &rarr; X is said to be a right-left f-derivation or (r, l)-f-derivation of X if it satisfies the identity d_f(x * y) = f(x) * d_f(y) &amp; d_f(x) * f(y) for all x, y ∈ X. Moreover, if d_f is both (l, r)- and (r, l)-f-derivation, it is said that d_f is an f-derivation, where f is an endomorphism. In the year 2007, Abujabal and Al-Shehri [12] defined and studied the notion of left derivation of BCI-algebras as follows: a self map D : X &rarr; X is said to be a left
derivation of $X$ if satisfying $D(x * y) = x * D(y) \land y * D(x)$ for all $x, y \in X$. Furthermore, in 2011 [13], Ilbira et al. have introduced the notion of symmetric bi-derivations in $BCI$-algebras. Following [13], a mapping $D(\cdot, \cdot) : X \times X \to X$ is said to be symmetric if $F(x, y) = F(y, x)$ holds for all pairs $x, y \in X$. A symmetric mapping $D(\cdot, \cdot) : X \times X \to X$ is called left-right symmetric bi-derivation (briefly, $(l, r)$-symmetric bi-derivation) if it satisfies the identity $D(x * y, z) = x * D(y, z) \land D(x, z) * y$ for all $x, y, z \in X$. $D$ is called right-left symmetric bi-derivation (briefly, $(r, l)$-symmetric bi-derivation) if it satisfies the identity $D(x * y, z) = x * D(y, z) \land D(x, z) * y$ for all $x, y, z \in X$. Moreover, if $D$ is both a $(l, r)$- and a $(r, l)$-symmetric bi-derivation, it is said that $D$ is a symmetric bi-derivation on $X$.

Motivated by the notion of symmetric bi-derivations [13] in the theory of $BCI$-algebras, in the present analysis, we introduced the notion of symmetric left bi-derivations on $BCI$-algebras and investigated related properties. Further, we obtain some results on componentwise regular and $d$-regular symmetric left bi-derivations. Finally, we characterize the notion of $p$-semisimple $BCI$-algebra $X$ by using the concept of symmetric left bi-derivation and show that, in a symmetric bi-derivation if and only if it is a symmetric bi-derivation.

2. Preliminaries

We begin with the following definitions and properties that will be needed in the sequel.

A nonempty set $X$ with a constant 0 and a binary operation $*$ is called a $BCI$-algebra if for all $x, y, z \in X$ the following conditions hold:

(I) \( ((x \ast y) \ast (x \ast z)) \ast (z \ast y) = 0 \),

(II) \( (x \ast (x \ast y)) \ast y = 0 \),

(III) \( x \ast x = 0 \),

(IV) \( x \ast y = 0 \) and \( y \ast x = 0 \) imply \( x = y \).

Define a binary relation $\leq$ on $X$ by letting $x \ast y = 0$ if and only if $x \leq y$. Then $(X, \leq)$ is a partially ordered set. A $BCI$-algebra $X$ satisfying $0 \leq x$ for all $x \in X$ is called $BCK$-algebra.

A $BCI$-algebra $X$ has the following properties for all $x, y, z \in X$.

(a1) \( x \ast 0 = x \).

(a2) \( (x \ast y) \ast z = (x \ast z) \ast y \).

(a3) \( x \leq y \) implies $x \ast z \leq y \ast z$ and $z \ast y \leq z \ast x$.

(a4) \( (x \ast z) \ast (y \ast z) \leq x \ast y \).

(a5) \( x \ast (x \ast (x \ast y)) = x \ast y \).

(a6) \( 0 \ast (x \ast y) = (0 \ast x) \ast (0 \ast y) \).

(a7) \( x \ast 0 = 0 \) implies $x = 0$.

For a $BCI$-algebra $X$, denote by $X_+$ (resp., $G(X)$) the $BCK$-part (resp., the $BCI$-$G$ part) of $X$; that is, $X_+$ is the set of all $x \in X$ such that $0 \leq x$ (resp., $G(X) := \{x \in X \mid 0 \ast x = x\}$). Note that $G(X) \cap X_+ = \{0\}$ (see [22]).

If $X_+ = \{0\}$, then $X$ is called a $p$-semisimple $BCI$-algebra. In a $p$-semisimple $BCI$-algebra $X$, the following hold.

(a8) \( (x \ast z) \ast (y \ast z) = x \ast y \).

(a9) \( 0 \ast (0 \ast x) = x \) for all $x \in X$.

(a10) \( x \ast (0 \ast y) = y \ast (0 \ast x) \).

(a11) \( x \ast y = 0 \) implies $x = y$.

(a12) \( x \ast a = x \ast b \) implies $a = b$.

(a13) \( a \ast x = b \ast x \) implies $a = b$.

(a14) \( (a \ast x) = x \).

(a15) \( (x \ast y) \ast (u \ast z) = (x \ast w) \ast (y \ast z) \).

Let $X$ be a $p$-semisimple $BCI$-algebra. We define addition “+” as $x + y = x \ast (0 \ast y)$ for all $x, y \in X$. Then $(X, +)$ is an abelian group with identity 0 and $x - y = x + y$. Conversely, let $(X, +)$ be an abelian group with identity 0, and let $x + y = x - y$. Then $X$ is a $p$-semisimple $BCI$-algebra and $x + y = x \ast (0 \ast y)$ for all $x, y \in X$ (see [6]).

For a $BCI$-algebra $X$, we denote $x \land y = y \ast (y \ast x)$, in particular $0 \ast (0 \ast x) = a_x$, and $L_p(X) := \{a \in X \mid a \ast a = 0 \Rightarrow a = x, \forall x \in X\}$. We call the elements of $L_p(X)$ the $p$-atoms of $X$. For any $a \in X$, let $V(a) := \{x \in X \mid a \ast x = 0\}$, which is called the branch of $X$ with respect to $a$. It follows that $x \ast y \in V(a \ast b)$ whenever $x \in V(a)$ and $y \in V(b)$ for all $x, y \in X$ and all $a, b \in L_p(X)$. Note that $L_p(X) = \{x \in X \mid a_x = x\}$, which is the $p$-semisimple part of $X$, and $X$ is a $p$-semisimple $BCI$-algebra if and only if $L_p(X) = X$ (see [23, Proposition 3.2]). Note also that $a_x \in L_p(X)$; that is, $0 \ast (0 \ast a_x) = a_x$, which implies that $a_x \ast y \in L_p(X)$ for all $y \in X$. It is clear that $G(X) \subseteq L_p(X)$, and $x \ast (x \ast a) = a \ast x \in L_p(X)$ for all $a \in L_p(X)$ and all $x \in X$. Let $D(\cdot, \cdot) : X \times X \to X$ be a symmetric mapping. Then for all $x \in X$, a mapping $d : X \to X$ defined by $d(x) = D(x, x)$ is called trace of $D$ [13]. For more details, refer to [3, 4, 6, 11, 22, 23].

3. Symmetric Left Bi-Derivations

The following definition introduces the notion of symmetric left bi-derivation for a $BCI$-algebra $X$.

Definition 1. A symmetric mapping $F(\cdot, \cdot) : X \times X \to X$ is called a symmetric left bi-derivation of $X$ if it satisfies the following identity:

\[ (\forall x, y, z \in X) \quad (F(x \ast y, z) = (x \ast F(y, z)) \land (y \ast F(x, z))) \] (1)

Example 2 (see [24]). Consider a $p$-semisimple $BCI$-algebra $X = \{0, 3, 4, 5\}$ with the following Cayley table:

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

(2)
Define a mapping \( F(\cdot, \cdot) : X \times X \to X \) by
\[
F(0,0) = F(3,3) = F(4,4) = F(5,5) = 0,
F(0,3) = F(3,0) = 3,
F(0,4) = F(4,0) = 4,
F(0,5) = F(5,0) = 5,
F(3,4) = F(4,3) = 5,
F(3,5) = F(5,3) = 4,
F(4,5) = F(5,4) = 3.
\]

It is routine to verify that \( F \) is a symmetric left bi-derivation of \( X \).

**Theorem 3.** Let \( F(\cdot, \cdot) : X \times X \to X \) be a symmetric left bi-derivation of \( X \). Then

1. \((\forall z \in X) (a \in G(X) \Rightarrow F(a, z) \in G(X))\).
2. \((\forall z \in X) (a \in L_p(X) \Rightarrow F(a, z) \in L_p(X))\).
3. \((\forall z \in X) (a + F(0, z) = 0 \Rightarrow F(a, z) = 0 + F(0, z))\).
4. \((\forall z \in X) (a \in L_p(X) \Rightarrow F(a, z) = a + F(0, z))\).

**Proof.**

1. Let \( a \in G(X) \). Then \( 0 \cdot a = a \), and so
\[
F(a, z) = F(0 \cdot a, z) = (0 \cdot F(a, z)) \land (a \cdot F(0, z)) = (a \cdot F(0, z)) \land ((a \cdot F(0, z)) \land (0 \cdot F(a, z))) = 0 \cdot F(a, z),
\]
   where \( 0 \cdot F(a, z) \in L_p(X) \). Hence \( F(a, z) \in G(X) \).

2. For any \( a \in L_p(X) \) implies \( a = 0 \cdot (0 \cdot a) \) and so
\[
F(a, z) = F(0 \cdot (0 \cdot a), z) = (0 \cdot F(0 \cdot a, z)) \land (0 \cdot a \cdot F(0, z)) = (0 \cdot a) \cdot F(0, z) = 0 \cdot (0 \cdot F(0, z)) \land (0 \cdot F(0, z)) = 0 \cdot F(0, z),
\]
   Since \( 0 \cdot F(a, z) \in L_p(X) \). Hence \( F(a, z) \in L_p(X) \).

3. By (2), we have \( F(a, z) \in L_p(X) \). Then
\[
F(a, z) = 0 \cdot (0 \cdot F(a, z)) = 0 + F(a, z).
\]

4. For any \( a \in L_p(X) \) and \( z \in X \), we have
\[
F(a, z) = F(0 \cdot a, z) = (a \cdot F(0, z)) \land (0 \cdot F(a, z)) = (a \cdot F(0, z)) \land ((0 \cdot F(a, z)) \land (a \cdot F(0, z))) = a \cdot F(0, z) = a \cdot (0 \cdot F(0, z)) = a + F(0, z).
\]

This completes the proof.

Using Theorem 3, we have the following corollary.

**Corollary 4.** Let \( F(\cdot, \cdot) : X \times X \to X \) be a symmetric left bi-derivation and \( d : X \to X \) be the trace of \( F \). Then

1. \((\forall a \in G(X)) (d(a) \in G(X))\).
2. \((\forall a \in L_p(X)) (d(a) \in L_p(X))\).

**Theorem 5.** Let \( F \) be a symmetric left bi-derivation of \( X \).

Then

1. \((\forall z \in X) (a, b \in L_p(X) \Rightarrow F(a + b, z) = a + F(b, z))\).
2. \((\forall z \in X) (a \in L_p(X) \Rightarrow F(a, z) = a \text{ if and only if } F(0, z) = 0)\).
3. \((\forall x, y, z \in X) (x \cdot F(y, z) = x \cdot F(y, z))\).
4. \((\forall x, y, z \in X) (x \cdot F(x, z) = y \cdot F(y, z))\).

**Proof.**

1. Let \( a, b \in L_p(X) \). Then
\[
F(a + b, z) = F(a \cdot (0 \cdot b), z) = a \cdot F(0 \cdot b, z) \land (0 \cdot b) \cdot F(a, z) = a \cdot F(0 \cdot b, z) \land (0 \cdot F(a, z)) = a \cdot (0 \cdot F(b, z)) = a + F(b, z).
\]

2. Suppose \( F(a, z) = a \) for all \( a \in L_p(X), z \in X \). It is clear that, for \( 0 \in L_p(X) \), we have \( F(0, z) = 0 \). Conversely let us assume that \( F(0, z) = 0 \); then by using Theorem 3(4), we have \( F(a, z) = a + F(0, z) = a + 0 = a \).

3. For any \( x, y, z \in X \), we have
\[
F(x \cdot F(y, z)) = (x \cdot F(y, z)) \land (y \cdot F(x, z)) = (y \cdot F(x, z)) \land ((y \cdot F(x, z)) \land (x \cdot F(y, z))) \leq x \cdot F(y, z).
\]

4. For any \( x, y, z \in X \), we have
\[
F(0, z) = F(x \cdot x, z) = (x \cdot F(x, z)) \land (x \cdot F(x, z)) = x \cdot F(x, z).
\]

Thus, we can write \( F(0, z) = x \cdot F(x, z) = y \cdot F(y, z) \) for any \( y \in X \). This completes the proof.

**Definition 6.** A symmetric left bi-derivation \( F(\cdot, \cdot) : X \times X \to X \) of a BCI-algebra \( X \) is said to be componentwise regular if \( F(0, z) = 0 \) for all \( z \in X \). In particular, \( F \) is called \( d \)-regular if \( F(0, 0) = d(0) = 0 \).

**Theorem 7.** Let \( F \) be a symmetric left bi-derivation of BCI-algebra \( X \). Then \( X \) is a BCK-algebra if and only if \( F \) is componentwise regular symmetric left bi-derivation.
Proof. Suppose $X$ is a $BCK$-algebra. Then for any $x, z \in X$, we have

$$F(0, z) = F(0 \ast x, z)$$
$$= (0 \ast F(x, z)) \wedge (x \ast F(0, z))$$
$$= 0 \wedge (x \ast F(0, z)) = 0. \quad (11)$$

Hence $F$ is componentwise regular.

Conversely, let $F$ be a componentwise regular symmetric left bi-derivation. Let for any $a \in L_p(X)$ be such that $a \neq 0$. Then

$$F(a \ast 0, 0) = F(a, 0) = 0. \quad (12)$$

But it is clear that

$$a \ast F(0, 0) \wedge 0 \ast F(a, 0) = a \ast 0 \wedge 0 \ast 0$$
$$= a \neq 0,$$

which is not possible as $F$ is a componentwise regular symmetric left bi-derivation. Thus $0$ is the unique $p$-atom.

Assume that for some $m \in X$, we have $0 \ast m \neq 0$, then $a_{0, m} = 0 \ast (0 \ast (0 \ast m)) = 0$, so $0 \ast m \in L_p(X)$, which is a contradiction. Henceforth, for all $m \in X$, we have $0 \ast m = 0$ implying thereby, $X$ is a $BCK$-algebra.

This completes the proof.

Theorem 8. Let $F$ be a componentwise regular symmetric left bi-derivation of a $BCI$-algebra $X$. Then

(1) Both $x$ and $F(x, z)$ belong to the same branch for all $x, z \in X$.

(2) $(\forall x, z \in X) (F(x, z) \leq x)$.

(3) $(\forall x, y, z \in X) (F(x, z) \ast y \leq x \ast F(y, z))$.

Proof. (1) For any $x, z \in X$, we get

$$0 = F(0, z) = F(a_x \ast x, z)$$
$$= (a_x \ast F(x, z)) \wedge (x \ast F(a_x, z))$$
$$= (x \ast F(a_x, z)) \ast ((x \ast F(a_x, z)) \ast (a_x \ast F(x, z)))$$
$$= a_x \ast F(x, z), \quad (14)$$

since $a_x \ast F(x, z) \in L_p(X)$. Hence $a_x \leq F(x, z)$, and so $F(x, z) \in V(a_x)$. Obviously, $x \in V(a_x)$.

(2) Since $F$ is componentwise regular, $F(0, z) = 0$. Then

$$F(x, z) = F(x \ast 0, z)$$
$$= (x \ast F(0, z)) \wedge (0 \ast F(x, z))$$
$$= (x \ast 0) \wedge (0 \ast F(x, z))$$
$$= (0 \ast F(x, z)) \ast ((0 \ast F(x, z)) \ast x)$$
$$\leq x. \quad (15)$$

(3) Since $F(x, z) \leq x$ for all $x, z \in X$ by (2), using (a3) we obtain

$$F(x, z) \ast y \leq x \ast y \leq x \ast F(y, z). \quad (16)$$

This completes the proof.

Next, we prove some results in a $p$-semisimple $BCI$-algebra.

Theorem 9. Let $F$ be a symmetric left bi-derivation of a $p$-semisimple $BCI$-algebra $X$; one has the following assertions.

(1) $(\forall x, y, z \in X) (F(x \ast y, z) = x \ast F(y, z)).$

(2) $(\forall x, y, z \in X) (F(x, z) \ast x = F(y, z) \ast y).$

(3) $(\forall x, y, z \in X) (F(x, z) \ast x = y \ast F(y, z)).$

Proof. (1) Let $X$ be a $p$-semisimple $BCI$-algebra. Then for any $x, y, z \in X$, we have

$$F(x \ast y, z) = (x \ast F(y, z)) \wedge (y \ast F(x, z)) = x \ast F(y, z). \quad (17)$$

(2) Let $x, y, z \in X$. Using (I), we have

$$(x \ast y) \ast (x \ast F(y, z)) \leq F(y, z) \ast y,$$

$$(y \ast x) \ast (y \ast F(x, z)) \leq F(x, z) \ast x. \quad (18)$$

These above inequalities can be rewritten as

$$((x \ast y) \ast (x \ast F(y, z))) \ast (F(y, z) \ast y) = 0,$$

$$((y \ast x) \ast (y \ast F(x, z))) \ast (F(x, z) \ast x) = 0. \quad (19)$$

Consequently, we get

$$((x \ast y) \ast (x \ast F(y, z))) \ast (F(y, z) \ast y)$$
$$= ((y \ast x) \ast (y \ast F(x, z))) \ast (F(x, z) \ast x) \quad (20)$$

Also, using Theorem 5(4) and (I), we obtain

$$(x \ast y) \ast F(x \ast y, z) = (y \ast x) \ast F(y \ast x, z)$$
$$\implies (x \ast y) \ast (x \ast F(y, z)) = (y \ast x) \ast (y \ast F(x, z)). \quad (21)$$

Since $X$ is a $p$-semisimple $BCI$-algebra, hence, by using (21) and (a12), the above (20) yields $F(x, z) \ast x = F(y, z) \ast y$. 

(3) We have $F(0, z) = x \ast F(x, z)$ by Theorem 5(4). Further, on letting $x = 0$, we get that $F(0, z) \ast 0 = F(y, z) \ast y$ implies $F(0, z) = F(y, z) \ast y$. Henceforth $F(y, z) \ast y = x \ast F(x, z)$, which amounts to say that $F(x, z) \ast x = y \ast F(y, z).$

This completes the proof.

Theorem 10. Let $X$ be a $p$-semisimple $BCI$-algebra. Then $F$ is a symmetric left bi-derivation if and only if it is a symmetric bi-derivation on $X$.\qed
Proof. Suppose that $F$ is a symmetric left bi-derivation on $X$. First, we show that $F$ is a $(r,l)$-symmetric bi-derivation on $X$. Let $x, y, z \in X$. Using Theorem 9(1) and (a14), we have
\[
F(x \ast y, z) = x \ast F(y, z) \\
= (x \ast 0) \ast F(y, z) \\
= (x \ast (F(0, z) \ast F(0, z))) \ast F(y, z) \\
= (x \ast ((x \ast F(x, z)) \ast (F(y, z) \ast y))) \\
\ast F(y, z) \\
= (x \ast F(y, z)) \\
\ast ((x \ast F(x, z)) \ast (F(y, z) \ast y)) \\
= (x \ast F(y, z) \ast (x \ast F(y, z))) \\
= (F(x, z) \ast y) \land (x \ast F(y, z)) .
\]
Hence $F$ is a $(r,l)$-symmetric bi-derivation on $X$.

Again, we show that $F$ is a $(l,r)$-symmetric bi-derivation on $X$. Let $x, y, z \in X$. Using Theorem 9(1), (3) and (a15), we have

\[
F(x \ast y, z) = x \ast F(y, z) \\
= (x \ast 0) \ast F(y, z) \\
= (x \ast (F(0, z) \ast F(0, z))) \ast F(y, z) \\
= (x \ast ((x \ast F(x, z)) \ast (F(y, z) \ast y))) \\
\ast F(y, z) \\
= (x \ast F(y, z)) \\
\ast ((x \ast F(y, z)) \ast (F(x, z) \ast y)) \\
= (F(x, z) \ast y) \land (x \ast F(y, z)) .
\]
Conversely, suppose that $F$ is a symmetric bi-derivation of $X$. As $F$ is a $(r,l)$-symmetric bi-derivation on $X$, then for any $x, y, z \in X$ and using (a14), we have
\[
F(x \ast y, z) = (x \ast F(y, z) \ast (F(x, z) \ast y)) \\
= (F(x, z) \ast y) \\
\ast ((F(x, z) \ast y) \ast (x \ast F(y, z))) \\
= x \ast F(y, z) \\
= (y \ast F(x, z)) \\
\ast ((y \ast F(x, z)) \ast (x \ast F(y, z))) \\
= (x \ast F(y, z) \ast (x \ast F(x, z)) .
\]
Hence $F$ is a symmetric left bi-derivation. This completes the proof.

Acknowledgments

The authors are grateful to the anonymous referee(s) for a careful checking of the details and for helpful comments that improved the present paper. G. Muhuiddin and Abdullah M. Al-roqi were partially supported by the Deanship of Scientific Research, University of Tabuk, Ministry of Higher Education, Saudi Arabia.

References


Submit your manuscripts at
http://www.hindawi.com