Research Article

Bayesian Inference of Burr Type VIII Distribution Based on Censored Samples

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This paper is concerned with estimation of the parameter of Burr type VIII distribution under a Bayesian framework using censored samples. The Bayes estimators and associated risks have been derived under the assumption of five priors and three loss functions. The comparison among the performance of different estimators has been made in terms of posterior risks. A simulation study has been conducted in order to assess and compare the performance of different estimators. The study proposes the use of inverse Levy prior based on quadratic loss function for Bayes estimation of the said parameter.

1. Introduction

Burr [1] introduced twelve different forms of the cumulative distribution functions of Burr distribution. Among these twelve distribution functions, Burr type X and Burr type XII have been discussed by most of the analysts, while the remaining density functions are still waiting for the attention of the researchers. Soliman [2] derived the Bayes estimates of the parameter of Burr type XII distribution relative to quadratic, LINEX, and generalized entropy loss functions. Based on Lindley's approximation, the comparisons were made between maximum likelihood and Bayes estimation. Monte Carlo simulation was also used. Joshi [3] obtained the Bayesian estimation of reliability of generalized Burr distribution with the help of a censored sample. The precautionary and squared error loss functions were used for estimation. The noninformative and beta prior distributions were assumed for posterior analysis. Asgharzadeh and Valiollahi [4] considered the uniformly minimum variance unbiased (UMVU) of Burr model under progressively type II censored samples; the Bayes and empirical Bayes estimates for the unknown parameter and the reliability function of the Burr model were derived. The Bayes and empirical Bayes estimates were obtained on the basis of absolute error and logarithmic loss functions. A numerical example and a Monte Carlo simulation study were used to illustrate the

applicability of the results. Saleem and Aslam [5] analyzed the parameter of the Rayleigh distribution under censored data. AL-Hussaini and Hussein [6] presented the maximum likelihood and Bayes estimators of the parameters, survival function (SF), and hazard rate function (HRF) for the three-parameter exponentiated Burr type XII distribution under type II censored scheme. Soliman et al. [7] dealt with Bayesian inference and prediction problems of the Burr type XII distribution based on progressive first failure censored data. Squared error loss function was proposed for estimation. Gibbs sampling procedure was applied to draw Markov chain Monte Carlo (MCMC) samples. A simulation study was carried out to compare the proposed Bayes estimators with the maximum likelihood estimators. A real life data set was also used to illustrate the results derived. Singh [8] discussed the uniformly minimum variance unbiased estimate (UMVUE) of the probability density function (pdf) of the Burr distribution. The UMVUE of the cumulative distribution function (CDF), p th quantile, and r th moment of the Burr distribution were also obtained. Feroze and Aslam [9] considered the Bayesian analysis of Gumbel type II distribution under doubly censored samples using different priors (informative and noninformative) and loss functions (symmetric and asymmetric). Bayesian credible intervals along with posterior predictive distributions were also derived.

One of the important distributions from Burr family of distributions is Burr type VIII distribution. This distribution can be used to model the lifetime data. Burr [1] has indicated that this distribution can be used as an alternative lifetime model in many cases. Its two-parametric case gives more flexibility as compared to exponential and Rayleigh distribution in order to model lifetime data. In this paper, the Bayesian inference of Burr type VIII distribution has been considered using different loss functions and priors on the basis of three censoring techniques.

The probability density function (pdf) of Burr type VIII distribution is

$$f(x, \psi) = \psi \left(\frac{2}{\pi} \right)^\psi \frac{e^x}{1 + e^{2x}} \{ \tan^{-1}(e^x) \}^{\psi-1}; \quad \psi > 0, x > 0. \quad (1)$$

The cumulative distribution function (CDF) of the distribution is

$$F(x, \psi) = \left\{ \left(\frac{2}{\pi} \right) \tan^{-1}(e^x) \right\}^\psi. \quad (2)$$

2. Prior Distributions

The prior distribution is a key part of Bayesian analysis and represents the information about an uncertain parameter that is combined with the probability distribution of new data to derive the posterior distribution, which in turn is used for future inferences and decision making. Therefore, prior subject-matter knowledge about a parameter is a vital aspect of the inference process. The prior distributions can be categorized as informative and noninformative priors. As their names suggest, the informative priors are used when the prior information regarding the current experiment is available (may be in the form of prior beliefs), while when such prior information is not available the use of noninformative priors becomes inevitable. We have assumed both informative and noninformative priors for the Bayesian analysis of the said parameter.

Among noninformative priors the uniform prior proposed by Laplace [10] and Jeffreys prior suggested by Jeffreys [11] are the most famous priors. We have assumed these priors along with some informative priors to derive the posterior distributions.

The uniform prior is assumed to be

$$p(\psi) \propto 1; \quad \psi > 0. \quad (3)$$

Jeffreys prior is defined to be

$$p(\psi) \propto \sqrt{|I(\psi)|},$$

$$\text{where } I(\psi) = -E \left[\frac{\partial^2 \ln f(x, \psi)}{\partial \psi^2} \right] = \frac{1}{\psi^2}; \quad (4)$$

$$\text{therefore } p(\psi) \propto \sqrt{|I(\psi)|} = \frac{1}{\psi}.$$

The informative prior for the parameter ψ is assumed to be gamma distribution as

$$p(\psi) \propto \psi^{a-1} e^{-b\psi} \quad a > 0, b > 0, \psi > 0. \quad (5)$$

The informative prior for the parameter ψ is assumed to be exponential distribution as

$$p(\psi) \propto e^{-c\psi}, \quad c > 0, \psi > 0. \quad (6)$$

The informative prior for the parameter ψ is assumed to be inverse Levy distribution as

$$p(\psi) \propto \psi^{-1/2} e^{-0.5d\psi}, \quad d > 0, \psi > 0. \quad (7)$$

The informative priors presented in (5)–(7) have been used in order to make our analysis more versatile. As the Burr type VIII distribution can be used to model the lifetime data, prior information available in such form can effectively be used. It should be noted that the gamma, exponential, and inverse Levy distributions can be used to model the lifetime data. So, these distributions can be used as prior distribution for the analysis of Burr type VIII distribution under Bayesian framework.

3. Bayesian Estimation under Left Censored Samples

Censoring is a useful technique when the value of a measurement or observation is only partially known. That is, it is used when an observed value is outside the range of a measuring instrument or the measure outside a range is not desired. It has a great usage in life testing experiments where continuity of the experiment till the last failure is not feasible. Censoring has many types. However, we have considered left, singly type II, and doubly type II censored samples for Bayesian inference of the parameter of Burr type VIII distribution.

Let $X_{m+1}, \dots, X_n$ be the last $n - m$ order statistics from a sample of size n from Burr type VIII distribution. Then the likelihood function for the $X_{m+1}, \dots, X_n$ observations is

$$L(\psi | x) \propto \{F(x_{m+1}, \psi)\}^m \prod_{i=m+1}^n f(x_i, \psi), \quad (8)$$

$$L(\psi | x)$$

$$\propto \psi^{n-m} e^{-\psi [\sum_{i=m+1}^n \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1} + m \ln \{(2/\pi) \tan^{-1}(e^{x_{m+1}})\}^{-1}]} \quad (9)$$

The details for the derivation of the likelihoods function can be seen from the appendix. Now the generalized expressions for posterior distributions, Bayesian estimators, posterior risks, posterior predictive distributions, posterior predictive intervals, and credible intervals have been presented in the following. The expressions under priors given in (3), (4), (5), (6), and (7) can be derived by putting $l_1 = 0, l_2 = 0, l_3 = 0, l_4 = 0; l_1 = 1, l_2 = 0, l_3 = 0, l_4 = 0; l_1 = 0, l_2 = 1, l_3 = 0, l_4 = 0; l_1 = 0, l_2 = 0, l_3 = 1, l_4 = 0$; and $l_1 = 0, l_2 = 0, l_3 = 1, l_4 = 1$, respectively, in the generalized results.

The generalized posterior distribution under left censored samples is

$$p(\psi | x) = \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \times \psi^{n-m-l_1+l_3(a-1)-0.5l_4} e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}}, \quad (10)$$

where

$$C_1(x) = \sum_{i=m+1}^n \ln \left\{ \left(\frac{2}{\pi} \right) \tan^{-1}(e^{x_i}) \right\}^{-1} + m \ln \left\{ \left(\frac{2}{\pi} \right) \tan^{-1}(e^{x_{m+1}}) \right\}^{-1}. \quad (11)$$

The details for the derivation of the posterior distribution can be seen from the appendix.

3.1. Loss Functions. The decision theory suggests that in order to select the best estimator a loss function must be specified and used to estimate the risk associated with each of the possible estimates. Since there is no definite analytical process that allows us to identify the proper loss function to be used, most of the analysts use the squared error loss function which is symmetrical and associates equal importance to the losses due to overestimation and underestimation of equal magnitude and obtain the posterior mean as Bayesian estimate. But in situations where the loss is asymmetric, we have to employ some asymmetric loss functions. So, here we have assumed some symmetric and asymmetric loss functions for the analysis of the parameter under study. The detail of these loss functions is as follows.

LINEX Loss Function (LLF). LINEX (linear exponential) loss function has been defined by Klebanov [12]. The expression of the loss function is $L(\psi_{LLF}, \psi) = b[e^{a(\psi_{LLF}-\psi)} - a(\psi_{LLF}-\psi) - 1]$.

For $a = b = 1$, we have $L(\psi_{LLF}, \psi) = e^{(\psi_{LLF}-\psi)} - (\psi_{LLF}-\psi) - 1$. Then the Bayes estimator based on this loss function is $\psi_{LLF} = -\ln[E(e^{-\psi})]$.

Precautionary Loss Function (PLF). Norström [13] introduced an asymmetric precautionary loss function (PLF) which can be presented as $L(\psi_{PLF}, \psi) = (\psi_{PLF} - \psi)^2 / \psi_{PLF}$, where $\psi_{PLF} = [E(\psi^2)]^{1/2}$ is the Bayes estimator under this loss function.

Quadratic Loss Function (QLF). The quadratic loss function can be defined as $L(\psi, \psi_{QLF}) = ((\psi - \psi_{QLF})/\psi)^2$. The Bayes estimator under QLF is given as $\psi_{QLF} = E(\psi^{-1})\{E(\psi^{-2})\}^{-1}$.

Bayes estimator and corresponding risk under using LINEX loss function are, respectively, presented as

$$\psi_{LLF} = -\{n-m+1-l_1+l_3(a-1)-0.5l_4\}$$

$$\times \ln \left\{ \frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{C_1(x) + l_2c + l_3b + 0.5dl_4 + 1} \right\},$$

$$\rho(\psi_{LLF}) = \frac{n-m+1-l_1+l_3(a-1)-0.5l_4}{C_1(x) + l_2c + l_3b + 0.5dl_4} + \{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \ln \left\{ \frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{C_1(x) + l_2c + l_3b + 0.5dl_4 + 1} \right\}. \quad (12)$$

The details for the derivation of the Bayes estimates and posterior risks under LLF can be seen from the appendix.

Bayes estimator and corresponding risk under precautionary loss function are, respectively, presented as

$$\psi_{PLF} = (\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \{n-m+2-l_1+l_3(a-1)-0.5l_4\})^{1/2} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1},$$

$$\rho(\psi_{PLF}) = 2 \left[(\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \{n-m+2-l_1+l_3(a-1)-0.5l_4\})^{1/2} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1} - (\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1}) \right]. \quad (13)$$

The details for the derivation of the Bayes estimates and posterior risks under PLF can be seen from the appendix.

Bayes estimator and corresponding risk under quadratic loss function are, respectively, presented as

$$\psi_{QLF} = \frac{n-m-1-l_1+l_3(a-1)-0.5l_4}{C_1(x) + l_2c + l_3b + 0.5dl_4},$$

$$\rho(\psi_{QLF}) = \frac{1}{n-m-l_1+l_3(a-1)-0.5l_4}. \quad (14)$$

The details for the derivation of the Bayes estimates and posterior risks under QLF can be seen from the appendix.

4. Bayesian Estimation under Singly Type II Censored Samples

Under type II censored samples the experiment is terminated after observing some fixed percentage of observations. The likelihood function for the singly type II censored samples can be defined as follows.

Let us observe “ n ” items for possible failure and only the first “ r ” failure times have been observed; that is, $x_1 < x_2 < \dots < x_r$ and remaining “ $n-r$ ” items are still working. Under the assumptions that the lifetimes of the items are independently and identically distributed random variables

following Burr type VIII distribution, the likelihood function for “ r ” observations is

$$\begin{aligned}
 L(\psi | x) & \propto \left\{ \prod_{i=1}^r f(x_i, \psi) \right\} \{1 - F(x_r, \psi)\}^{n-r}, \\
 L(\psi | x) & \propto \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \psi^r \\
 & \times e^{-\psi [\sum_{i=1}^r \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1} + j \ln \{(2/\pi) \tan^{-1}(e^{x_r})\}^{-1}]} .
 \end{aligned} \quad (15)$$

The generalized posterior distribution under singly type II censored samples is

$$\begin{aligned}
 p(\psi | x) & = \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\
 & \times \psi^{r-l_1+l_3(a-1)-0.5l_4} e^{-\psi \{C_2(x)+l_2c+l_3b+0.5dl_4\}} \Bigg) \\
 & \times \left(\Gamma(r+1-l_1+l_3(a-1)-0.5l_4) \right. \\
 & \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\
 & \times \{C_2(x) + l_2c + l_3b + 0.5dl_4\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \Bigg)^{-1} ; \\
 & \psi > 0,
 \end{aligned} \quad (17)$$

where $C_2(x) = \sum_{i=1}^r \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1} + j \ln \{(2/\pi) \tan^{-1}(e^{x_r})\}^{-1}$.

Bayes estimator and corresponding risk under LINEX loss function are, respectively, presented as

$$\begin{aligned}
 \psi_{\text{LLF}} & = -\ln \left\{ \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \right. \\
 & \times \{C_2(x) + l_2c + l_3b + 0.5dl_4 \\
 & \left. \left. + 1\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right) \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\
 & \times \{C_2(x) + l_2c + l_3b \\
 & \left. \left. + 0.5dl_4\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \right\} ,
 \end{aligned}$$

$$\begin{aligned}
 \rho(\psi_{\text{LLF}}) & = \left((r+1-l_1+l_3(a-1)-0.5l_4) \right. \\
 & \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\
 & \times \{C_2(x) + l_2c + l_3b \\
 & \left. \left. + 0.5dl_4\}^{-(r+2-l_1+l_3(a-1)-0.5l_4)} \right) \right. \\
 & \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\
 & \times \{C_2(x) + l_2c + l_3b \\
 & \left. \left. + 0.5dl_4\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \\
 & + \ln \left\{ \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \right. \\
 & \times \{C_2(x) + l_2c + l_3b + 0.5dl_4 \\
 & \left. \left. + 1\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right) \right. \\
 & \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\
 & \times \{C_2(x) + l_2c + l_3b \\
 & \left. \left. + 0.5dl_4\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \right\} .
 \end{aligned} \quad (18)$$

Bayes estimator and corresponding risk under precautionary loss function are, respectively, presented as

$$\begin{aligned}
 \psi_{\text{PLF}} & = \left\{ \left(\Gamma(r+3-l_1+l_3(a-1)-0.5l_4) \right. \right.
 \end{aligned}$$

$$\begin{aligned} & \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \left. + 0.5 d l_4 \right\}^{-(r+3-l_1+l_3(a-1)-0.5l_4)} \Bigg) \\ & \times \left(\Gamma(r+1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \quad \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \Bigg)^{-1} \Bigg\}^{1/2}, \\ & \rho(\psi_{PLF}) \\ & = 2 \left\{ \left(\Gamma(r+3-l_1+l_3(a-1)-0.5l_4) \right. \right. \\ & \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \quad \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r+3-l_1+l_3(a-1)-0.5l_4)} \Bigg) \\ & \quad \times \left(\Gamma(r+1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \quad \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \Bigg)^{-1} \Bigg\}^{1/2} \\ & - \left(\left(2(r+1-l_1+l_3(a-1)-0.5l_4) \right. \right. \\ & \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \quad \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r+2-l_1+l_3(a-1)-0.5l_4)} \Bigg) \\ & \quad \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\ & \quad \times \left\{ C_2(t) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \Bigg)^{-1} \\ & \quad \times \left[\Gamma(r-l_1+l_3(a-1)-0.5l_4) \right. \\ & \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\ & \quad \times \left\{ C_2(x) + l_2 c + l_3 b \right. \\ & \quad \quad \left. + 0.5 d l_4 \right\}^{-(r-l_1+l_3(a-1)-0.5l_4)} \Bigg]^2 \Bigg) \end{aligned}$$

$$\begin{aligned}
& \times \left(\Gamma(r+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \quad \times \Gamma(r-1-l_1+l_3(a-1)-0.5l_4) \\
& \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\
& \quad \times \{C_2(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(r-1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}.
\end{aligned} \tag{21}$$

5. Bayesian Estimation under Doubly Type II Censored Samples

Doubly type II censoring is used when the samples are censored at two test termination points; that is, the observations below and above a particular point cannot be either observed or feasible to be observed. The likelihood function under the doubly type II censored samples can be defined as follows.

Consider a random sample of size “ n ” from Burr type VIII distribution and let $x_w, \dots, x_s$ be the ordered observations that can only be observed. The remaining “ $w-1$ ” smallest observations and “ $n-s$ ” largest observations have been censored. Then the likelihood function for the type II doubly censored sample $\underline{x} = (x_w, \dots, x_s)$ can be written as

$$\begin{aligned}
L(\psi | x) & \propto \{F(x_w, \psi)\}^{w-1} \\
& \quad \times \{1 - F(x_s, \psi)\}^{n-s} \left\{ \prod_{i=r}^s f(x_i, \psi) \right\}, \\
L(\psi | x) & \propto \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \psi^k \\
& \quad \times e^{-\psi [\sum_{i=r}^s \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1}]} \\
& \quad \times e^{-\psi [(w-1) \ln \{(2/\pi) \tan^{-1}(e^{x_w})\}^{-1}]} \\
& \quad \times e^{-\psi [j \ln \{(2/\pi) \tan^{-1}(e^{x_s})\}^{-1}]}.
\end{aligned} \tag{22}$$

The generalized posterior distribution under doubly type II censored samples is

$$\begin{aligned}
p(\psi | x) & = \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right. \\
& \quad \times \psi^{k-l_1+l_3(a-1)-0.5l_4} \\
& \quad \left. \times e^{-\psi \{C_3(x) + l_2c + l_3b + 0.5dl_4\}} \right)
\end{aligned}$$

$$\begin{aligned}
& \times \left(\Gamma(k+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \quad \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}; \\
& \quad \psi > 0,
\end{aligned} \tag{23}$$

where $C_3(x) = \sum_{i=r}^s \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1} + (w-1) \ln \{(2/\pi) \tan^{-1}(e^{x_w})\}^{-1} + j \ln \{(2/\pi) \tan^{-1}(e^{x_s})\}^{-1}$. Bayes estimator and corresponding risk under LINEX loss function are, respectively, presented as

$$\begin{aligned}
\rho(\psi_{LLF}) & = \left((k+1-l_1+l_3(a-1)-0.5l_4 \right. \\
& \quad \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(k+2-l_1+l_3(a-1)-0.5l_4)} \right) \\
& \quad \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right. \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \\
& \quad + \ln \left\{ \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right. \right. \\
& \quad \quad \left. \left. + \{C_3(x) + l_2c + l_3b \right. \right. \\
& \quad \quad \left. \left. + 0.5dl_4 + 1\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right) \right. \\
& \quad \left. \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \right\}
\end{aligned}$$

$$\times \left\{ C_3(x) + l_2c + l_3b + 0.5dl_4 \right\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \Bigg)^{-1} \Bigg\} . \quad (24)$$

Bayes estimator and corresponding risk under precautionary loss function are, respectively, presented as

$$\begin{aligned} \psi_{\text{PLF}} = & \left\{ \left(\Gamma(k+3-l_1+l_3(a-1)-0.5l_4) \right. \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+3-l_1+l_3(a-1)-0.5l_4)} \right) \\ & \times \left(\Gamma(k+1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right) \Bigg\}^{-1/2}, \end{aligned}$$

$$\begin{aligned} \rho(\psi_{\text{PLF}}) = & 2 \left\{ \left(\Gamma(k+3-l_1+l_3(a-1)-0.5l_4) \right. \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+3-l_1+l_3(a-1)-0.5l_4)} \right) \\ & \times \left(\Gamma(k+1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right) \Bigg\}^{-1/2} \end{aligned}$$

$$\begin{aligned} & - \left(2(k+1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+2-l_1+l_3(a-1)-0.5l_4)} \right) \\ & \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}. \quad (25) \end{aligned}$$

Bayes estimator and corresponding risk under quadratic loss function are, respectively, presented as

$$\begin{aligned} \psi_{\text{QLF}} = & \left((k-1-l_1+l_3(a-1)-0.5l_4) \right. \\ & \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k-l_1+l_3(a-1)-0.5l_4)} \right) \\ & \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\ & \times \{C_3(x) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k-1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}, \quad (26) \end{aligned}$$

$$\begin{aligned} \rho(\psi_{\text{QLF}}) = & 1 - \left(\left[\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right. \right. \\ & \times \{C_3(t) + l_2c + l_3b \\ & \left. \left. + 0.5dl_4 \right\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right]^{-1} \end{aligned}$$

$$\begin{aligned}
& \times \left[\Gamma(k - l_1 + l_3(a - 1) - 0.5l_4) \right. \\
& \quad \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(k-l_1+l_3(a-1)-0.5l_4)} \left. \right]^2 \Bigg) \\
& \times \left(\Gamma(k + 1 - l_1 + l_3(a - 1) - 0.5l_4) \right. \\
& \quad \times \Gamma(k - 1 - l_1 + l_3(a - 1) - 0.5l_4) \\
& \quad \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \quad \left. + 0.5dl_4\}^{-(k-1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}. \tag{27}
\end{aligned}$$

6. Credible Intervals

Samaniego [14] has discussed that unlike the classical inference, the notion of interval estimation under Bayesian inference is simpler and manages to avoid potential conflicts with the observed data. The posterior distribution of the parameter ψ comprises the basis for all Bayesian inferences about ψ . The Bayesian counterpart of a confidence interval for ψ is called a credibility interval for ψ and is obtained from the posterior distribution by selecting an interval corresponding to the probability level desired. For example, any interval (ψ_L, ψ_U) for which $\int_{\psi_L}^{\psi_U} p(\psi | x) d\psi = 1 - \alpha$ is the $100(1 - \alpha)\%$ credibility interval for ψ , where $p(\psi | x)$ is the posterior density of θ and α is the level of significance. The credibility interval used most often is the central one in which the limits (ψ_L, ψ_U) are chosen to satisfy $\int_{-\infty}^{\psi_L} p(\psi | x) d\psi = \alpha/2 = \int_{\psi_U}^{\infty} p(\psi | x) d\psi$. Credibility intervals represent the statistician's posterior judgment about intervals that contain ψ with a given probability. They are clearly in harmony with the likelihood principle. So according to the above definition, the $100(1 - \alpha)\%$ credible intervals for ψ have been constructed under different censoring techniques as follows.

The generalized credible interval under left censored samples is

$$\begin{aligned}
& \Gamma(n - m + 1 - l_1 + l_3(a - 1) - 0.5l_4, \\
& \quad \psi_L \{C_1(x) + l_2c + l_3b + 0.5dl_4\})
\end{aligned}$$

$$\begin{aligned}
& \times (\Gamma(n - m + 1 - l_1 + l_3(a - 1) - 0.5l_4))^{-1} \\
& = 1 - \frac{\alpha}{2}, \\
& \Gamma(n - m + 1 - l_1 + l_3(a - 1) - 0.5l_4, \\
& \quad \psi_U \{C_1(x) + l_2c + l_3b + 0.5dl_4\}) \\
& \quad \times (\Gamma(n - m + 1 - l_1 + l_3(a - 1) - 0.5l_4))^{-1} \\
& = \frac{\alpha}{2}. \tag{28}
\end{aligned}$$

The generalized credible interval under singly type II censored samples is

$$\begin{aligned}
& \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \left(\Gamma(r + 1 - l_1 + l_3(a - 1) - 0.5l_4, \right. \right. \\
& \quad \psi_L \{C_2(x) + l_2c \\
& \quad \quad \left. + l_3b + 0.5dl_4\}) \right. \\
& \quad \times \left(\{C_2(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \right) \Bigg) \\
& \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right. \\
& \quad \times \left(\Gamma(r + 1 - l_1 + l_3(a - 1) - 0.5l_4) \right. \\
& \quad \times \left(\{C_2(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \right) \Bigg) \Bigg) \\
& = 1 - \frac{\alpha}{2}, \tag{29} \\
& \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \left(\Gamma(r + 1 - l_1 + l_3(a - 1) \right. \right. \\
& \quad - 0.5l_4, \psi_U \{C_2(x) + l_2c \\
& \quad \quad \left. + l_3b + 0.5dl_4\}) \right. \\
& \quad \times \left(\{C_2(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1} \right) \Bigg) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \times \left(\sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \right) \\
& \times \left(\Gamma(r+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \times \left(\{C_2(x) + l_2c + l_3b \right. \\
& \quad \left. \left. + 0.5dl_4\}^{(r+1-l_1+l_3(a-1)-0.5l_4)^{-1}} \right)^{-1} \right) \\
& = \frac{\alpha}{2}.
\end{aligned} \tag{30}$$

The generalized credible interval under doubly type II censored samples is

$$\begin{aligned}
& \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\
& \times \left(\Gamma(k+1-l_1+l_3(a-1)-0.5l_4, \right. \\
& \quad \left. \psi_L \{C_3(x) + l_2c + l_3b + 0.5dl_4\} \right. \\
& \quad \times \left(\{C_3(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(k+1-l_1+l_3(a-1)-0.5l_4)^{-1}} \right)^{-1} \right) \\
& \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\
& \times \left(\Gamma(k+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \quad \times \left(\{C_3(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(k+1-l_1+l_3(a-1)-0.5l_4)^{-1}} \right)^{-1} \right) \\
& = 1 - \frac{\alpha}{2},
\end{aligned} \tag{31}$$

$$\begin{aligned}
& \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\
& \times \left(\Gamma(k+1-l_1+l_3(a-1) \right. \\
& \quad \left. - 0.5l_4, \psi_U \{C_3(x) + l_2c + l_3b + 0.5dl_4\} \right)
\end{aligned}$$

$$\begin{aligned}
& \times \left(\{C_3(x) + l_2c + l_3b \right. \\
& \quad \left. \left. + 0.5dl_4\}^{(k+1-l_1+l_3(a-1)-0.5l_4)^{-1}} \right)^{-1} \right) \\
& \times \left(\sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \right) \\
& \times \left(\Gamma(k+1-l_1 \right. \\
& \quad \left. + l_3(a-1) - 0.5l_4) \right. \\
& \quad \times \left(\{C_3(x) + l_2c + l_3b \right. \\
& \quad \quad \left. \left. + 0.5dl_4\}^{(k+1-l_1+l_3(a-1)-0.5l_4)^{-1}} \right)^{-1} \right) \\
& = \frac{\alpha}{2}.
\end{aligned} \tag{32}$$

7. Posterior Predictive Distributions and Intervals

There are circumstances (e.g., in regression analysis or time series) where out-of-sample (e.g., data at future time points or under different conditions and covariates) predictions are the major interest; such predictions may be in circumstances where the explanatory variates take different values to those actually observed. In clinical trials comparing the efficacy of an established therapy with a new therapy, the interest may be in the predictive probability that a new patient will benefit from the new therapy. The predictive distribution contains the information about the independent future random observation given preceding observations (data at hand). In context of Bayesian inference the predictive distribution is referred to as the posterior predictive distribution. For more illustration, see Bolstad [15], Congdon [16], and Bansal [17]. The posterior predictive distribution can be defined as

$$g(y|x) = \int_{-\infty}^{\infty} p(\psi|x) f(y;\psi) d\psi, \tag{33}$$

where $p(\psi|x)$ is the posterior distribution, $f(y;\psi)$ is density for future observation, and $y = x_{n+1}$ is the future observation given the sample information $x = (x_1, x_2, \dots, x_n)$, from model (1). The posterior predictive distributions using different types of censored samples can be derived as follows.

The generalized posterior predictive distribution under left censored samples is

$$\begin{aligned}
p(y|x) &= \left((n-m+1-l_1 \right. \\
& \quad \left. + l_3(a-1) - 0.5l_4) \right)
\end{aligned}$$

$$\begin{aligned}
& \times e^y [C_1(x) + l_2c + l_3b \\
& \quad + 0.5dl_4]^{n-m+1-l_1+l_3(a-1)-0.5l_4}) \\
& \times \left((1 + e^{2y}) \tan^{-1}(e^y) \right. \\
& \quad \times [C_1(x) + l_2c + l_3b + 0.5dl_4 \\
& \quad \left. - \ln \left\{ \frac{2}{\pi} \tan^{-1}(e^y) \right\} \right]^{n-m+1-l_1+l_3(a-1)-0.5l_4})^{-1}; \\
& y > 0.
\end{aligned} \tag{34}$$

The generalized posterior predictive distribution under singly type II censored samples is

$$\begin{aligned}
p(y|x) &= \left((r+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \times e^y \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\
& \times [C_2(x) + l_2c + l_3b + 0.5dl_4 \\
& \quad \left. - \ln \left\{ \frac{2}{\pi} \tan^{-1}(e^y) \right\} \right]^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \\
& \times \left((1 + e^{2y}) \tan^{-1}(e^y) \right. \\
& \quad \times \sum_{j=0}^{n-r} (-1)^j \binom{n-r}{j} \\
& \quad \times \{C_2(x) + l_2c + l_3b \\
& \quad \left. + 0.5dl_4\}^{-(r+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}; \\
& y > 0.
\end{aligned} \tag{35}$$

The generalized posterior predictive distribution under doubly type II censored samples is

$$\begin{aligned}
p(y|x) &= \left((k+1-l_1+l_3(a-1)-0.5l_4) \right. \\
& \times e^y \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j}
\end{aligned}$$

$$\begin{aligned}
& \times [C_3(x) + l_2c + l_3b + 0.5dl_4 \\
& \quad - \ln \left\{ \frac{2}{\pi} \tan^{-1}(e^y) \right\}]^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \\
& \times \left((1 + e^{2y}) \tan^{-1}(e^y) \right. \\
& \quad \times \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \\
& \quad \times \{C_3(x) + l_2c + l_3b \\
& \quad \left. + 0.5dl_4\}^{-(k+1-l_1+l_3(a-1)-0.5l_4)} \right)^{-1}; \\
& y > 0.
\end{aligned} \tag{36}$$

The posterior predictive interval can be constructed by solving the following two equations:

$$\int_0^L g(y|x) dy = \frac{\alpha}{2}, \quad \int_U^\infty g(y|x) dy = \frac{\alpha}{2}, \tag{37}$$

where α is the level of significance.

The posterior predictive intervals cannot be obtained in the closed form, so the approximate solution of the limits has been obtained by iterative procedure.

8. Simulation Study

The simulation study has been carried out for $n = 20, 30, 50, 70, 100$, and 150 using $\psi \in \{2, 4, 6\}$. As single sample may not fully describe the behavior of the estimators, the results have been replicated sufficiently and average of the results has been presented in Tables 1, 2, 3, 4, 5, 6, and 7. The performance of the different estimates has been compared in terms of magnitude of posterior risks. The posterior risks have been presented in parenthesis in the tables.

It is immediate from Tables 1, 2, 3, 4, 5, 6, and 7 that estimated value of the parameter converges to the true value by increasing the sample size. The parameter is overestimated for the majority of the cases. A greater tendency of overestimation is observed for larger values of the parameter. This pattern is similar under each prior, loss function, and censoring scheme. The convergence of the estimated values of the parameter under informative priors is better than those under noninformative priors. In case of loss functions the convergence is better under LLF for smaller values of the parameter, while for larger values of the parameter the convergence is faster under QLF. As far as the censoring techniques are concerned, the better convergence is observed under left censored samples for LLF and PLF, while in case of QLF the estimates under singly type II censored samples have comparatively rapid convergence.

TABLE 1: Bayes estimators and posterior risks under uniform prior.

<i>n</i>	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
$\Psi = 2$									
20	2.16360	2.24633	2.11116	2.19235	2.26019	2.15047	2.18079	2.25540	2.13425
	(0.09240)	(0.08906)	(0.06250)	(0.07612)	(0.07236)	(0.05409)	(0.08355)	(0.07987)	(0.05465)
30	2.10850	2.18912	2.05740	2.13652	2.20263	2.09571	2.12525	2.19796	2.07990
	(0.06234)	(0.06009)	(0.04167)	(0.05135)	(0.04882)	(0.03606)	(0.05637)	(0.05389)	(0.03121)
50	2.08038	2.15993	2.02996	2.10803	2.17326	2.06776	2.09691	2.16865	2.05216
	(0.05368)	(0.05174)	(0.02500)	(0.04422)	(0.04204)	(0.02002)	(0.04854)	(0.04640)	(0.02223)
70	2.05895	2.13768	2.00905	2.08631	2.15087	2.04646	2.07531	2.14631	2.03102
	(0.04109)	(0.03960)	(0.01786)	(0.03385)	(0.03218)	(0.01545)	(0.03715)	(0.03551)	(0.01337)
100	2.01683	2.09395	1.97424	2.04167	2.10484	2.01100	2.02041	2.05749	1.99583
	(0.02499)	(0.02496)	(0.01250)	(0.02058)	(0.02029)	(0.01001)	(0.02259)	(0.02239)	(0.01111)
150	2.00807	2.08485	1.96665	2.02666	2.08937	2.00326	2.00605	2.03264	1.98815
	(0.01644)	(0.01654)	(0.00833)	(0.01354)	(0.01344)	(0.00667)	(0.01486)	(0.01483)	(0.00741)
$\Psi = 4$									
20	4.20964	4.37061	4.10762	4.26559	4.39758	4.18411	4.24309	4.38826	4.15254
	(0.15309)	(0.14755)	(0.10355)	(0.12611)	(0.11989)	(0.08961)	(0.13843)	(0.13232)	(0.09054)
30	4.18678	4.34688	4.08531	4.24243	4.37371	4.16139	4.22005	4.36443	4.12999
	(0.10328)	(0.09955)	(0.06903)	(0.08508)	(0.08089)	(0.05974)	(0.09339)	(0.08928)	(0.05170)
50	4.15175	4.31051	4.05113	4.20693	4.33711	4.12657	4.18474	4.32791	4.09543
	(0.08894)	(0.08572)	(0.04142)	(0.07326)	(0.06965)	(0.03317)	(0.08042)	(0.07687)	(0.03683)
70	4.15016	4.30886	4.04958	4.20532	4.33545	4.12499	4.18314	4.32625	4.09387
	(0.06807)	(0.06561)	(0.02959)	(0.05608)	(0.05331)	(0.02560)	(0.06155)	(0.05884)	(0.02216)
100	4.07290	4.22864	4.00663	4.12306	4.25064	4.06113	4.08013	4.15501	4.03049
	(0.04140)	(0.04135)	(0.02071)	(0.03410)	(0.03362)	(0.01658)	(0.03743)	(0.03710)	(0.01841)
150	4.05520	4.21026	4.00399	4.09275	4.21940	4.04550	4.05113	4.10483	4.01498
	(0.02724)	(0.02740)	(0.01380)	(0.02243)	(0.02227)	(0.01105)	(0.02462)	(0.02457)	(0.01228)
$\Psi = 6$									
20	6.50207	6.75070	6.34449	6.58849	6.79236	6.46263	6.55374	6.77796	6.41388
	(0.29086)	(0.28035)	(0.19674)	(0.23960)	(0.22779)	(0.17026)	(0.26301)	(0.25142)	(0.17202)
30	6.46677	6.71405	6.31004	6.55272	6.75548	6.42754	6.51815	6.74115	6.37905
	(0.19624)	(0.18915)	(0.13116)	(0.16166)	(0.15369)	(0.11351)	(0.17745)	(0.16962)	(0.09823)
50	6.41266	6.65787	6.25724	6.49789	6.69896	6.37376	6.46361	6.68475	6.32567
	(0.16898)	(0.16287)	(0.07870)	(0.13920)	(0.13234)	(0.06302)	(0.15280)	(0.14606)	(0.06998)
70	6.41020	6.65532	6.25485	6.49540	6.69639	6.37132	6.46114	6.68219	6.32325
	(0.12934)	(0.12466)	(0.05621)	(0.10654)	(0.10129)	(0.04865)	(0.11695)	(0.11180)	(0.04210)
100	6.29087	6.53142	6.18852	6.36834	6.56540	6.27268	6.30203	6.41769	6.22537
	(0.07867)	(0.07857)	(0.03935)	(0.06478)	(0.06387)	(0.03151)	(0.07111)	(0.07048)	(0.03497)
150	6.26353	6.50303	6.18443	6.32153	6.51714	6.24854	6.25724	6.34018	6.20141
	(0.05175)	(0.05207)	(0.02622)	(0.04262)	(0.04231)	(0.02100)	(0.04678)	(0.04668)	(0.02333)

The study also depicts that, in terms of posterior risks, the performance of the QLF is the best among all loss functions. The performance of each loss function has been negatively affected by the increasing values of the parameter. The magnitude values of risks associated with estimates using PLF and LLF are close to each other especially in large samples. So, we can replace these two loss functions by each other in the future studies. In comparison of priors, it has been observed that the informative priors have smaller risks than noninformative priors. The estimates based on inverse Levy prior are having the least amounts of posterior risks. The larger choice of the true parametric values has negatively

TABLE 2: Bayes estimators and posterior risks under Jeffreys prior.

n	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
$\Psi = 2$									
20	2.15700	2.23948	2.10472	2.18567	2.25330	2.14392	2.17414	2.24852	2.12774
	(0.09013)	(0.08687)	(0.06096)	(0.07425)	(0.07059)	(0.05276)	(0.08150)	(0.07791)	(0.05330)
30	2.10207	2.18245	2.05113	2.13001	2.19592	2.08932	2.11878	2.19126	2.07356
	(0.06081)	(0.05861)	(0.04064)	(0.05009)	(0.04762)	(0.03517)	(0.05499)	(0.05256)	(0.03044)
50	2.07404	2.15335	2.02377	2.10161	2.16664	2.06146	2.09052	2.16204	2.04591
	(0.05236)	(0.05047)	(0.02439)	(0.04313)	(0.04101)	(0.01953)	(0.04735)	(0.04526)	(0.02168)
70	2.05268	2.13117	2.00293	2.07996	2.14432	2.04022	2.06899	2.13977	2.02483
	(0.04008)	(0.03863)	(0.01742)	(0.03301)	(0.03139)	(0.01507)	(0.03624)	(0.03464)	(0.01304)
100	2.01068	2.08757	1.96822	2.03545	2.09843	2.00487	2.01425	2.05122	1.98975
	(0.02438)	(0.02435)	(0.01219)	(0.02007)	(0.01979)	(0.00976)	(0.02204)	(0.02184)	(0.01084)
150	2.00195	2.07850	1.96066	2.02049	2.08301	1.99716	1.99994	2.02645	1.98209
	(0.01604)	(0.01613)	(0.00813)	(0.01321)	(0.01311)	(0.00651)	(0.01449)	(0.01447)	(0.00723)
$\Psi = 4$									
20	4.19681	4.35729	4.09510	4.25259	4.38418	4.17136	4.23016	4.37488	4.13988
	(0.14933)	(0.14393)	(0.10101)	(0.12301)	(0.11695)	(0.08741)	(0.13503)	(0.12907)	(0.08831)
30	4.17403	4.33363	4.07286	4.22950	4.36038	4.14870	4.20719	4.35113	4.11741
	(0.10075)	(0.09711)	(0.06734)	(0.08299)	(0.07890)	(0.05827)	(0.09110)	(0.08708)	(0.05043)
50	4.13910	4.29737	4.03879	4.19411	4.32389	4.11399	4.17199	4.31472	4.08295
	(0.08675)	(0.08362)	(0.04040)	(0.07146)	(0.06794)	(0.03235)	(0.07844)	(0.07499)	(0.03593)
70	4.13752	4.29573	4.03724	4.19251	4.32224	4.11242	4.17039	4.31307	4.08139
	(0.06640)	(0.06400)	(0.02886)	(0.05470)	(0.05200)	(0.02497)	(0.06004)	(0.05739)	(0.02161)
100	4.06049	4.21575	3.99442	4.11049	4.23769	4.04875	4.06770	4.14235	4.01821
	(0.04039)	(0.04034)	(0.02020)	(0.03326)	(0.03279)	(0.01618)	(0.03651)	(0.03618)	(0.01795)
150	4.04284	4.19743	3.99179	4.08028	4.20654	4.03317	4.03879	4.09232	4.00275
	(0.02657)	(0.02673)	(0.01346)	(0.02188)	(0.02172)	(0.01078)	(0.02402)	(0.02397)	(0.01198)
$\Psi = 6$									
20	6.48226	6.73013	6.32516	6.56842	6.77167	6.44294	6.53377	6.75730	6.39433
	(0.28372)	(0.27346)	(0.19191)	(0.23372)	(0.22220)	(0.16608)	(0.25655)	(0.24524)	(0.16779)
30	6.44706	6.69359	6.29081	6.53275	6.73489	6.40795	6.49829	6.72061	6.35961
	(0.19142)	(0.18450)	(0.12794)	(0.15768)	(0.14991)	(0.11072)	(0.17309)	(0.16546)	(0.09582)
50	6.39312	6.63758	6.23817	6.47809	6.67854	6.35434	6.44391	6.66438	6.30640
	(0.16483)	(0.15887)	(0.07676)	(0.13578)	(0.12909)	(0.06147)	(0.14905)	(0.14247)	(0.06826)
70	6.32657	6.56848	6.17324	6.41065	6.60902	6.28819	6.37684	6.59500	6.24075
	(0.12616)	(0.12160)	(0.05483)	(0.10393)	(0.09880)	(0.04745)	(0.11408)	(0.10905)	(0.04106)
100	6.14588	6.38089	6.04589	6.22157	6.41409	6.12811	6.15679	6.26978	6.08189
	(0.07673)	(0.07664)	(0.03838)	(0.06319)	(0.06230)	(0.03074)	(0.06936)	(0.06875)	(0.03411)
150	6.07266	6.30486	6.03308	6.12889	6.31854	6.05813	6.06656	6.14697	6.01243
	(0.05048)	(0.05079)	(0.02558)	(0.04158)	(0.04127)	(0.02048)	(0.04563)	(0.04554)	(0.02275)

affected the performance of each prior. The increased censoring rate results in slower convergence of the estimates and the inflated posterior risks. It is a natural consequence of the censoring that can be prevented by increased sample size. It is interesting to note that, for each prior and loss function, the

posterior risks under singly type II censoring are smaller than those under other censoring techniques.

The Bayesian interval estimation exhibited a similar pattern as observed in case of point estimation. All the credible intervals are skewed toward the right. The credible

TABLE 3: Bayes estimators and posterior risks under exponential prior.

n	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
$\Psi = 2$									
20	2.13477 (0.08792)	2.21640 (0.08474)	2.08304 (0.05947)	2.16315 (0.07242)	2.23008 (0.06885)	2.12182 (0.05146)	2.15174 (0.07950)	2.22535 (0.07599)	2.10582 (0.05199)
30	2.08041 (0.05931)	2.15996 (0.05717)	2.02999 (0.03964)	2.10806 (0.04886)	2.17329 (0.04645)	2.06779 (0.03431)	2.09694 (0.05363)	2.16868 (0.05127)	2.05219 (0.02969)
50	2.05267 (0.05108)	2.13116 (0.04923)	2.00292 (0.02379)	2.07995 (0.04207)	2.14431 (0.04000)	2.04022 (0.01905)	2.06898 (0.04618)	2.13976 (0.04415)	2.02482 (0.02115)
70	2.03152 (0.03909)	2.10920 (0.03768)	1.98229 (0.01699)	2.05852 (0.03220)	2.12222 (0.03062)	2.01920 (0.01470)	2.04766 (0.03535)	2.11772 (0.03379)	2.00397 (0.01272)
100	1.98996 (0.02378)	2.06606 (0.02375)	1.94794 (0.01189)	2.01447 (0.01958)	2.07681 (0.01931)	1.98421 (0.00952)	1.99350 (0.02149)	2.03008 (0.02130)	1.96924 (0.01057)
150	1.98132 (0.01564)	2.05708 (0.01574)	1.94045 (0.00793)	1.99966 (0.01288)	2.06154 (0.01279)	1.97658 (0.00635)	1.97933 (0.01414)	2.00556 (0.01411)	1.96167 (0.00705)
$\Psi = 4$									
20	4.15357 (0.14566)	4.31239 (0.14039)	4.05290 (0.09852)	4.20877 (0.11999)	4.33901 (0.11407)	4.12837 (0.08526)	4.18657 (0.13171)	4.32980 (0.12590)	4.09722 (0.08614)
30	4.13101 (0.09827)	4.28898 (0.09472)	4.03089 (0.06568)	4.18592 (0.08095)	4.31544 (0.07696)	4.10595 (0.05684)	4.16384 (0.08886)	4.30629 (0.08494)	4.07498 (0.04919)
50	4.09645 (0.08462)	4.25309 (0.08156)	3.99717 (0.03941)	4.15089 (0.06971)	4.27934 (0.06627)	4.07160 (0.03156)	4.12900 (0.07652)	4.27026 (0.07314)	4.04088 (0.03504)
70	4.09488 (0.06477)	4.25146 (0.06243)	3.99564 (0.02815)	4.14930 (0.05335)	4.27770 (0.05072)	4.07004 (0.02436)	4.12742 (0.05857)	4.26862 (0.05598)	4.03933 (0.02108)
100	4.01865 (0.03939)	4.17231 (0.03935)	3.95326 (0.01970)	4.06814 (0.03244)	4.19402 (0.03198)	4.00703 (0.01578)	4.02578 (0.03561)	4.09966 (0.03530)	3.97680 (0.01751)
150	4.00118 (0.02592)	4.15418 (0.02607)	3.95066 (0.01313)	4.03824 (0.02134)	4.16319 (0.02119)	3.99161 (0.01051)	3.99717 (0.02343)	4.05015 (0.02338)	3.96150 (0.01168)
$\Psi = 6$									
20	6.41546 (0.27675)	6.66078 (0.26675)	6.25998 (0.18720)	6.50073 (0.22798)	6.70189 (0.21674)	6.37654 (0.16200)	6.46644 (0.25025)	6.68767 (0.23922)	6.32844 (0.16367)
30	6.38063 (0.18672)	6.62461 (0.17997)	6.22599 (0.12480)	6.46543 (0.15381)	6.66549 (0.14623)	6.34192 (0.10800)	6.43132 (0.16884)	6.65135 (0.16139)	6.29407 (0.09346)
50	6.32724 (0.16078)	6.56918 (0.15497)	6.17389 (0.07488)	6.41133 (0.13244)	6.60972 (0.12592)	6.28886 (0.05996)	6.37751 (0.14538)	6.59570 (0.13897)	6.24141 (0.06658)
70	6.26137 (0.12306)	6.50080 (0.11861)	6.10962 (0.05348)	6.34459 (0.10137)	6.54092 (0.09638)	6.22339 (0.04629)	6.31113 (0.11128)	6.52704 (0.10637)	6.17644 (0.04006)
100	6.08255 (0.07485)	6.31513 (0.07476)	5.98359 (0.03744)	6.15746 (0.06164)	6.34799 (0.06077)	6.06497 (0.02998)	6.09335 (0.06766)	6.20517 (0.06706)	6.01921 (0.03328)
150	6.01008 (0.04924)	6.23989 (0.04954)	5.97091 (0.02495)	6.06574 (0.04055)	6.25343 (0.04025)	5.99570 (0.01998)	6.00405 (0.04451)	6.08363 (0.04442)	5.95047 (0.02219)

intervals under informative priors are more specific than those under noninformative priors on the basis of each censoring technique. The minimum widths of credible intervals have been observed under inverse Levy prior. In addition, the

credible intervals under singly type II censored samples have smaller widths as compared to other censoring techniques. Similar trends have been observed as far as the posterior predictive intervals are concerned.

TABLE 4: Bayes estimators and posterior risks under gamma prior.

n	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
$\Psi = 2$									
20	2.17527 (0.07926)	2.25845 (0.07640)	2.12255 (0.05361)	2.20418 (0.06529)	2.27239 (0.06207)	2.16207 (0.04640)	2.19255 (0.07167)	2.26757 (0.06851)	2.14576 (0.04688)
30	2.11988 (0.05347)	2.20094 (0.05154)	2.06850 (0.03574)	2.14805 (0.04405)	2.21452 (0.04188)	2.10702 (0.03093)	2.13672 (0.04835)	2.20982 (0.04622)	2.09112 (0.02677)
50	2.09161 (0.04605)	2.17159 (0.04438)	2.04091 (0.02144)	2.11941 (0.03793)	2.18499 (0.03606)	2.07892 (0.01717)	2.10823 (0.04164)	2.18035 (0.03980)	2.06323 (0.01907)
70	2.07006 (0.03524)	2.14922 (0.03397)	2.01989 (0.01532)	2.09757 (0.02903)	2.16248 (0.02760)	2.05750 (0.01326)	2.08651 (0.03187)	2.15789 (0.03046)	2.04198 (0.01147)
100	2.02771 (0.02144)	2.10525 (0.02141)	1.98489 (0.01072)	2.05268 (0.01765)	2.11620 (0.01740)	2.02185 (0.00859)	2.03131 (0.01938)	2.06859 (0.01921)	2.00660 (0.00953)
150	2.01890 (0.01410)	2.09610 (0.01419)	1.97726 (0.00715)	2.03760 (0.01161)	2.10065 (0.01153)	2.01407 (0.00572)	2.01687 (0.01275)	2.04361 (0.01272)	1.99888 (0.00636)
$\Psi = 4$									
20	4.23236 (0.13132)	4.39420 (0.12657)	4.12978 (0.08882)	4.28861 (0.10817)	4.42131 (0.10284)	4.20668 (0.07687)	4.26599 (0.11874)	4.41194 (0.11351)	4.17495 (0.07766)
30	4.20938 (0.08860)	4.37034 (0.08539)	4.10736 (0.05922)	4.26532 (0.07298)	4.39731 (0.06938)	4.18384 (0.05125)	4.24282 (0.08011)	4.38798 (0.07658)	4.15228 (0.04435)
50	4.17416 (0.07629)	4.33377 (0.07353)	4.07299 (0.03553)	4.22963 (0.06284)	4.36051 (0.05975)	4.14883 (0.02845)	4.20732 (0.06898)	4.35126 (0.06594)	4.11753 (0.03159)
70	4.17256 (0.05839)	4.33211 (0.05628)	4.07143 (0.02538)	4.22801 (0.04810)	4.35884 (0.04573)	4.14725 (0.02196)	4.20571 (0.05280)	4.34960 (0.05047)	4.11596 (0.01901)
100	4.09488 (0.03552)	4.25146 (0.03547)	4.02825 (0.01776)	4.14531 (0.02925)	4.27358 (0.02884)	4.08304 (0.01423)	4.10215 (0.03210)	4.17743 (0.03182)	4.05224 (0.01579)
150	4.07708 (0.02336)	4.23298 (0.02351)	4.02560 (0.01184)	4.11484 (0.01924)	4.24217 (0.01910)	4.06733 (0.00948)	4.07299 (0.02112)	4.12698 (0.02108)	4.03665 (0.01053)
$\Psi = 6$									
20	6.53716 (0.24950)	6.78713 (0.24048)	6.37873 (0.16877)	6.62405 (0.20553)	6.82902 (0.19540)	6.49750 (0.14605)	6.58910 (0.22561)	6.81453 (0.21566)	6.44849 (0.14756)
30	6.50166 (0.16833)	6.75028 (0.16225)	6.34409 (0.11251)	6.58808 (0.13867)	6.79193 (0.13183)	6.46222 (0.09737)	6.55332 (0.15221)	6.77753 (0.14550)	6.41347 (0.08426)
50	6.44726 (0.14495)	6.69379 (0.13971)	6.29101 (0.06751)	6.53295 (0.11940)	6.73511 (0.11352)	6.40815 (0.05406)	6.49849 (0.13107)	6.72082 (0.12529)	6.35981 (0.06003)
70	6.38015 (0.11094)	6.62412 (0.10693)	6.22552 (0.04822)	6.46495 (0.09139)	6.66500 (0.08689)	6.34145 (0.04173)	6.43084 (0.10032)	6.65086 (0.09590)	6.29360 (0.03611)
100	6.19793 (0.06748)	6.43493 (0.06740)	6.09709 (0.03375)	6.27426 (0.05557)	6.46841 (0.05479)	6.18002 (0.02703)	6.20893 (0.06100)	6.32288 (0.06046)	6.13340 (0.03000)
150	6.12409 (0.04439)	6.35826 (0.04466)	6.08418 (0.02249)	6.18080 (0.03656)	6.37206 (0.03629)	6.10943 (0.01801)	6.11794 (0.04013)	6.19903 (0.04004)	6.06335 (0.02001)

9. Real Life Example

This section covers the analysis of real life data set regarding the breaking strengths of 64 single carbon fibers of length 10, presented by Lawless [18]. The idea has been to see whether the results and properties of the Bayes estimators, explored by simulation study, are applicable to a real life situation. The amounts of posterior risks associated with each estimate have been presented in parenthesis in Tables 8, 9, and 10.

The results regarding the analysis of the real life data displayed the characteristics of the estimates parallel to those observed under simulation study. The inverse Levy prior has been found to be the most suitable prior and QLF the best working loss function. The results under real life data also indicated that the estimates under singly type II censored samples are more efficient under each prior and loss function. So, the information lost from the right end may not be as serious as from the left tail.

TABLE 5: Bayes estimators and posterior risks under inverse Levy prior.

n	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
$\Psi = 2$									
20	2.15285	2.23518	2.10068	2.18147	2.24897	2.13980	2.16996	2.24420	2.12365
	(0.07731)	(0.07452)	(0.05230)	(0.06369)	(0.06055)	(0.04526)	(0.06991)	(0.06683)	(0.04572)
30	2.09803	2.17826	2.04718	2.12592	2.19170	2.08531	2.11470	2.18705	2.06957
	(0.05216)	(0.05028)	(0.03486)	(0.04297)	(0.04085)	(0.03017)	(0.04717)	(0.04509)	(0.026110)
50	2.07005	2.14921	2.01988	2.09757	2.16247	2.05750	2.08650	2.15788	2.04197
	(0.04492)	(0.04329)	(0.02092)	(0.03700)	(0.03518)	(0.01675)	(0.04061)	(0.03882)	(0.01860)
70	2.04873	2.12707	1.99908	2.07596	2.14020	2.03630	2.06501	2.13566	2.02094
	(0.03438)	(0.03314)	(0.01494)	(0.02832)	(0.02692)	(0.01293)	(0.03109)	(0.02972)	(0.01119)
100	2.00682	2.08355	1.96444	2.03153	2.09440	2.00102	2.01038	2.04728	1.98592
	(0.02091)	(0.02088)	(0.01046)	(0.01722)	(0.01698)	(0.00838)	(0.01890)	(0.01873)	(0.00930)
150	2.00673	2.08346	1.96534	2.02531	2.08798	2.00192	2.00471	2.03128	1.98682
	(0.01376)	(0.01384)	(0.00697)	(0.01133)	(0.01125)	(0.00558)	(0.01243)	(0.01241)	(0.00620)
$\Psi = 4$									
20	4.18874	4.34891	4.08723	4.24442	4.37575	4.16333	4.22203	4.36647	4.13193
	(0.12809)	(0.12346)	(0.08664)	(0.10552)	(0.10032)	(0.07498)	(0.11583)	(0.11072)	(0.07575)
30	4.16600	4.32530	4.06503	4.22137	4.35199	4.14073	4.19910	4.34276	4.10949
	(0.08642)	(0.08330)	(0.05776)	(0.07119)	(0.06768)	(0.04999)	(0.07814)	(0.07470)	(0.04326)
50	4.13114	4.28911	4.03102	4.18605	4.31558	4.10608	4.16397	4.30643	4.07510
	(0.07441)	(0.07173)	(0.03466)	(0.06130)	(0.05828)	(0.02775)	(0.06729)	(0.06432)	(0.03082)
70	4.12956	4.28747	4.02948	4.18445	4.31393	4.10451	4.16237	4.30478	4.07354
	(0.05696)	(0.05490)	(0.02475)	(0.04692)	(0.04461)	(0.02142)	(0.05150)	(0.04923)	(0.01854)
100	4.05268	4.20765	4.02703	4.10259	4.22954	4.07060	4.05988	4.13439	4.03989
	(0.03464)	(0.03460)	(0.01733)	(0.02853)	(0.02813)	(0.01388)	(0.03132)	(0.03104)	(0.01540)
150	4.05250	4.20746	4.00132	4.09003	4.21659	4.04280	4.04843	4.10209	4.01231
	(0.02279)	(0.02293)	(0.01155)	(0.01877)	(0.01863)	(0.00925)	(0.02060)	(0.02056)	(0.01027)
$\Psi = 6$									
20	6.46980	6.71719	6.31300	6.55579	6.75865	6.43055	6.52120	6.74431	6.38204
	(0.24337)	(0.23458)	(0.16462)	(0.20048)	(0.19060)	(0.14246)	(0.22007)	(0.21037)	(0.14393)
30	6.43467	6.68072	6.27872	6.52019	6.72195	6.39563	6.48579	6.70769	6.34738
	(0.16420)	(0.15826)	(0.10975)	(0.13526)	(0.12859)	(0.09498)	(0.14847)	(0.14193)	(0.08219)
50	6.38083	6.62482	6.22618	6.46563	6.66570	6.34212	6.43153	6.65156	6.29427
	(0.14139)	(0.13628)	(0.06585)	(0.11647)	(0.11073)	(0.05273)	(0.12785)	(0.12221)	(0.05855)
70	6.31440	6.55586	6.16137	6.39833	6.59632	6.27610	6.36458	6.58232	6.22875
	(0.10822)	(0.10431)	(0.04703)	(0.08915)	(0.08475)	(0.04070)	(0.09786)	(0.09354)	(0.03522)
100	6.13406	6.36862	6.09523	6.20961	6.40175	6.16118	6.14495	6.25773	6.11471
	(0.06582)	(0.06574)	(0.03292)	(0.05421)	(0.05344)	(0.02637)	(0.05950)	(0.05897)	(0.02926)
150	6.08716	6.31992	6.04748	6.14353	6.33363	6.07259	6.08105	6.16165	6.02679
	(0.04330)	(0.04356)	(0.02194)	(0.03566)	(0.03540)	(0.01757)	(0.03914)	(0.03906)	(0.01952)

10. Conclusions and Recommendations

The study has been carried out to investigate the performance of the Bayesian point and interval estimators of the parameters for Burr type VIII distribution based on five priors, three loss functions, and three censoring techniques. From the above analysis it can be concluded that amount of risk decreases and rate of convergence of estimates towards

the true value increases with the increase in sample size. The performance of the inverse Levy prior has been the best among all priors under all cases. On the other hand, the quadratic loss function has been found to be the most efficient loss function under each prior and every censoring technique. Similarly, the estimates under singly type II censoring scheme are associated with the minimum amount of risks. So, on the whole, the use of inverse Levy prior along with

TABLE 6: 95% credible intervals under different priors and censoring techniques for $\Psi = 2$.

n	Left censored			Singly type II censored			Doubly type II censored		
	LL	UL	UL – LL	LL	UL	UL – LL	LL	UL	UL – LL
Uniform prior									
20	1.56781	2.75938	1.19157	1.73877	2.84372	1.10495	1.73962	2.90945	1.16983
30	1.61913	2.59787	0.97874	1.79641	2.70600	0.90960	1.78866	2.72517	0.93650
50	1.62627	2.53449	0.90822	1.80163	2.66082	0.85919	1.79136	2.66332	0.87196
70	1.66166	2.45624	0.79458	1.83777	2.58974	0.75197	1.82279	2.59834	0.77556
100	1.70699	2.32667	0.61968	1.85560	2.41714	0.56155	1.83423	2.40771	0.57348
150	1.75676	2.25937	0.50262	1.88471	2.34154	0.45683	1.85867	2.33087	0.47219
Jeffreys prior									
20	1.56858	2.74542	1.17685	1.73708	2.80267	1.06559	1.73746	2.86972	1.13226
30	1.61875	2.58540	0.96664	1.79389	2.66752	0.87362	1.78579	2.68849	0.90270
50	1.62554	2.52254	0.89700	1.79889	2.62316	0.82427	1.78830	2.62765	0.83935
70	1.66030	2.44505	0.78476	1.83458	2.55341	0.71884	1.79885	2.54065	0.74180
100	1.70467	2.31670	0.61202	1.85182	2.38369	0.53187	1.79008	2.33228	0.54221
150	1.75374	2.25015	0.49640	1.88049	2.30946	0.42898	1.79945	2.24150	0.44204
Exponential prior									
20	1.55362	2.71593	1.16230	1.71997	2.74490	1.02493	1.72025	2.78576	1.06552
30	1.60306	2.55776	0.95470	1.77605	2.61266	0.83660	1.76796	2.61904	0.85109
50	1.60971	2.49562	0.88591	1.78096	2.56925	0.78830	1.77039	2.64202	0.87163
70	1.64399	2.41905	0.77506	1.81620	2.50101	0.68482	1.78077	2.49203	0.71126
100	1.68773	2.29219	0.60446	1.83315	2.33487	0.50173	1.77199	2.31768	0.54570
150	1.73618	2.22645	0.49027	1.86144	2.26223	0.40079	1.78120	2.25684	0.47564
Gamma prior									
20	1.62347	2.72707	1.10360	1.77884	2.78763	1.00880	1.77576	2.78477	1.00901
30	1.66664	2.57312	0.90648	1.83130	2.65756	0.82626	1.82029	2.62195	0.80166
50	1.67102	2.51219	0.84117	1.83474	2.61473	0.77998	1.82142	2.64617	0.82475
70	1.70210	2.43802	0.73592	1.86815	2.54765	0.67951	1.82981	2.49792	0.66811
100	1.74075	2.31468	0.57393	1.88157	2.38174	0.50017	1.81750	2.32602	0.50852
150	1.78615	2.25166	0.46551	1.90782	2.31004	0.40222	1.82464	2.26706	0.44242
Inverse Levy prior									
20	1.60787	2.69784	1.08996	1.76125	2.73853	0.97728	1.75811	2.70183	0.94372
30	1.65039	2.54567	0.89528	1.81304	2.61087	0.79783	1.80206	2.54396	0.74190
50	1.65467	2.48544	0.83077	1.81640	2.56882	0.75242	1.80314	2.56750	0.76436
70	1.68532	2.41214	0.72682	1.84939	2.50299	0.65360	1.81138	2.42371	0.61233
100	1.72340	2.29024	0.56684	1.86256	2.34007	0.47751	1.79911	2.25700	0.45789
150	1.77685	2.23660	0.45976	1.89728	2.27883	0.38156	1.81447	2.20874	0.39427

quadratic loss function under singly type II censored samples can be preferred to estimate the parameter of Burr type VIII distribution.

Appendix

Derivation of Different Expressions/Formulae

The pdf of the Burr type VIII distribution given in (1) can be written as

$$f(x, \psi) \propto \psi e^{-\psi \{(2/\pi) \tan^{-1}(e^x)\}^{-1}}. \quad (\text{A.1})$$

The CDF of the Burr type VIII distribution used in (2) can be written as

$$F(x, \psi) \propto e^{-\psi \{(2/\pi) \tan^{-1}(e^x)\}^{-1}}. \quad (\text{A.2})$$

The likelihood function presented in (8) can be expressed as

$$L(\psi | x) \propto \left\{ e^{-\psi \{(2/\pi) \tan^{-1}(e^{x_{m+1}})\}^{-1}} \right\}^m \times \prod_{i=m+1}^n \psi e^{-\psi \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1}},$$

TABLE 7: 95% posterior predictive intervals under different priors and censoring techniques for $\Psi = 2$.

n	Left censored			Singly type II censored			Doubly type II censored		
	LL	UL	UL – LL	LL	UL	UL – LL	LL	UL	UL – LL
Uniform prior									
20	0.49523	13.47175	12.97652	0.54924	13.88351	13.33428	0.54950	14.20442	13.65491
30	0.51144	12.68323	12.17179	0.56744	13.21114	12.64370	0.56500	13.30473	12.73974
50	0.51370	12.37380	11.86010	0.56909	12.99057	12.42147	0.56585	13.00277	12.43692
70	0.52488	11.99177	11.46689	0.58051	12.64354	12.06303	0.57578	12.68553	12.10975
100	0.53920	11.35919	10.81999	0.58614	11.80088	11.21474	0.57939	11.75484	11.17545
150	0.55492	11.03062	10.47570	0.59534	11.43179	10.83645	0.58711	11.37969	10.79258
Jeffreys prior									
20	0.49548	13.40360	12.90812	0.54870	13.68310	13.13440	0.54882	14.01045	13.46163
30	0.51132	12.62235	12.11103	0.56665	13.02328	12.45663	0.56409	13.12565	12.56157
50	0.51347	12.31546	11.80199	0.56823	12.80670	12.23848	0.56488	12.82862	12.26374
70	0.52445	11.93714	11.41269	0.57950	12.46617	11.88667	0.56821	12.40388	11.83566
100	0.53846	11.31051	10.77205	0.58495	11.63757	11.05262	0.56544	11.38658	10.82113
150	0.55396	10.98561	10.43164	0.59400	11.27517	10.68117	0.56840	10.94338	10.37497
Exponential prior									
20	0.49075	13.25962	12.76887	0.54330	13.40106	12.85776	0.54339	13.60054	13.05716
30	0.50637	12.48741	11.98104	0.56101	12.75544	12.19443	0.55846	12.78659	12.22813
50	0.50847	12.18403	11.67556	0.56256	12.54351	11.98094	0.55922	12.89878	12.33956
70	0.51930	11.81020	11.29091	0.57369	12.21035	11.63665	0.56250	12.16650	11.60400
100	0.53311	11.19085	10.65774	0.57905	11.39922	10.82017	0.55973	11.31530	10.75557
150	0.54842	10.86990	10.32148	0.58798	11.04458	10.45660	0.56264	11.01827	10.45563
Gamma prior									
20	0.51282	13.31401	12.80119	0.56189	13.60967	13.04778	0.56092	13.59571	13.03479
30	0.52645	12.56240	12.03595	0.57846	12.97465	12.39619	0.57499	12.80080	12.22581
50	0.52784	12.26493	11.73709	0.57955	12.76555	12.18600	0.57534	12.91904	12.34370
70	0.53765	11.90282	11.36517	0.59010	12.43805	11.84795	0.57799	12.19526	11.61727
100	0.54986	11.30065	10.75079	0.59434	11.62805	11.03371	0.57411	11.35602	10.78191
150	0.56420	10.99298	10.42878	0.60264	11.27800	10.67536	0.57636	11.06816	10.49180
Inverse Levy prior									
20	0.50789	13.17130	12.66342	0.55634	13.36996	12.81362	0.55535	13.19078	12.63544
30	0.52132	12.42838	11.90706	0.57270	12.74670	12.17400	0.56923	12.42004	11.85081
50	0.52267	12.13433	11.61166	0.57376	12.54141	11.96765	0.56957	12.53496	11.96539
70	0.53235	11.77647	11.24412	0.58418	12.22001	11.63583	0.57217	11.83295	11.26078
100	0.54438	11.18133	10.63695	0.58834	11.42461	10.83627	0.56830	11.01905	10.45075
150	0.56126	10.91945	10.35819	0.59931	11.12563	10.52632	0.57315	10.78344	10.21029

$$L(\psi | x) \propto \psi^{n-m} e^{-\psi [\sum_{i=m+1}^n \ln \{(2/\pi) \tan^{-1}(e^{x_i})\}^{-1}]} \\ \times e^{-\psi [m \ln \{(2/\pi) \tan^{-1}(e^{x_{m+1}})\}^{-1}]}, \\ L(\psi | x) \propto \psi^{n-m} e^{-\psi C_1(t)},$$

(A.3)

The generalized (joint) version of priors presented in (3)–(7) can be written as

$$p(\psi) \propto \psi^{-l_1 + l_3(a-1) - 0.5l_4} e^{-\psi \{l_2c + l_3b + 0.5dl_4\}}. \quad (\text{A.5})$$

Based on the density, kernel of the posterior distribution can be given as

where

$$C_1(x) = \sum_{i=m+1}^n \ln \left\{ \left(\frac{2}{\pi} \right) \tan^{-1}(e^{x_i}) \right\}^{-1} \\ + m \ln \left\{ \left(\frac{2}{\pi} \right) \tan^{-1}(e^{x_{m+1}}) \right\}^{-1}. \quad (\text{A.4})$$

$$p(\psi | x) \propto \psi^{n-m-l_1 + l_3(a-1) - 0.5l_4} e^{-\psi \{C_1(x) + l_2c + l_3b + 0.5dl_4\}}, \\ p(\psi | x) = \frac{\psi^{n-m-l_1 + l_3(a-1) - 0.5l_4} e^{-\psi \{C_1(x) + l_2c + l_3b + 0.5dl_4\}}}{\int_0^\infty \psi^{n-m-l_1 + l_3(a-1) - 0.5l_4} e^{-\psi \{C_1(x) + l_2c + l_3b + 0.5dl_4\}} d\psi}. \quad (\text{A.6})$$

TABLE 8: Bayes estimates and posterior risks under real life data.

Priors	Left censored			Singly type II censored			Doubly type II censored		
	LLF	PLF	QLF	LLF	PLF	QLF	LLF	PLF	QLF
10% censored									
Uniform	47.24555 (3.30822)	48.72147 (2.49507)	46.64106 (0.01786)	47.87348 (2.86304)	49.02215 (2.28683)	47.50956 (0.01724)	47.62094 (3.01607)	48.91817 (2.43653)	47.15113 (0.01760)
Jeffreys	46.66228 (3.26701)	47.87151 (2.48894)	46.01342 (0.01763)	47.33033 (2.82738)	48.21597 (2.28121)	46.83223 (0.01702)	47.03304 (2.97850)	48.06477 (2.43055)	46.51663 (0.01738)
Exponential	46.08621 (3.21329)	47.03637 (2.42312)	45.39423 (0.01734)	46.79335 (2.78088)	47.42304 (2.22089)	46.16456 (0.01674)	46.45240 (2.92952)	47.22627 (2.36627)	45.89067 (0.01709)
Gamma	46.40313 (2.93552)	47.35982 (2.33481)	45.70638 (0.01585)	47.11513 (2.54049)	47.74915 (2.13995)	46.48201 (0.01530)	46.77183 (2.67628)	47.55102 (2.28004)	46.20624 (0.01561)
Inverse Levy	46.53661 (2.88725)	47.52780 (2.29642)	45.83237 (0.01558)	47.25066 (2.49872)	47.91850 (2.10476)	46.61013 (0.01505)	46.90637 (2.63227)	47.71967 (2.24255)	46.33360 (0.01536)
20% censored									
Uniform	48.88463 (3.40898)	50.33114 (2.57077)	48.34057 (0.01848)	49.75404 (2.94916)	50.92134 (2.37172)	49.35048 (0.01785)	49.40203 (3.12592)	50.34688 (2.50748)	48.98630 (0.01828)
Jeffreys	47.86867 (3.34670)	49.27538 (2.56094)	47.14291 (0.01805)	48.74232 (2.90939)	49.75933 (2.35381)	48.22909 (0.01749)	48.59545 (3.04704)	49.44202 (2.49551)	48.29985 (0.01804)
Exponential	47.13807 (3.28574)	48.35455 (2.48695)	46.45451 (0.01766)	48.09022 (2.85620)	48.68251 (2.27935)	47.41832 (0.01708)	47.50274 (2.99525)	48.04644 (2.40593)	47.55169 (0.01771)
Gamma	47.42154 (2.99705)	48.43907 (2.38132)	46.48440 (0.01610)	48.00535 (2.58490)	48.93807 (2.19134)	47.30361 (0.01556)	47.70787 (2.72857)	48.24894 (2.30718)	47.67193 (0.01590)
Inverse Levy	47.22389 (2.91876)	48.27313 (2.32932)	46.52757 (0.01579)	47.91474 (2.52577)	48.90824 (2.13919)	47.34369 (0.01522)	47.79353 (2.66669)	48.24738 (2.26701)	46.99236 (0.01553)

TABLE 9: 95% credible intervals under real life data.

Priors	Left censored			Singly type II censored			Doubly type II censored		
	LL	UL	UL – LL	LL	UL	UL – LL	LL	UL	UL – LL
10% censored									
Uniform	43.68060	50.81050	7.12990	44.55706	51.18990	6.63284	44.21704	51.02484	6.80780
Jeffreys	43.11961	50.20496	7.08535	44.03463	50.62604	6.59140	43.65041	50.41568	6.76526
Exponential	42.57279	49.59964	7.02685	43.52486	50.06184	6.53698	43.09770	49.80711	6.70941
Gamma	43.04499	49.76127	6.71627	43.99110	50.23916	6.24806	43.56540	49.97826	6.41286
Inverse Levy	43.20620	49.86702	6.66082	44.15242	50.34889	6.19647	43.72641	50.08633	6.35992
20% censored									
Uniform	42.72716	52.83739	10.11023	43.22736	52.78174	9.55438	43.26458	52.76883	9.50425
Jeffreys	42.25671	52.09809	9.84138	42.72587	52.09732	9.37145	42.83694	51.57464	8.73770
Exponential	41.89867	51.40512	9.50645	42.34254	51.34485	9.00231	42.41062	50.82486	8.41424
Gamma	42.50686	51.06047	8.55360	42.91141	51.31689	8.40548	42.89212	50.76187	7.86975
Inverse Levy	42.74227	50.65512	7.91285	43.63056	51.40093	7.77038	43.07501	50.82409	7.74908

The complete version of the posterior distribution can be presented as

$$p(\psi | x) = \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \times \psi^{n-m-l_1+l_3(a-1)-0.5l_4} \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}}. \quad (A.7)$$

The Bayes estimator under the LLF can be derived by using the following formula:

$$\psi_{LLF} = -\ln[E(e^{-\psi})], \quad (A.8)$$

TABLE 10: 95% posterior predictive intervals under real life data.

Priors	Left censored			Singly type II censored			Doubly type II censored		
	LL	UL	UL – LL	LL	UL	UL – LL	LL	UL	UL – LL
10% censored									
Uniform	13.79766	248.06530	234.26764	14.07452	249.91759	235.84308	13.96711	249.11174	235.14463
Jeffreys	13.62046	245.10895	231.48849	13.90949	247.16474	233.25524	13.78813	246.13772	232.34960
Exponential	13.44773	242.15368	228.70595	13.74847	244.41022	230.66175	13.61354	243.16658	229.55304
Gamma	13.59689	242.94278	229.34589	13.89574	245.27592	231.38018	13.76128	244.00217	230.24089
Inverse Levy	13.64781	243.45907	229.81126	13.94670	245.81164	231.86494	13.81213	244.52978	230.71765
20% censored									
Uniform	13.49650	257.96091	244.46442	13.65450	257.68922	244.03472	13.66625	257.62619	243.95994
Jeffreys	13.34789	254.35153	241.00364	13.49609	254.34777	240.85168	13.53117	251.79596	238.26479
Exponential	13.23479	250.96833	237.73354	13.37500	250.67409	237.29908	13.39651	248.13541	234.73890
Gamma	13.42691	249.28570	235.85879	13.55470	250.53758	236.98288	13.54860	247.82788	234.27928
Inverse Levy	13.50127	247.30671	233.80544	13.78186	250.94788	237.16602	13.60637	248.13165	234.52528

where

$$\begin{aligned}
 E(e^{-\psi}) &= \frac{\{C_1(t) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \\
 &\quad \times \int_0^\infty \psi^{n-m-l_1+l_3(a-1)-0.5l_4} \\
 &\quad \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4+1\}} d\psi, \\
 E(e^{-\psi}) &= \{(C_1(x) + l_2c + l_3b + 0.5dl_4) \\
 &\quad \times (C_1(x) + l_2c + l_3b + 0.5dl_4 \\
 &\quad + 1)^{-1}\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}.
 \end{aligned}
 \tag{A.9}$$

Therefore,

$$\begin{aligned}
 \psi_{LLF} &= -\ln[E(e^{-\psi})] \\
 &= -\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \\
 &\quad \times \ln\left\{\frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{C_1(x) + l_2c + l_3b + 0.5dl_4 + 1}\right\}.
 \end{aligned}
 \tag{A.10}$$

The formula for the derivation of posterior risk under LLF is

$$\rho(\psi_{LLF}) = E(\psi) - \psi_{LLF}, \tag{A.11}$$

where

$$\begin{aligned}
 E(\psi) &= \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \\
 &\quad \times \int_0^\infty \psi^{n-m+1-l_1+l_3(a-1)-0.5l_4} \\
 &\quad \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}} d\psi, \\
 E(\psi) &= \frac{n-m+1-l_1+l_3(a-1)-0.5l_4}{C_1(x) + l_2c + l_3b + 0.5dl_4}.
 \end{aligned}
 \tag{A.12}$$

Therefore, the posterior risk under LLF can be given as

$$\begin{aligned}
 \rho(\psi_{LLF}) &= \frac{n-m+1-l_1+l_3(a-1)-0.5l_4}{C_1(x) + l_2c + l_3b + 0.5dl_4} \\
 &\quad + \{n-m+1-l_1+l_3(a-1)-0.5l_4\} \\
 &\quad \times \ln\left\{\frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{C_1(x) + l_2c + l_3b + 0.5dl_4 + 1}\right\}.
 \end{aligned}
 \tag{A.13}$$

The formula for the derivation of the Bayes estimator under PLF is

$$\psi_{PLF} = [E(\psi^2)]^{1/2}, \tag{A.14}$$

where

$$E(\psi^2) = \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \times \int_0^\infty \psi^{n-m+2-l_1+l_3(a-1)-0.5l_4} \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}} d\psi, \quad (\text{A.15})$$

$$E(\psi^2) = \{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \{n-m+2-l_1+l_3(a-1)-0.5l_4\} \times (\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^2)^{-1}. \quad (\text{A.16})$$

Therefore,

$$\psi_{\text{PLF}} = [E(\psi^2)]^{1/2} = (\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \{n-m+2-l_1+l_3(a-1)-0.5l_4\})^{1/2} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1}. \quad (\text{A.17})$$

The formula for the derivation of posterior risk under PLF is

$$\psi_{\text{PLF}} = 2(\psi_{\text{PLF}} - E(\psi)). \quad (\text{A.18})$$

Therefore, the posterior risk under PLF is

$$\rho(\psi_{\text{PLF}}) = 2[(\{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times \{n-m+2-l_1+l_3(a-1)-0.5l_4\})^{1/2} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1} - \{n-m+1-l_1+l_3(a-1)-0.5l_4\} \times (C_1(x) + l_2c + l_3b + 0.5dl_4)^{-1}]. \quad (\text{A.19})$$

The Bayes estimator under QLF can be derived by using the following formula:

$$\psi_{\text{QLF}} = E(\psi^{-1}) \{E(\psi^{-2})\}^{-1}, \quad (\text{A.20})$$

where

$$E(\psi^{-1}) = \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \times \int_0^\infty \psi^{n-m-1-l_1+l_3(a-1)-0.5l_4} \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}} d\psi,$$

$$E(\psi^{-1}) = \frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{n-m-l_1+l_3(a-1)-0.5l_4},$$

$$E(\psi^{-2}) = \frac{\{C_1(x) + l_2c + l_3b + 0.5dl_4\}^{n-m+1-l_1+l_3(a-1)-0.5l_4}}{\Gamma(n-m+1-l_1+l_3(a-1)-0.5l_4)} \times \int_0^\infty \psi^{n-m-2-l_1+l_3(a-1)-0.5l_4} \times e^{-\psi\{C_1(x)+l_2c+l_3b+0.5dl_4\}} d\psi,$$

$$E(\psi^{-2}) = \{C_1(x) + l_2c + l_3b + 0.5dl_4\}^2 \times \{n-m-l_1+l_3(a-1)-0.5l_4\} \times \{n-m-1-l_1+l_3(a-1)-0.5l_4\}^{-1}. \quad (\text{A.21})$$

Hence, the Bayes estimator under the QLF is

$$\psi_{\text{QLF}} = E(\psi^{-1}) \{E(\psi^{-2})\}^{-1} = \frac{C_1(x) + l_2c + l_3b + 0.5dl_4}{n-m-1-l_1+l_3(a-1)-0.5l_4}. \quad (\text{A.22})$$

Similarly, the expression for the posterior risk under QLF is

$$\rho(\psi_{\text{QLF}}) = 1 - \frac{\{E(\psi^{-1})\}^2}{E(\psi^{-2})} = \frac{1}{n-m-l_1+l_3(a-1)-0.5l_4}. \quad (\text{A.23})$$

The derivation of Bayes estimators and posterior risks under other censoring schemes can be obtained in a similar manner.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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