Research Article

On $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -Continuous Functions in Biminimal Structure Spaces

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We introduce the notion of $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous functions and some other forms of continuity in biminimal structure spaces. Some new characterizations and several fundamental properties of $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous functions are obtained.

1. Introduction

Weak continuity due to Levine [1] is one of the most important weak forms of continuity in topological spaces. Rose [2] has introduced the notion of subweakly continuous functions and investigated the relationships between subweak continuity and weak continuity. In [3], Baker obtained several properties of subweak continuity which are analogous to results in [4]. Njåstad [5] introduced a weak form of open sets called α -sets. In [6], the author showed that connectedness is preserved under weakly α -continuous surjections. Mashhour et al. [7] have called strongly semicontinuous α -continuous and obtained several properties of such functions. In [8], they stated without proofs that α -continuity implies θ -continuity and is independent of almost continuity in the sense of Singal [9]. On the other hand, in 1980 Maheshwari and Thakur [10] defined α -irresolute and obtained several properties of α -irresolute functions. Levine [11] defined the notions of semiopen sets and semicontinuity in topological spaces. Maheshwari and Prasad [12] extended the notions of semiopen sets and semicontinuity to the bitopological setting. Bose [13] further investigated many properties of semiopen sets and semicontinuity in bitopological spaces. Mashhour et al. [7] introduced the notions of preopen sets and precontinuity in topological spaces. Jelić [14] generalized the notions of preopen sets and precontinuity to the setting of bitopological spaces. The purpose of the present paper is to introduce the notion of $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous functions in

biminimal structure spaces and investigate the properties of these functions.

2. Preliminaries

Definition 1 (see [15]). Let X be a nonempty set and $\mathcal{P}(X)$ the power set of X . A subfamily m_X of $\mathcal{P}(X)$ is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X) , we denote a nonempty set X with an m -structure m_X on X and it is called an m -space. Each member of m_X is said to be m_X -open, and the complement of an m_X -open set is said to be m_X -closed.

Definition 2 (see [16]). Let X be a nonempty set and m_X an m -structure on X . For a subset A of X , the m_X -closure of A and the m_X -interior of A are defined as follows:

- (1) $m_X \text{Cl}(A) = \cap \{F : A \subseteq F, X - F \in m_X\}$;
- (2) $m_X \text{Int}(A) = \cup \{U : U \subseteq A, U \in m_X\}$.

Lemma 3 (see [16]). Let X be a nonempty set and m_X a minimal structure on X . For subset A and B of X , the following properties hold:

- (1) $m_X \text{Cl}(X - A) = X - m_X \text{Int}(A)$ and $m_X \text{Int}(X - A) = X - m_X \text{Cl}(A)$.
- (2) If $(X - A) \in m_X$, then $m_X \text{Cl}(A) = A$ and if $A \in m_X$, then $m_X \text{Int}(A) = A$.

- (3) $m_X Cl(\emptyset) = \emptyset$, $m_X Cl(X) = X$, $m_X Int(\emptyset) = \emptyset$, and $m_X Int(X) = X$.
- (4) If $A \subseteq B$, then $m_X Cl(A) \subseteq m_X Cl(B)$ and $m_X Int(A) \subseteq m_X Int(B)$.
- (5) $A \subseteq m_X Cl(A)$ and $m_X Int(A) \subseteq A$.
- (6) $m_X Cl(m_X Cl(A)) = m_X Cl(A)$ and $m_X Int(m_X Int(A)) = m_X Int(A)$.

Definition 4 (see [16]). An m -structure m_X on a nonempty set X is said to have property $\mathcal{B}$ if the union of any family of subsets belonging to m_X belongs to m_X .

Lemma 5 (see [17]). Let X be a nonempty set and m_X an m -structure on X satisfying property $\mathcal{B}$. For a subset A of X , the following properties hold:

- (1) $A \in m_X$ if and only if $m_X Int(A) = A$.
- (2) A is m_X -closed if and only if $m_X Cl(A) = A$.
- (3) $m_X Int(A) \in m_X$ and $m_X Cl(A)$ is m_X -closed.

Definition 6. Let X be a nonempty set and m_X^1, m_X^2 minimal structures on X . The triple (X, m_X^1, m_X^2) is called a *bispace* (briefly *bi m -space*) [18] or *biminimal structure space* (briefly *bimspace*) [19].

Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X . The m_X -closure of A and the m_X -interior of A with respect to m_X^i are denoted by $m_X^i Cl(A)$ and $m_X^i Int(A)$, respectively, for $i = 1, 2$. Also $i, j = 1, 2$ and $i \neq j$.

Definition 7 (see [20]). A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i, j) - m_X - α -open (resp., (i, j) - m_X -semiopen, (i, j) - m_X -preopen) if $A \subseteq m_X^i Int(m_X^j Cl(m_X^i Int(A)))$ (resp., $A \subseteq m_X^i Cl(m_X^j Int(A))$, $A \subseteq m_X^i Int(m_X^j Cl(A))$).

Lemma 8. Let (X, m_X^1, m_X^2) be a biminimal structure space and $\{A_k : k \in \mathcal{K}\}$ a family of subsets of X .

- (1) If A_k is (i, j) - m_X - α -open for each $k \in \mathcal{K}$, then $\cup_{k \in \mathcal{K}} A_k$ is (i, j) - m_X - α -open.
- (2) If A_k is (i, j) - m_X - α -closed for each $k \in \mathcal{K}$, then $\cap_{k \in \mathcal{K}} A_k$ is (i, j) - m_X - α -closed.

Definition 9. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X . Then the m_X^{ij} - α -closure of A and the m_X^{ij} - α -interior of A are defined as follows:

- (1) $m_X^{ij} Cl_{\mathcal{A}}(A) = \cap \{F : A \subseteq F, F \text{ is } (i, j)\text{-}m_X\text{-}\alpha\text{-closed}\}$;
- (2) $m_X^{ij} Int_{\mathcal{A}}(A) = \cup \{U : U \subseteq A, U \text{ is } (i, j)\text{-}m_X\text{-}\alpha\text{-open}\}$.

Lemma 10. Let (X, m_X^1, m_X^2) be a biminimal structure space. For a subset A of X , the following properties hold:

- (1) $m_X^{ij} Cl_{\mathcal{A}}(A)$ is (i, j) - m_X - α -closed;
- (2) $m_X^{ij} Int_{\mathcal{A}}(A)$ is (i, j) - m_X - α -open;

- (3) A is (i, j) - m_X - α -closed if and only if $m_X^{ij} Cl_{\mathcal{A}}(A) = A$;
- (4) A is (i, j) - m_X - α -open if and only if $m_X^{ij} Int_{\mathcal{A}}(A) = A$.

Lemma 11. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset of X . Then $x \in m_X^{ij} Cl_{\mathcal{A}}(A)$ if and only if $U \cap A \neq \emptyset$ for every (i, j) - m_X - α -open set U containing x .

Lemma 12. Let (X, m_X^1, m_X^2) be a biminimal structure space and A a subset A of X

- (1) $X - m_X^{ij} Int_{\mathcal{A}}(A) = m_X^{ij} Cl_{\mathcal{A}}(X - A)$;
- (2) $X - m_X^{ij} Cl_{\mathcal{A}}(A) = m_X^{ij} Int_{\mathcal{A}}(X - A)$.

Proof. (1) By Lemma 10, $m_X^{ij} Cl_{\mathcal{A}}(A)$ is (i, j) - m_X - α -closed. Then $X - m_X^{ij} Cl_{\mathcal{A}}(A)$ is (i, j) - m_X - α -open. On the other hand, $X - m_X^{ij} Cl_{\mathcal{A}}(X - A) \subseteq A$, and hence $X - m_X^{ij} Cl_{\mathcal{A}}(X - A) \subseteq X - m_X^{ij} Int_{\mathcal{A}}(A)$. Conversely, let $x \in m_X^{ij} Int_{\mathcal{A}}(A)$. Then there exists an (i, j) - m_X - α -open U such that $x \in U \subseteq A$. Then $X - U$ is (i, j) - m_X - α -closed and $X - A \subseteq X - U$. Since $x \notin X - U$, $x \notin m_X^{ij} Cl_{\mathcal{A}}(X - A)$, and hence $m_X^{ij} Int_{\mathcal{A}}(A) \subseteq X - m_X^{ij} Cl_{\mathcal{A}}(X - A)$. Therefore, $X - m_X^{ij} Int_{\mathcal{A}}(A) = m_X^{ij} Cl_{\mathcal{A}}(X - A)$.

(2) This follows from (1) immediately. $\square$

3. Characterizations of $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -Continuous Functions

Definition 13. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is said to be $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous at point $x \in X$ if for each τ_i -open set V of Y containing $f(x)$, there exists an (i, j) - m_X - α -open set U containing x such that $f(U) \subseteq V$.

A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is said to be $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous if f has this property at each point of X .

Theorem 14. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. For a function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$, the following properties are equivalent:

- (1) f is $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous at $x \in X$;
- (2) $x \in m_X^{ij} Int_{\mathcal{A}}(f^{-1}(V))$ for every τ_i -open set V containing $f(x)$;
- (3) $x \in f^{-1}(\overline{f(A)})$ for every subset A of X with $x \in m_X^{ij} Cl_{\mathcal{A}}(A)$;
- (4) $x \in f^{-1}(\overline{B})$ for every subset B of Y with $x \in m_X^{ij} Cl_{\mathcal{A}}(f^{-1}(B))$;
- (5) $x \in m_X^{ij} Int_{\mathcal{A}}(f^{-1}(B))$ for every subset B of Y with $x \in f^{-1}(Int(B))$;
- (6) $x \in f^{-1}(F)$ for every τ_i -closed set F of Y with $x \in m_X^{ij} Cl_{\mathcal{A}}(f^{-1}(F))$.

Proof. (1) $\Rightarrow$ (2): Let $V \in \tau_i$ containing $f(x)$. Then there exists an (i, j) - m_X - α -open set containing x such that $f(U) \subseteq V$. Thus, $x \in U \subseteq f^{-1}(V)$. Hence, $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$.

(2) $\Rightarrow$ (3): Let A be any subset of X such that $x \in m_X^{ij} \mathcal{A}(A)$, and let $V \in \tau_i$ containing $f(x)$. Then $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$. There exists an (i, j) - m_X - α -open set U such that $x \in U \subseteq f^{-1}(V)$. Since $x \in m_X^{ij} \text{Cl}_{\mathcal{A}}(A)$, by Lemma 11, $U \cap A \neq \emptyset$ and $\emptyset \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. Since $V \in \tau_i$ containing $f(x)$, $f(x) \in \overline{f(A)}$, and hence $x \in f^{-1}(\overline{f(A)})$.

(3) $\Rightarrow$ (4): Let B be any subset of Y and $x \in m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(B))$. Then by (3), $x \in f^{-1}(\overline{f(f^{-1}(B))}) \subseteq f^{-1}(\overline{B})$. Hence, we have $x \in f^{-1}(\overline{B})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $x \notin m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(B))$. Then $x \in X - m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(B)) = m_X^{ij} \text{Cl}_{\mathcal{A}}(X - f^{-1}(B)) = m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(Y - B))$. By (4), we have $x \in f^{-1}(\overline{Y - B}) = f^{-1}(Y - \text{Int}(B)) = X - f^{-1}(\text{Int}(B))$. Hence, $x \notin f^{-1}(\text{Int}(B))$.

(5) $\Rightarrow$ (6): Let F be any τ_i -closed set of Y such that $x \notin f^{-1}(F)$. Then $x \in X - f^{-1}(F) = f^{-1}(Y - F) = f^{-1}(\text{Int}(Y - F))$. By (5), $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(Y - F)) = m_X^{ij} \text{Int}_{\mathcal{A}}(X - f^{-1}(F)) = X - m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F))$. Hence, $x \notin m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F))$.

(6) $\Rightarrow$ (2): Let $x \in X$ and $V \in \tau_i$ containing $f(x)$. Suppose that $x \notin m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$. Then $x \in X - m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V)) = m_X^{ij} \text{Cl}_{\mathcal{A}}(X - f^{-1}(V)) = m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(Y - V))$. By (6), $x \in f^{-1}(Y - V) = X - f^{-1}(V)$. Hence, $x \notin f^{-1}(V)$. This contradicts the hypothesis.

(2) $\Rightarrow$ (1): Let $V \in \tau_i$ containing $f(x)$. Then by (2), $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$, and hence there exists an (i, j) - m_X - α -open set U containing x such that $f(U) \subseteq V$. Therefore, $f(U) \subseteq V$, and hence f is $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous at x . $\square$

Theorem 15. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. For a function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$, the following properties are equivalent:

- (1) f is $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous;
- (2) $f^{-1}(V)$ is (i, j) - m_X - α -open for every τ_i -open set V in Y ;
- (3) $f(m_X^{ij} \text{Cl}_{\mathcal{A}}(A)) \subseteq \overline{f(A)}$ for every subset A of X ;
- (4) $m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(B)) \subseteq f^{-1}(\overline{B})$ for every subset B of Y ;
- (5) $f^{-1}(\text{Int}(B)) \subseteq m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(B))$ for every subset B of Y ;
- (6) $f^{-1}(F)$ is (i, j) - m_X - α -closed for every τ_i -closed set F of Y .

Proof. (1) $\Rightarrow$ (2): Let V be any τ_i -open set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists an (i, j) - m_X - α -open set U containing x such that $f(U) \subseteq V$, and hence $x \in U \subseteq f^{-1}(V)$. Hence, we have $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$. Therefore, $f^{-1}(V) \subseteq m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(V))$. This shows that $f^{-1}(V)$ is (i, j) - m_X - α -open.

(2) $\Rightarrow$ (3): Let A be any subset of X . Since $Y - \overline{f(A)}$ is τ_i -open and by (2), we have $f^{-1}(Y - \overline{f(A)}) = m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(Y - \overline{f(A)}))$. Hence, $X - f^{-1}(\overline{f(A)}) = m_X^{ij} \text{Int}_{\mathcal{A}}(X - f^{-1}(\overline{f(A)})) = X - m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(\overline{f(A)}))$. Therefore, $m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(\overline{f(A)})) = f^{-1}(\overline{f(A)})$. Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\overline{f(A)})$ and hence $m_X^{ij} \text{Cl}_{\mathcal{A}}(A) \subseteq m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(\overline{f(A)})) = f^{-1}(\overline{f(A)})$. Therefore, $f(m_X^{ij} \text{Cl}_{\mathcal{A}}(A)) \subseteq \overline{f(A)}$.

(3) $\Rightarrow$ (4): Let B be any subset of Y . Then by (3), we have $f(m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(B))) \subseteq \overline{f(f^{-1}(B))} \subseteq \overline{B}$. Hence, $m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(B)) \subseteq f^{-1}(\overline{B})$.

(4) $\Rightarrow$ (5): Let B be any subset of Y and $x \in f^{-1}(\text{Int}(B))$. Then $f(x) \in \text{Int}(B)$, and hence $f(x) \notin Y - \text{Int}(B) = \overline{Y - B}$. Therefore, $x \notin f^{-1}(\overline{Y - B})$. By (4), we have $x \notin m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(Y - B))$, and hence $x \notin X - m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(B))$. Therefore, $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(B))$.

(5) $\Rightarrow$ (6): Let F be any τ_i -closed set of Y such that $x \notin f^{-1}(F)$. Then $x \in X - f^{-1}(F) = f^{-1}(Y - F) = f^{-1}(\text{Int}(Y - F))$. By (5), $x \in m_X^{ij} \text{Int}_{\mathcal{A}}(f^{-1}(Y - F)) = m_X^{ij} \text{Int}_{\mathcal{A}}(X - f^{-1}(F)) = X - m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F))$. Therefore, $x \notin m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F))$, and hence $m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F)) \subseteq f^{-1}(F)$. This shows that $f^{-1}(F)$ is (i, j) - m_X - α -closed.

(6) $\Rightarrow$ (1): Let F be any τ_i -closed set of Y such that $x \in m_X^{ij} \text{Cl}_{\mathcal{A}}(f^{-1}(F))$. Then by (6), we have $x \in f^{-1}(F)$. Hence, by Theorem 14(6), f is $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous. $\square$

Definition 16. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is said to be

- (1) $\mathcal{M}_{\mathcal{S}}^{(i,j)}$ -continuous (resp., $\mathcal{M}_{\mathcal{S}}^{(j,i)}$ -continuous) if $f^{-1}(V)$ is (i, j) - m_X -semiopen (resp., (j, i) - m_X -semiopen) in X for each τ_i -open set V of Y ;
- (2) $\mathcal{M}_{\mathcal{S}}^{(i,j)}$ -continuous or (i, j) - m -precontinuous [21] if $f^{-1}(V)$ is (i, j) - m_X -preopen in X for each τ_i -open set V of Y .

Lemma 17. Let A be a subset of a biminimal structure space (X, m_X^1, m_X^2) . Then A is (i, j) - m_X - α -open in (X, m_X^1, m_X^2) if and only if A is (j, i) - m_X -semiopen and (i, j) - m_X -preopen in (X, m_X^1, m_X^2) .

Proof. Let A be (i, j) - m_X - α -open in (X, m_X^1, m_X^2) . By the definition of (i, j) - m_X - α -open sets, we have $A \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(A))$ and $A \subseteq m_X^j \text{Cl}(m_X^i \text{Int}(A))$. Therefore, we obtain A is (j, i) - m_X -semiopen and (i, j) - m_X -preopen in (X, m_X^1, m_X^2) .

Conversely, let A be (j, i) - m_X -semiopen and (i, j) - m_X -preopen in (X, m_X^1, m_X^2) . Since A is (j, i) - m_X -semiopen, $A \subseteq m_X^j \text{Cl}(m_X^i \text{Int}(A))$, and hence it follows from A is (i, j) - m_X -preopen that $A \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(A)) \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(m_X^i \text{Int}(m_X^j \text{Cl}(m_X^i \text{Int}(A)))) = m_X^i \text{Int}(m_X^j \text{Cl}(m_X^i \text{Int}(A)))$. Therefore, A is (i, j) - m_X - α -open in (X, m_X^1, m_X^2) . $\square$

Theorem 18. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is $\mathcal{M}_{\mathcal{A}}^{(i,j)}$ -continuous if and only if f is $\mathcal{M}_{\mathcal{P}}^{(i,j)}$ -continuous and $\mathcal{M}_{\mathcal{S}}^{(j,i)}$ -continuous.

Proof. This is an immediate consequence of Lemma 17. $\square$

Definition 19. Let (X, m_X^1, m_X^2) be a biminimal structure space and (Y, τ_1, τ_2) a bitopological space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is said to be $\mathcal{M}_{\mathcal{P}}^{(i,j)}$ -continuous if f is $\mathcal{M}_{\mathcal{P}}^{(i,j)}$ -continuous and for every $U, V \in \tau_i$ such that $[m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U)))] \cap [m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(V)))] \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U \cap V)))$.

Theorem 20. Let (X, m_X^1, m_X^2) be a biminimal structure space, and let m_X^1, m_X^2 have property $\mathcal{B}$, and let (Y, τ_1, τ_2) be a bitopological space. A function $f : (X, m_X^1, m_X^2) \rightarrow (Y, \tau_1, \tau_2)$ is $\mathcal{M}_{\mathcal{P}^*}^{(i,j)}$ -continuous if and only if for every $U, V \in \tau_i$ such that

$$\begin{aligned} (1) \quad & f^{-1}(V) \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(V))) \text{ and} \\ (2) \quad & m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U \cap V))) = m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U))) \cap m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(V))). \end{aligned}$$

Proof. It is obvious that f is $\mathcal{M}_{\mathcal{P}}^{(i,j)}$ -continuous if and only if f satisfies (1). We assume that f is $\mathcal{M}_{\mathcal{P}^*}^{(i,j)}$ -continuous and show equality (2). For any $U, V \in \tau_i$, it follows from (1) that $f^{-1}(U \cap V) \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U))) \cap m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(V)))$. Since the intersection of two (i, j) - m_X -regular open sets is (i, j) - m_X -regular open, we obtain $m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U \cap V))) \subseteq m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(U))) \cap m_X^i \text{Int}(m_X^j \text{Cl}(f^{-1}(V)))$. Hence, equality (2) holds. $\square$

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References

- [1] N. Levine, "A decomposition of continuity in topological spaces," *The American Mathematical Monthly*, vol. 68, pp. 44–46, 1961.
- [2] D. A. Rose, "Weak continuity and almost continuity," *International Journal of Mathematics and Mathematical Sciences*, vol. 7, no. 2, pp. 311–318, 1984.
- [3] C. W. Baker, "Properties of subweakly continuous functions," *Yokohama Mathematical Journal*, vol. 32, no. 1-2, pp. 39–43, 1984.
- [4] T. Noiri, "On weakly continuous mappings," *Proceedings of the American Mathematical Society*, vol. 46, pp. 120–124, 1974.
- [5] O. Njåstad, "On some classes of nearly open sets," *Pacific Journal of Mathematics*, vol. 15, pp. 961–970, 1965.
- [6] T. Noiri, "Weakly α -continuous functions," *International Journal of Mathematics and Mathematical Sciences*, vol. 10, no. 3, pp. 483–490, 1987.
- [7] A. S. Mashhour, M. E. Abd El-Monsef, and S. N. El-Deeb, "On precontinuous and weak precontinuous mappings," *Proceedings of the Physico-Mathematical Society of Egypt*, vol. 53, pp. 47–53, 1982.
- [8] A. S. Mashhour, I. A. Hasanein, and S. N. El-Deeb, " α -continuous and α -open mappings," *Acta Mathematica Hungarica*, vol. 41, no. 3-4, pp. 213–218, 1983.
- [9] M. K. Singal and A. R. Singal, "Almost-continuous mappings," *Yokohama Mathematical Journal*, vol. 16, pp. 63–73, 1968.
- [10] S. N. Maheshwari and S. S. Thakur, "On α -irresolute mappings," *Tamkang Journal of Mathematics*, vol. 11, no. 2, pp. 209–214, 1980.
- [11] N. Levine, "Semi-open sets and semi-continuity in topological spaces," *The American Mathematical Monthly*, vol. 70, pp. 36–41, 1963.
- [12] S. N. Maheshwari and R. Prasad, "Semi-open sets and semicontinuous functions in bitopological spaces," *Mathematicae Notae*, vol. 26, pp. 29–37, 1977-1978.
- [13] S. Bose, "Semi-open sets, semicontinuity and semi-open mappings in bitopological spaces," *Bulletin of the Calcutta Mathematical Society*, vol. 73, no. 4, pp. 237–246, 1981.
- [14] M. Jelić, "A decomposition of pairwise continuity," *Journal of the Institute of Mathematics & Computer Sciences (Mathematics Series)*, vol. 3, no. 1, pp. 25–29, 1990.
- [15] V. Popa and T. Noiri, "On M -continuous functions," *Analele Universitatii, "Dunărea De Jos" din Galați, Seria II. Matematica-Fizica-Mecanica Teoretica*, vol. 18, no. 23, pp. 31–41, 2000.
- [16] H. Maki, K. Chandrasekhara Rao, and A. Nagoor Gani, "On generalizing semi-open sets and preopen sets," *Pure and Applied Mathematica Sciences*, vol. 49, no. 1-2, pp. 17–29, 1999.
- [17] V. Popa and T. Noiri, "A unified theory of weak continuity for functions," *Rendiconti del Circolo Matematico di Palermo. Serie II*, vol. 51, no. 3, pp. 439–464, 2002.
- [18] T. Noiri, "The further unified theory for modifications of g -closed sets," *Rendiconti del Circolo Matematico di Palermo*, vol. 57, no. 3, pp. 411–421, 2008.
- [19] C. Boonpok, "Biminimal structure spaces," *International Mathematical Forum*, vol. 5, no. 13–16, pp. 703–707, 2010.
- [20] C. Boonpok, "M-Continuous functions on biminimal structure spaces," *Far East Journal of Mathematical Sciences*, vol. 43, no. 1, pp. 41–58, 2010.
- [21] C. Carpintero, N. Rajesh, and E. Rosas, " m -preopen sets in biminimal spaces," *Demonstratio Mathematica*, vol. 45, no. 4, pp. 953–961, 2012.