Research Article

Hom-Lie Triple System and Hom-Bol Algebra Structures on Hom-Maltsev and Right Hom-Alternative Algebras

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Every multiplicative Hom-Maltsev algebra has a natural multiplicative Hom-Lie triple system structure. Moreover, there is a natural Hom-Bol algebra structure on every multiplicative Hom-Maltsev algebra and on every multiplicative right (or left) Hom-alternative algebra.

1. Introduction

The study of Lie triple systems (Lts) on their own as algebraic objects started from Jacobson’s work [1] and developed further by, for example, Lister [2], Yamaguti [3], and other mathematicians. Lts constitute examples of ternary algebras. If \((g, [\cdot, \cdot, \cdot])\) is a Lie algebra, then \((g, [\cdot, \cdot, \cdot])\) is a Lts, where \([x, y, z] = [[x, y], z]\) (see [1, 4, 5]). Another construction of Lts from binary algebras is the one from Maltsev algebras found by Loos [6].

Maltsev algebras were introduced by Maltsev [7] in a study of commutator algebras of alternative algebras and also as a study of tangent algebras to local smooth Moufang loops. Maltsev used the name “Moufang-Lie algebras” for these nonassociative algebras while Sagle [8] introduced the term “Malcev algebras.” Equivalent defining identities of Maltsev algebras are pointed out in [8].

Alternative algebras, Maltsev algebras, and Lts (among other algebras) received a twisted generalization in the development of the theory of Hom-algebras during these latest years. The forerunner of the theory of Hom-algebras is the Hom-Lie algebra introduced by Hartwig et al. in [9] in order to describe the structure of some deformation of the Witt algebra and the Virasoro algebra. It is well-known that Lie algebras are related to associative algebras via the commutator bracket construction. In the search of a similar construction for Hom-Lie algebras, the notion of a Hom-associative algebra is introduced by Makhlouf and Silvestrov in [10], where it is proved that a Hom-associative algebra gives rise to a Hom-Lie algebra via the commutator bracket construction. Since then, various Hom-type structures are considered (see, e.g., [11–23]). Roughly speaking, Hom-algebraic structures are corresponding ordinary algebraic structures whose defining identities are twisted by a linear self-map. A general method for constructing a Hom-type algebra from the ordinary type of algebra with a linear self-map is given by Yau in [24].

In [11, 21], \(n\)-ary Hom-algebra structures generalizing \(n\)-ary algebras of Lie type or associative type were considered. In particular, generalizations of \(n\)-ary Nambu or Nambu-Lie algebras, called \(n\)-ary Hom-Nambu and Hom-Nambu-Lie algebras, respectively, were introduced in [11] while Hom-Jordan algebras were defined in [18] and Hom-Lie triple systems (Hom-Lts) were introduced in [21] (another definition of a Hom-Jordan algebra is given in [20]). It is shown [21] that Hom-Lts are ternary Hom-Nambu algebras with additional properties and that Hom-Lts arise also from Hom-Jordan triple systems or from other Hom-type algebras.

Motivated by the relationships between some classes of binary algebras and some classes of binary-ternary algebras, a study of Hom-type generalization of binary-ternary algebras is initiated in [16] with the definition of Hom-Akivis algebras.
Further, Hom-Lie-Yamaguti algebras are considered in [14] and Hom-Bol algebras [12] are defined as a twisted generalization of Bol algebras which are introduced and studied in [25–27] as infinitesimal structures tangent to smooth Bol loops (some aspects of the theory of Bol algebras are discussed in [28–30]).

In this paper, we will be concerned with right (or left) Hom-alternative algebras, Hom-Maltsev algebras, Hom-Lts, and Hom-Bol algebras. We extend Loos’ construction of Lts from Maltsev algebras ([6], Satz 1) to the Hom-algebra setting (Section 3). Specifically, we prove (Theorem 14) that every multiplicative Hom-Maltsev algebra is naturally a multiplicative Hom-Lts by a suitable definition of the ternary operation. As a tool in the proof of this fact, we point out a kind of compatibility relation between the original binary operation of a given Hom-Maltsev algebra and the ternary operation mentioned above (Lemma 13). Moreover, we obtain that every multiplicative Hom-Maltsev algebra has a natural Hom-Bol algebra structure (Theorem 17). In [31] Mikheev proved that every right alternative algebra has a natural (left) Bol algebra structure. In [29] Hentzel and Peresi proved that not only a right alternative algebra but also a left alternative algebra has left Bol algebra structure. In Section 4 we prove that the Hom-analogue of these results holds. Specifically, every multiplicative right (or left) Hom-alternative algebra is a Hom-Bol algebra (Theorem 23). It could be observed that the methods used in the proof of results in [6, 26, 30, 31] cannot be reported in the Hom-algebra setting at the present stage of the theory of Hom-algebras. In Section 5 we specify Theorem 23 to recover the construction of left Bol algebras from right alternative algebras (Theorem 26; one observes that, in our proof, we use essentially some fundamental properties of right alternative algebras). In Section 2 we recall some basic definitions and facts about Hom-algebras. We define the Hom-Jordan associator of a given Hom-algebra and point out that every Hom-algebra is a Hom-triple system with respect to the Hom-Jordan associator. This observation is used in the proof of Theorem 23.

All vector spaces and algebras are meant over an algebraically closed ground field \( \mathbb{K} \) of characteristic 0.

2. Some Basics on Hom-Algebras

We first recall some relevant definitions about binary and ternary Hom-algebras. In particular, we recall the notion of a Hom-Maltsev algebra as well as some of its equivalent defining identities. Although various types of \( n \)-ary Hom-algebras are introduced and discussed in [11, 21], for our purpose, we will consider ternary Hom-algebras (ternary Hom-Nambu algebras and Hom-Lts) and Hom-Bol algebras. For fundamentals on Hom-algebras, one may refer, for example, to [9–11, 13, 17, 24, 32]. Some aspects of the theory of binary Hom-algebras are considered in [33], while some classes of binary-ternary Hom-algebras are defined and discussed in [12, 14, 16].

**Definition 1.** (i) A **Hom-algebra** is a triple \((A, \ast, \alpha)\) in which \( A \) is a \( \mathbb{K} \)-vector space, \( \ast : A \times A \to A \) a bilinear map (the binary operation), and \( \alpha : A \to A \) a linear map (the twisting map).

The Hom-algebra \( A \) is said to be **multiplicative** if \( \alpha(x \ast y) = \alpha(x) \ast \alpha(y) \) for all \( x, y \in A \).

(ii) The **Hom-Jacobian** in \((A, \ast, \alpha)\) is the trilinear map \( J_\alpha : A \times A \times A \to A \) defined as \( J_\alpha (x, y, z) := \alpha(x \ast y \ast z) - \alpha(x) \ast (\alpha(y) \ast z) \), where \( \alpha(x) \ast y \ast z \) denotes the sum over cyclic permutation of \( x, y, z \).

(iii) The **Hom-associator** of a Hom-algebra \((A, \ast, \alpha)\) is the trilinear map as: \( A^{\otimes 3} \to A \) defined as \( \alpha(x, y, z) = (x \ast y) \ast \alpha(z) - \alpha(x) \ast (y \ast z) \).

Remark 2. If \( \alpha = \text{id} \) (the identity map), then a Hom-algebra \((A, \ast, \alpha)\) reduces to an ordinary algebra \((A, \ast)\), the Hom-Jacobian \( J_\alpha \) is the ordinary Jacobian \( J \), and the Hom-associator is the usual associator for the algebra \((A, \ast)\). One observes that, in general, the map \( \alpha \) is not always injective nor surjective (see [13, 15] for discussions on the subject). So, for example, a given algebra can be twisted into zero algebra and some properties of Hom-algebras may not be valid for corresponding ordinary algebras.

As for ordinary algebras, to each Hom-algebra \( A = (A, \ast, \alpha) \) are attached two Hom-algebras: the **commutator** Hom-algebra \( A^\prime = (A, [\cdot, \cdot], \alpha) \), where \([x, y] = x \ast y - y \ast x\) (the commutator of \( x \) and \( y \)), and the plus Hom-algebra \( A^\ast = (A, \ast, \alpha) \), where \( x \ast y = x \ast y + y \ast x \) (the Jordan product) for all \( x, y \in A \).

For our purpose, we provide the following.

**Definition 3.** The **Hom-Jordan associator** of a Hom-algebra \((A, \ast, \alpha)\) is the trilinear map \( J_\ast : A^{\otimes 3} \to A \) defined as \( J_\ast (x, y, z) = (x \ast y) \ast \alpha(z) - \alpha(x) \ast (y \ast z) \), where \( \ast \) is the Jordan product on \( A \).

If \( \alpha = \text{id} \), the Hom-Jordan associator reduces to the usual Jordan associator.

**Definition 4.** (i) A **Hom-Lie algebra** is a Hom-algebra \((A, \ast, \alpha)\) such that the binary operation \( \ast \) is anticommutative and the Hom-Jacobi identity

\[
J_\alpha (x, y, z) = 0 \tag{1}
\]

holds for all \( x, y, z \) in \( A \) ([9]).

(ii) A **Hom-Maltsev algebra** is a Hom-algebra \((A, \ast, \alpha)\) such that the binary operation \( \ast \) is anticommutative and that the Hom-Maltsev identity

\[
J_\alpha (\alpha(x), \alpha(y), x \ast z) = J_\alpha (x, y, z) \ast \alpha^2 (x) \tag{2}
\]

holds for all \( x, y, z \) in \( A \) ([20]).

(iii) A **Hom-Jordan algebra** is a Hom-algebra \((A, \ast, \alpha)\) such that \((A, \ast)\) is a commutative algebra and the Hom-Jordan identity

\[
as(x \ast x, \alpha(y), \alpha(x)) = 0 \tag{3}
\]

is satisfied for all \( x, y \) in \( A \) ([20]).

(iv) A Hom-algebra \((A, \ast, \alpha)\) is called a right Hom-alternative algebra if

\[
as(x, y, y) = 0 \tag{4}
\]
for all \(x, y\) in \(A\). A Hom-algebra \((A, *, \alpha)\) is called a left Hom-alternative algebra if

\[
\alpha(x, y, z) = 0
\]

for all \(x, y, z\) in \(A\). A Hom-algebra \((A, *, \alpha)\) is called a Hom-alternative algebra if it is both right and left Hom-alternative [18].

**Remark 5.** When \(\alpha = \text{id}\), the Hom-Jacobi identity (1) is the usual Jacobi identity \(J(x, y, z) = 0\). Likewise, for \(\alpha = \text{id}\), the Hom-Maltsev identity (2) reduces to the Maltsev identity \(J(x, y, z) = J(x, y, z) \ast x\). Therefore a Lie (resp., Maltsev) algebra \((A, *)\) may be seen as a Hom-Lie (resp., Hom-Maltsev) algebra with the identity map as the twisting map. Also Hom-Maltsev algebras generalize Hom-Lie algebras in the same way that Maltsev algebras generalize Lie algebras. For \(\alpha = \text{id}\) in the Hom-Jordan identity, we recover the usual Jordan identity. Observe that the definition of the Hom-Jordan identity in [20] is slightly different from the one formerly given in [18].

Hom-Maltsev algebras are introduced in [20] in connection with a study of Hom-alternative algebras introduced in [18]. In fact it is proved ([20], Theorem 3.8) that every Hom-alternative algebra is Hom-Maltsev admissible; that is, the commutator Hom-algebra of any Hom-alternative algebra is a Hom-Maltsev algebra (this is the Hom-analogue of Maltsev's construction of Maltsev algebras as commutator algebras of alternative algebras [7]). This result is also mentioned in [16], Section 4, using an approach via Hom-Akivis algebras (this approach is close to the one of Maltsev in [7]). Also, every Hom-alternative algebra is Hom-Maltsev admissible; that is, its plus Hom-algebra is a Hom-Jordan algebra ([20]). Examples of Hom-alternative algebras and Hom-Jordan algebras could be found in [18, 20]. An example of a right Hom-alternative algebra that is not left Hom-alternative is given in [23].

Equivalent to (2) defining identities of Hom-Maltsev algebras are found in [20] where, in particular, it is shown that the identity

\[
J_\alpha(x, y, z) + J_\alpha(w, x, z) = J_\alpha(x, y, z) \ast x + J_\alpha(x, y, z) \ast 2w
\]

is equivalent to (2) in any anticommutative Hom-algebra \((A, *, \alpha)\) ([20], Proposition 2.7). In [34], it is proved that, in any anticommutative Hom-algebra \((A, *, \alpha)\), the Hom-Maltsev identity (2) is equivalent to

\[
J_\alpha(x, y, z) + J_\alpha(w, x, z) = J_\alpha(x, y, z) \ast x + J_\alpha(x, y, z) \ast 2w
\]

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\[
J_\alpha(x, y, z) + J_\alpha(w, x, z) = J_\alpha(x, y, z) \ast x + J_\alpha(x, y, z) \ast 2w
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\[
J_\alpha(x, y, z) + J_\alpha(w, x, z) = J_\alpha(x, y, z) \ast x + J_\alpha(x, y, z) \ast 2w
\]
Hom-triple system is just the Hom-algebra \((A, [], \alpha)\). With this vision of a Hom-triple system, it is shown [14] that every multiplicative non-Hom-associative algebra (i.e., not necessarily Hom-associative algebra) has a natural Hom-triple system structure if defining \([x, y, z] = [x, y, \alpha(z)] = as(x, y, z) + as(y, x, z)\). We note here that we get the same result if defining another ternary operation on a given Hom-algebra. Specifically, we have the following result.

**Proposition 9.** Let \(\mathcal{A} = (A, \ast, \alpha)\) be a multiplicative Hom-algebra. Define on \(\mathcal{A}\) the ternary operation
\[
(x, y, z) = as^\ast(y, z, x)
\]
for all \(x, y,\) and \(z \in A\). Then \((A, \ast, \alpha)\) is a multiplicative Hom-triple system.

**Proof.** A proof follows from the straightforward checking of identities (9) and (10) for "\((\ast, \alpha)\" using the commutativity of the Jordan product "\(\ast\)."

Since our results here depend on multiplicativity, in the rest of this paper we assume that all Hom-algebras (binary or ternary) are multiplicative and while dealing with the binary operation "\(\ast\" and where there is no danger of confusion, we will use juxtaposition in order to reduce the number of braces; that is, for example, \(xy \ast \alpha(z)\) means \((x \ast y) \ast \alpha(z)\).

Various results and constructions related to Hom-Lts are given in [21]. In particular, it is shown that every Lts \(L\) can be twisted along any self-morphism of \(L\) into a multiplicative Hom-Lts. For our purpose we just mention the following result.

**Proposition 10 (see [21]).** Let \((A, \ast)\) be a Mal'tsev algebra and \(\alpha: A \to A\) an algebra morphism. Then \(A_\alpha := (A, [\mathcal{L}], \alpha)\) is a multiplicative Hom-Lts, where \([x, y, z]_\alpha = \alpha(2xy \ast z - yz \ast x - zx \ast y)\), for all \(x, y,\) and \(z \in A\).

One observes that the product \([x, y, z] = 2xy \ast z - yz \ast x - zx \ast y\) is the one defined in [6] providing a Mal'tsev algebra \((A, \ast)\) with a Lts structure. A construction describing another view of Proposition 10 above will be given in Section 3 (see Proposition 16) via Hom-Mal'tsev algebras. For the time being, we point out the following slight generalization of the result above, producing a sequence of multiplicative Hom-Lts from a given Mal'tsev algebra.

**Proposition 11.** Let \((A, \ast)\) be a Mal'tsev algebra and \(\alpha: A \to A\) an algebra morphism. Let \(\alpha^n = \mathrm{id}\) and, for any integer \(n \geq 1\), \(\alpha^n = \alpha \circ \alpha^{n-1}\). If one defines on \(A\) a trilinear operation \([\mathcal{L}], \alpha^n\) by
\[
[x, y, z]_{\alpha^n} = \alpha^n(2xy \ast z - yz \ast x - zx \ast y)
\]
for all \(x, y,\) and \(z \in A\), then \((A, [\mathcal{L}], \alpha^n)\) is a multiplicative Hom-Lts.

**Proof.** Let \([x, y, z] = 2xy \ast z - yz \ast x - zx \ast y\) and then \([x, y, z]_{\alpha^n} = \alpha^n([x, y, z])\). We shall use the fact that \((A, [\mathcal{L}], \alpha^n)\) is a Lts [6]. Identities (9) and (10) for \([x, y, z]_{\alpha^n}\) are quite obvious. Next,
\[
[\alpha^n(x), \alpha^n(y), [u, v, w]_{\alpha^n}] = [\alpha^n(x), \alpha^n(y)]_{\alpha^n} + [\alpha^n(u), \alpha^n(v), [x, y, w]]_{\alpha^n} + [\alpha^n(u), \alpha^n(v), [x, y, w]]_{\alpha^n} + [\alpha^n(u), \alpha^n(v), [x, y, w]]_{\alpha^n}
\]
and so (11) holds for \([\mathcal{L}], \alpha^n\). Thus \((A, [\mathcal{L}], \alpha^n)\) is a multiplicative Hom-Lts.

In [12] we defined a Hom-Bol algebra as a twisted generalization of a (left) Bol algebra. For the introduction and original studies of Bol algebras, we refer to [25–27] (the definition identities of left Bol algebras are recalled in Section 5 of the present paper). Bol algebras are further considered in, for example, [29, 30].

**Definition 12 (see [12]).** A Hom-Bol algebra is a quadruple \((A, [\mathcal{L}], \langle \rangle, \alpha)\) in which \(A\) is a vector space, \(\langle \rangle\) a binary operation, \(\langle \rangle\) a ternary operation on \(A\), and \(\alpha: A \to A\) a linear map such that
\[
(HB1) \alpha([x, y]) = [\alpha(x), \alpha(y)].
\]
\[
(HB2) \alpha((x, y, z)) = (\alpha(x), \alpha(y), \alpha(z)).
\]
\[
(HB3) [x, y, z] = -[y, x, z].
\]
\[
(HB4) [x, y, z] = -([y, x, z]).
\]
\[
(HB5) [\sigma_{x,y,z} (x, y, z) = 0.\]
\[
(HB6) (\sigma_{x,y,z} (x, y, u, v) = [(x, y, u), \alpha^2(v)] + [\alpha^2(u), (x, y, v)] + (\alpha(u), \alpha(v), [x, y]) - [[\alpha(u), \alpha(v)], [\alpha(x), \alpha(y)]].
\]
\[
(HB7) ([\alpha^2(x), \alpha^2(y), (u, v, w)] = ([x, y, u], \alpha^2(v), \alpha^2(w)) + ([\alpha^2(u), (x, y, v)], \alpha^2(w)) + ([\alpha^2(u), \alpha^2(v), (x, y, w)] for all \(u, v, w\) and \(x, y, z \in A\).
\]

Identities (HB1) and (HB2) mean the multiplicativity of \((A, [\mathcal{L}], \langle \rangle, \alpha)\). It is built into our definition for convenience. One observes that for \(\alpha = \mathrm{id}\) identities (HB3)–(HB7) reduce to the defining identities of a (left) Bol algebra [25] (see also [29, 30]). If \([x, y] = 0\) for all \(x, y \in A\), then \((A, [\mathcal{L}], \langle \rangle, \alpha)\) becomes a (multiplicative) Hom-Lts \((A, [\mathcal{L}], \alpha^2)\).
Construction results and some examples of Hom-Bol algebras are given in [12]. In particular, Hom-Bol algebras can be constructed from Mal’tsev algebras. The Hom-analogues of the construction of Bol algebras from Mal’tsev algebras [25] or from right alternative algebras [31] (see also [29]) are considered in this paper.

3. Hom-Lts and Hom-Bol Algebras from Hom-Maltsev Algebras

In this section, we prove that every multiplicative Hom-Maltsev algebra has a natural multiplicative Hom-Lts structure (Theorem 14) and, moreover, a natural Hom-Bol algebra structure (Theorem 17). Theorem 14 could be seen as the Hom-analogue of Loos’ result ([6], Satz 1) although his proof cannot be reproduced here. Besides identities (6) and (7), Lemma 13 below is a tool in the proof of this result. Theorem 17 could be seen as the Hom-analogue of a construction by Mikheev [25] of Bol algebras from Mal’tsev algebras. Proposition 16 is another view of a result in [21] (see Proposition 10 above).

In his work [6], Loos considered in a Mal’tsev algebra $(A, *)$ the following ternary operation:

$$\{x, y, z\} = 2xy * z - yz * x - zx * y.$$  \hspace{1cm} (15)

Then $(A, \{,,\})$ turns out to be a Lts. This result, in the Hom-algebra setting, looks as in Theorem 14 below. Similarly as in the Loos construction, our investigations are based on the following ternary operation in a Hom-Mal’tsev algebra $(A,*,\alpha)$:

$$\{x, y, z\}_\alpha = 2xy * \alpha (z) - yz * \alpha (x) - zx * \alpha (y).$$  \hspace{1cm} (16)

From (16) it clearly follows that $\{,,\}_\alpha$ can also be written as

$$\{x, y, z\}_\alpha = -I_\alpha (x, y, z) + 3xy * \alpha (z).$$  \hspace{1cm} (17)

One observes that when $\alpha = \text{id}$, we recover product (15). First, we prove the following.

**Lemma 13.** Let $(A,*,\alpha)$ be a Hom-Mal’tsev algebra. If one defines on $(A,*,\alpha)$ a ternary operation $\{,,\}_\alpha$ by (16), then

$$\{\alpha (x), \alpha (y), u * v\}_\alpha = \alpha^2 (u) * \{x, y, v\}_\alpha + \{x, u, v\}_\alpha * \alpha^2 (v) - I_\alpha (\alpha (u), \alpha (v), x * y)$$ \hspace{1cm} (18)

for all $u, v, x, y$ in $A$.

**Proof.** Let us write (7) as

$$-I_\alpha (\alpha (x), \alpha (y), u * v) = -I_\alpha (x, y, u) * \alpha^2 (v) + \alpha^2 (u) * (-I_\alpha (x, y, v)) + 3I_\alpha (\alpha (u), \alpha (v), x * y) - I_\alpha (\alpha (u), \alpha (v), x * y).$$ \hspace{1cm} (19)

That is,

$$-I_\alpha (\alpha (x), \alpha (y), u * v) = -I_\alpha (x, y, u) * \alpha^2 (v) + \alpha^2 (u) * (-I_\alpha (x, y, v)) + 3\alpha (u) \alpha (v) * \alpha (x * y) + 3(\alpha (v) * xy)$$ \hspace{1cm} (20)

$$+ \alpha^2 (u) * 3(xy * \alpha (u)) + \alpha^2 (v)$$

$$- I_\alpha (\alpha (u), \alpha (v), x * y).$$

Therefore, by multiplicativity, we have

$$-I_\alpha (\alpha (x), \alpha (y), u * v) + 3\alpha (x) \alpha (y) * \alpha (u * v)$$

$$= (-I_\alpha (x, y, u) + 3\alpha (y) * \alpha (u)) * \alpha^2 (v) + \alpha^2 (u)$$

$$- I_\alpha (\alpha (u), \alpha (v), x * y)$$ \hspace{1cm} (21)

and so, we get (18) by (17).

We now prove the following.

**Theorem 14.** Let $(A,*,\alpha)$ be a multiplicative Hom-Mal’tsev algebra. If one defines on $(A,*,\alpha)$ a ternary operation $\{,,\}_\alpha$ by (16), then $(A,\{,,\}_\alpha,\alpha^2)$ is a multiplicative Hom-Lts.

**Proof.** We must prove the validity of (9), (10), and (11) for operation (16) in the Hom-Mal’tsev algebra $(A,*,\alpha)$.

First observe that the multiplicativity of $(A,*,\alpha)$ implies that $\alpha^2 (x, y, z) = \{\alpha^2 (x), \alpha^2 (y), \alpha^2 (z)\}_\alpha$ with $x, y, z$ in $A$.

From the skew-symmetry of $\{,,\}$ and $I_\alpha (x, y, z)$, it clearly follows from (17) that $\{x, y, z\}_\alpha = -\{y, x, z\}_\alpha$ which is (9) for $\{,,\}_\alpha$.

Next, using (17) and the skew-symmetry of $I_\alpha (x, y, z)$ where applicable, we compute

$$\{x, y, z\}_\alpha + \{y, z, x\}_\alpha + \{z, x, y\}_\alpha$$

$$= -I_\alpha (x, y, z) + 3xy * \alpha (z) + yz * \alpha (x) + zx * \alpha (y)$$

$$+ \alpha (x) - I_\alpha (z, x, y) + 3zx * \alpha (y)$$

$$= -3I_\alpha (x, y, z) + 3I_\alpha (x, y, z) = 0$$ \hspace{1cm} (22)

and thus $\sigma_{x,y,z}[x, y, z]_\alpha = 0$, so we get (10) for $\{,,\}_\alpha$.

Consider now $\{\alpha^2 (x), \alpha^2 (y), \{u, v, w\}_\alpha\}_\alpha$ in $(A,*,\alpha)$. Then

$$\{\alpha^2 (x), \alpha^2 (y), \{u, v, w\}_\alpha\}_\alpha = \{\alpha^2 (x), \alpha^2 (y), 2uv * \alpha (u) - vw * \alpha (u) - uu * \alpha (v)\}_\alpha$$ \hspace{1cm} (by (16))

$$= \{\alpha^2 (x), \alpha^2 (y), 2uv * \alpha (u)\}_\alpha - \{\alpha^2 (x), \alpha^2 (y)\}_\alpha - \{\alpha^2 (x), \alpha^2 (y), uu * \alpha (v)\}_\alpha$$

$$= \{\alpha (x), \alpha (y), 2u * v\}_\alpha * \alpha^3 (w) + \alpha^2 (2u * v)$$
\[\begin{align*}
\{\alpha(x),\alpha(y),\alpha(w)\}_\alpha - J_\alpha(\alpha(2u*v),\alpha^2(w),\alpha^3(u)) - \alpha^2(v*u) \cdot \{\alpha(x),\alpha(y),\alpha(u)\}_\alpha - J_\alpha(\alpha(v\cdot w),\alpha^2(u),\alpha(x*y)) & = 0, \\
\{\alpha(x),\alpha(y),\alpha(w)\}_\alpha - J_\alpha(\alpha(2u*v),\alpha^2(w),\alpha^3(u)) & = 0, \\
2\{x, y, u\}_\alpha & = 2\{x, y, v\}_\alpha - \alpha^3(u) - \alpha^2(w*u) \cdot \alpha(\{x, y, v\}_\alpha + 2\alpha^2(u) \\
\{\alpha(x),\alpha(y),\alpha(w)\}_\alpha - J_\alpha(\alpha(v\cdot w),\alpha^2(u),\alpha(x*y)) & = 0. \\
\{\alpha(x),\alpha(y),\alpha(w)\}_\alpha & = 0.
\end{align*}\]

In this latest expression, denote by \(N(u, v, w, x, y)\) the expression in "\(\cdots\)"; to conclude, we proceed to show that \(N(u, v, w, x, y) = 0\).

Observe first that, by (6), we have

\[\begin{align*}
J_\alpha(\alpha(u),x*y,\alpha(u)) & = \alpha^2(\alpha(u)) \\
+ J_\alpha(\alpha(v),x*y,\alpha(u)) & = \alpha^2(\alpha(u)).
\end{align*}\]

That is,

\[\begin{align*}
J_\alpha(\alpha(u),x*y,\alpha(u)) & = \alpha^2(\alpha(u)) \\
+ J_\alpha(\alpha(v),x*y,\alpha(u)) & = \alpha^2(\alpha(u)).
\end{align*}\]

With this observation, the expression \(N(u, v, w, x, y)\) is transformed as follows:

\[\begin{align*}
N(u, v, w, x, y) & = 2 \left[ -J_\alpha(\alpha(u),\alpha(v),x*y) \\
* \alpha^3(w) - J_\alpha(\alpha(u*v),\alpha^2(w),\alpha(x*y)) \right].
\end{align*}\]
Proof. anyself-morphism 

Therefore, we obtain that (11) holds for "

Remark 15. In the proof of his result, Loos ([6], Satz 1) used essentially the fact that the left translations \(L(x)\) in a Mal'tsev algebra \((A,*)\) are derivations with respect to the ternary operation "\(\cdot\)" defined by (15). Unfortunately, for Hom-Mal'tsev algebras such a tool is still not available at hand.

From [20] (Theorem 2.12) we know that any Mal'tsev algebra \(A\) can be twisted into a Hom-Mal'tsev algebra along any linear self-map of \(A\). Consistent with this result, we recall the following method for constructing Hom-Lts which, in fact, is a result in [21] (see also Propositions 10 and 11 above) but using a Hom-Mal'tsev algebra construction in our proof (as a consequence of Theorem 14).

Proposition 16. Let \((A,\ast)\) be a Mal'tsev algebra and \(\alpha\) any self-morphism of \((A,\ast)\). If one defines on \((A,\ast,\alpha)\) a ternary operation \("\cdot, \cdot\)_{\alpha}\) by

\[
\alpha(x, y, z) = \alpha(x) \ast \alpha(y) \ast \alpha(z),
\]

then \((A,\ast,\alpha,\alpha^2)\) is a multiplicative Hom-Lts.

Proof. One knows ([20], Theorem 2.12) that, from \((A,\ast)\) and any self-morphism \(\alpha\) of \((A,\ast)\), we get a (multiplicative) Hom-Mal'tsev algebra \((A,\ast,\alpha,\alpha)\), where \(x\ast y = \alpha(x \ast y)\) for all \(x, y\) in \(A\). Next, if one defines on \((A,\ast,\alpha,\alpha)\) a ternary operation \(\alpha(x, y, z) = 2(\alpha(x, y, z) - \alpha(x) \ast \alpha(y) \ast \alpha(z))\)

then, by Theorem 14, \((A,\ast,\alpha,\alpha^2)\) is a Hom-Lts and "\(\cdot, \cdot\)_{\alpha}\) is expressed through "\(\ast\)" as

\[
\alpha(x, y, z) = 2\alpha(x) \ast \alpha(y) \ast \alpha(z)
\]

Observe that though constructed in quite a different way, the operation "\(\cdot, \cdot\)_{\alpha}\) in Proposition 16 above coincides with "\(\cdot, \cdot\)_{\alpha}\) in Proposition 11 for \(n = 2\).

Combining Lemma 13 and Theorem 14, we get the following result.

Theorem 17. Let \((A,\ast,\alpha)\) be a multiplicative Hom-Mal'tsev algebra. If one defines on \((A,\ast,\alpha)\) a ternary operation \("\cdot, \cdot\)_{\alpha}\) by

\[
\alpha(x, y, z) = \alpha(x) \ast \alpha(y) \ast \alpha(z),
\]

where "\(\cdot, \cdot\)_{\alpha}\) is defined by (17), then \((A,\ast,\cdot, \cdot\)_{\alpha}\) is a Hom-Bol algebra.

Proof. Definition (30) and Theorem 14 imply that \((A,\ast,\cdot, \cdot)_{\alpha}\), \(\alpha^2\) is a multiplicative Hom-Lts; that is, (HB4), (HB5), and (HB7) hold for \((A,\ast,\cdot, \cdot\)_{\alpha}\). Now, (HB1), (HB2), and (HB3) are, respectively, the multiplicativity and skew-symmetry of "\(\ast\)"; next, we are done if we prove (HB6) for \((A,\ast,\cdot, \cdot\)_{\alpha}\), \(\alpha\).

From (17) and multiplicativity we have

\[
\alpha(u, v) = \alpha(u) \ast \alpha(v) \ast \alpha(u \ast v)
\]

and then (18) takes the form

\[
\alpha(x, y, z) = \alpha(x) \ast \alpha(y) \ast \alpha(z)
\]

Multiplying by \(1/3\) each member of this latter equality and using (30), we get

\[
\alpha(x, y, z) = \alpha(x) \ast \alpha(y) \ast \alpha(z)
\]
which is (HB6) for \((A,\ast,(\cdot)_\alpha,\alpha)\). So \((A,\ast,(\cdot)_\alpha,\alpha)\) is a Hom-Bol algebra.

\[\square\]

**Example 18.** Let \(A\) be a vector space with basis \(\{e_1,e_2,e_3,e_4\}\). From [20] (Example 2.13) we know that if one considers the linear map \(\alpha : A \to A\) given by

\[
\alpha(e_1) = e_1 + e_3; \\
\alpha(e_2) = 2e_2 + 2e_4; \\
\alpha(e_3) = -e_3; \\
\alpha(e_4) = -2e_4
\]

and the multiplication table given by

\[
e_1 \ast e_2 = -2e_2 - 2e_4 = (-e_2 \ast e_1); \\
e_1 \ast e_3 = e_3 = (-e_3 \ast e_1); \\
e_1 \ast e_4 = -2e_4 = (-e_4 \ast e_1); \\
e_2 \ast e_3 = -4e_4 = (-e_3 \ast e_2)
\]

(only nonzero products are specified), then \((A,\ast,\alpha)\) is a multiplicative Hom-Maltsev algebra. It is observed that \((A,\ast,\alpha)\) is not a Hom-Lie algebra nor a Maltsev algebra.

Now, by (17) and (30), one checks that the only nonzero ternary products \((x,y,z)\) on \(A\) with respect to the basis elements are

\[
(e_1,e_2,e_1)_\alpha = -4e_2 = (-e_2,e_1,e_1)_\alpha; \\
(e_1,e_3,e_1)_\alpha = -e_3 = (-e_3,e_1,e_1)_\alpha; \\
(e_1,e_4,e_1)_\alpha = -4e_4 = (-e_4,e_1,e_1)_\alpha.
\]

By Theorem 17 we get that \((A,\ast,(\cdot)_\alpha,\alpha)\) is a Hom-Bol algebra.

Since any Hom-algebraic is Hom-Maltsev admissible ([20], Theorem 3.8), from Theorem 17 we have the following.

**Corollary 19.** Let \((A,\ast,\alpha)\) be a multiplicative Hom-alternative algebra. Then \((A,\cdot,\cdot,\ast,\alpha)\) is a Hom-Bol algebra, where \((x,y,z)\) is Hom-Bol algebra, where \((x,y,z)\) is defined by

\[
three{x,y,z}_\alpha := -(1/3)(2[[x,y],\alpha(z)] - [[y,z],\alpha(x)] - [[z,x],\alpha(y)]),
\]

for all \(x,y,z \in A\).

The aim of Section 4 is a generalization of Corollary 19 to multiplicative right (or left) Hom-alternative algebras.

Various constructions of Hom-Lts are offered in [21] starting from either Hom-associative algebras, Hom-Lie algebras, Hom-Jordan triple systems, ternary totally Hom-associative algebras, Maltsev algebras, or alternative algebras. In practice, it is easier to construct Hom-Lts or Hom-Bol algebras from well-known (binary) algebras such as alternative algebras or Maltsev algebras. From this point of view, our construction results (Theorem 14, Proposition 16, and Theorem 17) have rather a theoretical feature (the extension to Hom-algebra setting of Loos’ result [6] and a result by Mikheev [25]) than a practical method for constructing Hom-Lts or Hom-Bol algebras. However, it could be of some interest to get a Hom-Lts or a Hom-Bol algebra from a given Hom-Maltsev algebra without resorting to the corresponding Maltsev algebra.

### 4. Hom-Lts and Hom-Bol Algebras from Right (or Left) Hom-Alternative Algebras

In this section we prove that every multiplicative right (or left) Hom-associative algebra has a natural Hom-Bol algebra structure (and, subsequently, a natural Hom-Lts structure). This is the Hom-analogue of a result by Mikheev [31] and by Hentzel and Peresi [29] although with a different scheme of proof.

First we recall some few basic properties of right Hom-alternative algebras which would be useful in the sequel.

**Lemma 20** (see [18]). If \((A,\ast,\alpha)\) is a Hom-algebra, then the following statements are equivalent.

(i) \((A,\ast,\alpha)\) is right Hom-associative.

(ii) \((A,\ast,\alpha)\) satisfies

\[
as(x,y,z) = -as(x,z,y)
\]

for all \(x,y,z \in A\).

(iii) \((A,\ast,\alpha)\) satisfies

\[
\alpha(x) \ast (yz + yz) = xy \ast \alpha(z) + xz \ast \alpha(y)
\]

for all \(x,y,z \in A\).

Observe that if \((A,\ast,\alpha)\) is a right Hom-associative algebra, then \((A,\ast_{\text{op}},\alpha)\) is a left Hom-associative algebra, where \(x_{\text{op}} y := y \ast x\). So the mirrors of (37) and (38) hold for \((A,\ast_{\text{op}},\alpha)\):

\[
as(x,y,z) = -as(x,y,z),
\]

\[
((x_{\text{op}} y) + (y_{\text{op}} x))_{\text{op}} \alpha(z) = \alpha(x)_{\text{op}} (y_{\text{op}} z) + \alpha(y)_{\text{op}} (x_{\text{op}} z).
\]

Now we have the following.

**Lemma 21.** In any multiplicative right Hom-alternative algebra \((A,\ast,\alpha)\), the identity

\[
as([u,v],\alpha(x),\alpha(y)) = [as((u,x,y),\alpha^2(v)) + [\alpha^2(u),as(v,x,y)]]
\]

holds for all \(x,y,z \in A\).
Proof. The identity
\[
\text{as}(uv, \alpha(x), \alpha(y)) = \text{as}(u, x, y)\alpha^2(v) + \alpha^2(u)\text{as}(v, x, y)
\]
holds in any right Hom-alternative algebra (see [23], Theorem 7.1(7.1.1c)). Next, in this identity, switching \(u\) and \(v\), we have
\[
\text{as}(v, x, y)\alpha^2(u) + \alpha^2(v)\text{as}(u, x, y) - \text{as}(\alpha(u), \alpha(v), [x, y]).
\]
(43)

Then, subtracting memberwise this latter equality from the one above and using the linearity of \(\text{as}\), we get (41).

Note that in the case when \((A, \cdot, \alpha)\) is a left Hom-alternative algebra, identity (41) reads as
\[
\text{as}(\alpha(x), \alpha(y), [u, v]) = [\text{as}(x, y, u), \alpha^2(v)] - [\text{as}(\alpha(v), \alpha(u), [x, y]),
\]
(44)

and so we get (46).

(ii) Proceeding as above, but using (40) and then (39), one gets (47).

We are now in a position to prove the main result of this section.

Theorem 23. Let \((A, \cdot, \alpha)\) be a multiplicative right (resp., left) Hom-alternative algebra. If one defines on \(A\) a ternary operation "(,,)" by (46) (resp., (47)), then \((A,(,,),\alpha^2)\) is a Hom-Lts and \((A,[[,],(,,),\alpha)\) is a Hom-Bol algebra.

Proof. We prove the theorem for a multiplicative right Hom-alternative algebra \((A, \cdot, \alpha)\) (the proof of the left case is the mirror of the right one).

Identities (HB1) and (HB2) follow from the multiplicity of \((A, \cdot, \alpha)\). Identities (HB3) and (HB4) are obvious from the definition of "[,,]\) and "(,,)\; identity (HB5) follows from Proposition 9.

In [22] Yau showed that if, on a multiplicative Hom-Jordan algebra \((A,\circ,\alpha)\) (the proof of the left case is the mirror of the right one),

\[
\text{as}(y, z, x) = 2\text{as}(x, y, z)
\]

then \((A,[[,],(,,),\alpha)\) is a Hom-Lts (see [22], Corollary 4.1). Now, observe that \([x, y, z] = 2\text{as}(x, y, z)\); that is, \([x, y, z] = 2(x, y, z)\). Therefore, since every multiplicative right Hom-alternative algebra is Hom-Jordan admissible
(see [23], Theorem 4.3), we conclude that \((A, (\cdot), \alpha^2)\) is a multiplicative Hom-\(\Lambda\)s and so identity (HB7) holds for \((A, [\cdot], (\cdot), \alpha)\).

Next, \((A, [\cdot], (\cdot), \alpha)\) is a Hom-Bol algebra if we prove that (HB6) additionally holds.

Write (46) as
\[
-2 as (z, x, y) = (x, y, z) - [[x, y], \alpha(z)].
\]

Multiplying each member of (41) by \(-2\) and next using (50), we get
\[
(\alpha(x), \alpha(y), [u, v]) - [[\alpha(x), \alpha(y)], \alpha([u, v])]
\]
\[
= ([x, y, u] - [[x, y], \alpha(u)], \alpha^2(v)]
\]
\[
+ [\alpha^2(u), (x, y, v)] - [[x, y], \alpha(v)]
\]
\[
+ (\alpha(u), [x, y], \alpha(v))
\]
\[
- ([\alpha(u), [x, y]], \alpha^2(v))
\]
\[
- (\alpha(v), [x, y], \alpha(u))
\]
\[
+ [\alpha(v), [x, y]], \alpha^2(u)].
\]

That is,
\[
(\alpha(x), \alpha(y), [u, v]) = ([x, y, u], \alpha^2(v)]
\]
\[
+ [\alpha^2(u), (x, y, v)]
\]
\[
- ([x, y], \alpha(u), \alpha(v))
\]
\[
+ (\alpha(v), [x, y], \alpha(u))
\]
\[
+ \alpha([[x, y], [u, v]]).
\]

Observe that
\[
- ([x, y], \alpha(u), \alpha(v)) + ([x, y], \alpha(v), \alpha(u))
\]
\[
= (\alpha(u), [x, y], \alpha(v)) + ([x, y], \alpha(v), \alpha(u))
\]
\[
= - (\alpha(v), \alpha(u), [x, y]).
\]

Therefore, (52) now reads
\[
(\alpha(x), \alpha(y), [u, v]) = ([x, y, u], \alpha^2(v)]
\]
\[
+ [\alpha^2(u), (x, y, v)]
\]
\[
+ (\alpha(u), \alpha(v), [x, y])
\]
\[
- \alpha([[u, v], [x, y]]).
\]

and so (HB6) holds for \((A, [\cdot], (\cdot), \alpha)\). Thus we conclude that \((A, [\cdot], (\cdot), \alpha)\) is a Hom-Bol algebra. One gets the same result in the case when \((A, *, \alpha)\) is a multiplicative left Hom-alternative algebra and essentially using (47) and (44). This finishes the proof.

---

**Example 24.** Let \(A\) be a five-dimensional vector space with basis \(\{e, u, v, w, z\}\) and let \(\alpha : A \to A\) be a linear map given by
\[
\alpha(e) = e + u + v;
\]
\[
\alpha(u) = -u;
\]
\[
\alpha(v) = -v;
\]
\[
\alpha(w) = -w;
\]
\[
\alpha(z) = -z.
\]

Define on \(A\) a binary operation “\(*\)” by
\[
e * e = e + u + v;
\]
\[
e * u = -v;
\]
\[
e * w = -w + z;
\]
\[
e * z = -z;
\]
\[
u * e = -u;
\]
\[
z * e = -z
\]

(again, only nonzero products are specified). Then \((A, *, \alpha)\) is a multiplicative right Hom-alternative algebra (see [23], Example 2.9). Then, using \([x, y] = x * y - y * x\) and (46), one could find (although the computation is somewhat lengthy) all the nonzero products “\([\cdot, \cdot]\)” and “\((\cdot, \cdot)\)” with respect to the basis elements \(e, u, v, w, z\) of \(A\). We just point out that they are nonzero products; for example, \([e, u] = u - v, [e, w] = -w + z, [e, u, e] = -u - v,\) and \([e, w, e] = -w - z.\) Therefore, Theorem 23 implies that \((A, [\cdot], (\cdot), \alpha)\) is a Hom-Bol algebra.

---

**5. The Construction of Bol Algebras from Right Alternative Algebras Revisited**

As already mentioned in Section 2, for \(\alpha = \text{id}\) in Definition 12 we get the definition of a left Bol algebra.

**Definition 25** (see [25, 27]). A left Bol algebra \((A, [\cdot], (\cdot))\) is a triple \((A, [\cdot], (\cdot))\) in which \(A\) is a vector space, “\([\cdot, \cdot]\)” a binary operation, and “\((\cdot, \cdot)\)” a ternary operation on \(A\) such that

\begin{enumerate}
  \item [(B1)] \([x, y] = -[y, x],\)
  \item [(B2)] \((x, y, z) = -(y, x, z),\)
  \item [(B3)] \(\sigma_{x,y,z}(x, y, z) = 0,\)
  \item [(B4)] \([x, y, [u, v]] = ([x, y, u], v) + [u, (x, y, v)] + (u, v, [x, y]) - [[u, v], [x, y]],\)
  \item [(B5)] \((x, y, (u, v, w)) = ((x, y, u), v, w) + (u, v, (x, y, w)) + (u, v, (x, y, w)),\)
\end{enumerate}

for all \(u, v, w, x, y, z \in A.\)

In this section we show how the construction of Hom-Bol algebras from right or left Hom-alternative algebras described in Section 4 can be specified to the ordinary untwisted case of construction of (left) Bol algebras from right or left alternative algebras ([29, 31]). In fact, for \(\alpha = \text{id}\) in Theorem 23 and specifying the right alternative case, we get the following.
Theorem 26. Let \((A, \ast)\) be a right alternative algebra. If one defines on \(A\) a ternary operation \(\langle \cdot, \cdot \rangle\) by
\[
(x, y, z) = \langle [x, y], z \rangle - 2\text{as}(z, x, y),
\]
where \(\text{as}(u, v, w) = uv \ast w - u \ast (v \ast w)\), then \((A, \langle \cdot, \cdot \rangle)\) is a left Bol algebra.

Proof. Identities (B1) and (B2) are obvious. For \(\alpha = \text{id}\), the Hom-Jordan associator (see Definition 3) reduces to the usual Jordan associator \(\text{as}(u, v, w) = (u \ast v) \ast w - (u \ast (v \ast w))\) in \((A, \ast)\). The fact that (B3) and (B5) hold in \((A, \ast)\) follows from the equality \(\text{as}(x, y, z) = \text{as}^l(y, z, x)\) that holds in right alternative algebras (the untwisted form of (46)) and from that right alternative algebras are Jordan admissible [36].

Therefore \((A, \text{as}(\cdot, \cdot, \cdot))\) is a left Bol algebra.

Remark 27. (i) The process of constructing left Bol algebras from right alternative algebras described in Theorem 26 above is different from the ones given in [29, 31]. In our approach here, we rely essentially on fundamental properties of right alternative algebras (see, e.g., [36, 37]) without subsidiary constructions.

(ii) If \((A, \ast)\) is a left alternative algebra, it is also possible to get a natural left Bol algebra structure on \((A, \ast)\). Indeed, one needs to consider the counterparts of \((x, y, z)\) and (63) that looks, respectively, as
\[
(x, y, z) = \langle [x, y], z \rangle - \langle x, z, y \rangle,
\]
(the untwisted version of (47)) and
\[
\text{as}(x, y, [u, v]) = \text{as}(x, y, u) \ast v + [u, \text{as}(x, y, v)]
\]
\[
\text{as}(x, y, u) \ast v + [u, \text{as}(x, y, v)]
\]
\[
\text{as}(x, y, u) \ast v + [u, \text{as}(x, y, v)]
\]
Next one proceeds as in Theorem 26 observing that a left alternative algebra is also Jordan-admissible (see [36], Theorem 2, for right alternative algebras).

Conflicts of Interest

The authors declare that they have no conflicts of interests.

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