Research Article

The Projective Character Tables of a Solvable Group $2^6 : (6 \times 2)$

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The Chevalley–Dickson simple group $G_2(4)$ of Lie type G_2 over the Galois field $\text{GF}(4)$ and of order $251596800 = 2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$ has a class of maximal subgroups of the form $2^{4+6} : (A_5 \times 3)$, where 2^{4+6} is a special 2-group with center $Z(2^{4+6}) = 2^4$. Since 2^4 is normal in $2^{4+6} : (A_5 \times 3)$, the group $2^{4+6} : (A_5 \times 3)$ can be constructed as a nonsplit extension group of the form $\overline{G} = 2^{4+} (2^6 : (A_5 \times 3))$. Two inertia factor groups, $H_1 = 2^6 : (A_5 \times 3)$ and $H_2 = 2^6 : (6 \times 2)$, are obtained if $\overline{G}$ acts on 2^4 . In this paper, the author presents a method to compute all projective character tables of H_2 . These tables become very useful if one wants to construct the ordinary character table of $\overline{G}$ by means of Fischer–Clifford theory. The method presented here is very effective to compute the irreducible projective character tables of a finite soluble group of manageable size.

1. Introduction

The Chevalley–Dickson simple group $G_2(4)$ of Lie type G_2 over the Galois field $\text{GF}(4)$ and of order $251596800 = 2^{12} \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 13$ has exactly eight conjugacy classes of maximal subgroups [1]. One of these classes of maximal subgroups of index 1365 in $G_2(4)$ is a 2-local subgroup of the form $2^{4+6} : (A_5 \times 3)$, where 2^{4+6} is a special 2-group with center $Z(2^{4+6}) = 2^4$. Also, it can be mentioned that $2^{4+6} : (A_5 \times 3)$ is a parabolic subgroup of $G_2(4)$.

The group $2^{4+6} : (A_5 \times 3)$ is the normalizer $N_{G_2(4)}((2A)^4)$ [1] of an elementary abelian 2-group $(2A)^4$ of order 16 in $G_2(4)$, where the generators of $(2A)^4$ are 4 commuting involutions found in the class of involutions $2A$ of $G_2(4)$. Hence, we can construct $2^{4+6} : (A_5 \times 3)$ as an extension group of $N = 2^4$ by $G = 2^6 : (A_5 \times 3)$. Using a permutation representation of degree 416 of $2^{4+6} : (A_5 \times 3)$ found in [2], we can easily verify with the help of the computer algebra system GAP [3] that $2^{4+6} : (A_5 \times 3)$ exists as a nonsplit extension group $\overline{G} = 2^{4+} (2^6 : (A_5 \times 3))$. In fact, from the character table of $2^{4+6} : (A_5 \times 3)$ uploaded in the GAP library, we can deduce that 2^6 is also normal in

$2^{4+6} : (A_5 \times 3)$. Therefore, with the aid of GAP, it can be easily shown that the group $2^{4+6} : (A_5 \times 3)$ is also isomorphic to a nonsplit extension of the form $2^{6+} (2^4 : (A_5 \times 3))$.

Since $N_{G_2(4)}((2A)^4) \cong \overline{G}$, the action of $G = 2^6 : (A_5 \times 3)$ on $N = 2^4$ splits N into two orbits of lengths 1 and 15. By Brauer's theorem in [4], G also acts on $\text{Irr}(N)$ with two orbits of lengths 1 and 15 and the corresponding inertia factor subgroups are of the forms $H_1 = G$ and $H_2 = 2^6 : (6 \times 2)$. If one wants to construct the ordinary character table of $\overline{G}$ by the technique of Fischer–Clifford matrices [5], the irreducible ordinary characters $\text{Irr}(H_1)$ of H_1 and either the ordinary irreducible characters $\text{Irr}(H_2)$ or a set of irreducible projective characters $\text{IrrProj}(H_2, \alpha^{-1})$ of H_2 with associated nontrivial factor set α^{-1} are needed. Readers are referred to [6] on a survey of Fischer–Clifford theory.

From the uploaded character table in the GAP library, we found that $\overline{G}$ has 42 conjugacy classes. Hence, by Fischer–Clifford theory, $|\text{Irr}(\overline{G})| = |\text{Irr}(G)| + |\text{IrrProj}(H_2, \alpha^{-1})| = 42$. Therefore, $|\text{IrrProj}(H_2, \alpha^{-1})| = 16$, where $|\text{Irr}(G)| = 26$. Since $|\text{Irr}(H_2)| = 30$, certainly a set of

irreducible projective characters with nontrivial factor set α^{-1} of H_2 is required in the construction of $\text{Irr}(\overline{G})$ via Fischer-Clifford theory.

Using computations in GAP, the Schur multiplier $M(H_2) \cong 2^4$ of H_2 is identified as a group isomorphic to the elementary abelian 2-group 2^4 . Therefore, the group H_2 can have up to 15 sets of distinct irreducible projective characters $\text{IrrProj}(H_2, \alpha_i^{-1})$, $i = 2, 3, 4, \dots, 16$, such that $\alpha^2 \sim 1$. In this paper, the author presents a method (based on the work done in [7, 8]) to compute all the distinct irreducible projective character tables of H_2 . The author will also show how to choose the appropriate set $\text{IrrProj}(H_2, \alpha^{-1})$ for H_2 if one wants to construct the set $\text{Irr}(\overline{G})$ using the technique of Fischer-Clifford matrices.

A proof is given of a result, which states that the number of irreducible projective characters $|\text{IrrProj}(G, \alpha^{-1})|$ of a finite group G associated with some factor set α^{-1} is always less or equal to the number of the ordinary irreducible characters $|\text{Irr}(G)|$ of G . Based on the above-mentioned proof, GAP codes are developed to find the number $|\text{IrrProj}(H_2, \alpha_i^{-1})|$ of irreducible projective characters of H_2 associated with each factor set α_i^{-1} in a cohomology class $[\alpha_i]$ of $M(H_2)$. Interested readers are referred to [9–13] for definitions on concepts in ordinary and projective character theory. Computations are carried out with the aid of the computer algebra systems MAGMA [14] and GAP [3], and the notation of ATLAS [1] is mostly used.

2. Preliminary Results on Projective Characters

In this section, a brief overview of relevant projective character theory pertaining to our study is given. G will always denote a finite group. Also, a proof of a proposition is given which states that the number of irreducible projective characters of a finite group G associated with some factor set α is always less or equal to the number of the ordinary irreducible characters of G .

Definition 1. A function $\alpha: G \times G \longrightarrow \mathbb{C}^*$ is called a factor set of G if $\alpha(xy, z)\alpha(x, y) = \alpha(x, yz)\alpha(y, z)$ for all $x, y, z \in G$.

The set of all equivalence classes of factor sets of G forms a finite abelian group, called the Schur multiplier, and is denoted by $M(G)$.

Definition 2. A projective representation of a group G of degree n over the complex numbers is a map $P: G \longrightarrow \text{GL}(n, \mathbb{C})$, such that

- (i) $P(1) = I_n$
- (ii) Given $x, y \in G$, there exists $\alpha(x, y) \in \mathbb{C}^*$ such that $P(x)P(y) = \alpha(x, y)P(xy)$

The map α is called the factor set associated with P .

Let P be a projective representation of G with factor set α . Define $\kappa(g) = \text{Trace}(P(g))$ for all $g \in G$. Then, κ is called a projective character of G . We say that κ is irreducible if P is, and κ has a factor set α , where α is the factor set of P .

Let $\text{IrrProj}(G, \alpha)$ denote the set of irreducible projective characters of G associated with the factor set α . An element $x \in G$ is said to be α -regular if $\alpha(x, g) = \alpha(g, x)$ for all $g \in C_G(x)$. It is well known that $g \in G$ is α -regular if and only if $\kappa(g) \neq 0$ for some $\kappa \in \text{IrrProj}(G, \alpha)$ or equivalently that g is α -irregular if and only if $\kappa(g) = 0$ for all $\kappa \in \text{IrrProj}(G, \alpha)$. The number of irreducible projective characters with factor set α equals the number of α -regular classes of a group G . Projective characters also satisfy the usual orthogonality relations and have analogues to ordinary characters.

As we will see later in Section 4, Definition 3 and Remark 1 will play an important role in the computation of the irreducible projective characters of H_2 .

Definition 3. A group R is a representation group for G if there exists a homomorphism π from R onto G such that (i) $A = \ker(\pi) \cong M(G)$ and (ii) $A \leq Z(R) \cap R'$.

Remark 1. A covering group C for G will normally be a quotient of R by a subgroup B of A . If A/B has order n , we sometimes refer to the covering group as a n -fold cover of G . Projective representations of G are found in the representation group R for all the equivalence classes of factor sets in $M(G)$. However, in an n -fold cover C of G , only the n equivalence classes which C covers will be represented [7].

The following proposition (see [15, 16]) is useful to determine the number of irreducible projective characters of a group G associated with a certain factor set α . It tells us also under which condition $|\text{IrrProj}(G, \alpha)|$ is strictly less than $|\text{Irr}(G)|$. In Section 4, GAP codes (based on Proposition 1) will be used to compute the number $|\text{IrrProj}(G, \alpha_i^{-1})|$ of all irreducible projective characters of H_2 found in a set $\text{IrrProj}(G, \alpha_i^{-1})$.

Proposition 1. Let $R = M(G)$. G be a representation group of a finite group G , where $M(G)$ denotes the Schur multiplier of G . Then, the number of irreducible characters $\text{Irr}(R)$ of R which lie over a linear character $\theta \in \text{Irr}(M(G))$ is less or equal to $|\text{Irr}(G)|$.

Proof. The number of irreducible characters $\text{Irr}(R)$ of R which lies over a linear character $\theta \in \text{Irr}(M(G))$ is given by $\sum_{\chi \in \text{Irr}(R)} \langle \chi \downarrow_{M(G)}, \theta \rangle / \chi(1)$. It is known that the quantity $\sum_{\chi \in \text{Irr}(R)} \chi(x) / \chi(1) \geq 0$ for each $x \in M(G)$, and it is nonzero if x is a commutator in R . For any $\theta \in \text{Irr}(M(G))$, we have

$$\sum_{x \in M(G)} \sum_{\chi \in \text{Irr}(R)} \frac{\chi(x)\theta(x^{-1})}{\chi(1)} \leq \sum_{x \in M(G)} \sum_{\chi \in \text{Irr}(R)} \frac{\chi(x)}{\chi(1)} \quad (1)$$

$$= |M(G)| |[g]_{R/M(G)}|,$$

where $|[g]_{R/M(G)}|$ is the number of conjugacy classes of $R/M(G) \cong G$. The last equality follows because the irreducible characters of R with $M(G)$ in their kernels are precisely those which contain the trivial character on the restriction to $M(G)$. Hence, $(1/|M(G)|) \sum_{x \in M(G)} \sum_{\chi \in \text{Irr}(R)} \chi(x)\theta(x^{-1}) / \chi(1) = \sum_{\chi \in \text{Irr}(R)} (1/|M(G)|) \sum_{x \in M(G)} \chi(x)\theta(x^{-1}) / \chi(1)$

TABLE 1: Action of G on N and $\text{Irr}(N)$.

	Action of G on N	Action of G on $\text{Irr}(N)$
Number and size of orbits Q_i	$ O_1 = 1$ $ O_2 = 15$	$ O_1 = 1$ $ O_2 = 15$
Structure of stabilizers P_i and H_i	$P_1 = 2^6 : (A_5 \times 3)$ $P_2 = 2^6 : (6 \times 2)$	$H_1 = 2^6 : (A_5 \times 3)$ $H_2 = 2^6 : (6 \times 2)$
Size of stabilizers P_i and H_i	$ P_1 = 11520$ $ P_2 = 768$	$ H_1 = 11520$ $ H_2 = 768$
Number of conjugacy classes $[g]$ of P_i and H_i	$ [g]_{P_1} = 26$ $ [g]_{P_2} = 30$	$ [g]_{H_1} = 26$ $ [g]_{H_2} = 30$

$\chi(1) = \sum_{\chi \in \text{Irr}(R)} \langle \chi \downarrow_{M(G)}, \theta \rangle / \chi(1) \leq |[g]_{R/M(G)}| = |\text{Irr}(G)|$. Furthermore, if there is a nonidentity element $x \in M(G) \setminus \ker(\theta)$ which is a commutator in R , then the inequality becomes strict. $\square$

3. Action of $2^6 : (A_5 \times 3)$ on 2^4 and $\text{Irr}(2^4)$

As explained in the introductory section, the split extension $2^{4+6} : (A_5 \times 3)$ can be constructed as a nonsplit extension group $\bar{G} = 2^4(2^6 : (A_5 \times 3))$ of an elementary abelian group $N = 2^4$ by $G = 2^6 : (A_5 \times 3)$. Since $\bar{G}$ is the normalizer of N in $G_2(4)$, the action of $\bar{G}$ on N gives rise to two orbits of lengths 1 and 15. It follows that the action of G on N will also result in two orbits of lengths 1 and 15 with corresponding point stabilizers $P_1 = G$ and $P_2 = 2^6 : (6 \times 2)$. Then, by Brauer's theorem [4], G also has two orbits of lengths 1 and 15 on $\text{Irr}(N)$ with corresponding inertia factor subgroups $H_1 = G$ and $H_2 = 2^6 : (6 \times 2)$. See Table 1 for a summary of the action (which is self-dual) of $2^6 : (A_5 \times 3)$ on 2^4 and $\text{Irr}(2^4)$, respectively.

4. Projective Character Tables of Inertia Factor H_2

In this section, all the sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ of irreducible projective characters of H_2 with associated factor sets α_i^{-1} will be computed. Appropriate GAP codes will be used to assist in the computational aspects of determining the sets $\text{IrrProj}(H_2, \alpha_i^{-1})$. Readers are referred to [7, 8] for a background on the computational techniques being used in this section.

Since the Schur multiplier of $M(H_2) \cong 2^4$ is isomorphic to the elementary abelian group 2^4 of order 16, we obtain that $M(H_2)$ contains 15 cohomology classes $[\alpha_i]$ of order 2 and the trivial class [1]. Hence, there exist 15 sets of projective characters $\text{IrrProj}(H_2, \alpha_i^{-1})$ lying above the nontrivial factor sets $\alpha_i^{-1} \in [\alpha_i]$, $i = 2, 3, 4, \dots, 16$, such that $\alpha_i^2 \sim 1$. Note that the order $|H_2| = 2^8 \cdot 3$ of H_2 is the product of powers of two primes, and hence, by the Burnside's $p^a q^b$ theorem [17], the group H_2 is solvable. Since H_2 is of relative small order and it is solvable, the following GAP codes [18] (based on Proposition 1) give the number of irreducible projective characters of H_2 contained in each set $\text{IrrProj}(H_2, \alpha_i^{-1})$:

```
gap > h := H2
gap > f := EpimorphismSchurCover(h)
```

```
gap > f := InverseGeneralMapping(Isomorphism
    PGroup(Source(f))) * f
gap > z := Kernel(f)
gap > x := Source(f)
gap > List(Irr(z), lambda -> Number(Irr(x), chi -> not
    IsZero(ScalarProduct(RestrictedClassFunction(chi, z),
    lambda))))
[30, 18, 26, 14, 16, 4, 16, 4, 16, 4, 16, 4, 16, 4, 16, 4]
```

From the output of the above GAP codes, the number of irreducible projective characters $|\text{IrrProj}(H_2, \alpha_i^{-1})|$ of H_2 contained in each set $\text{IrrProj}(H_2, \alpha_i^{-1})$ is 30, 26, 18, 16, 16, 16, 16, 16, 14, 4, 4, 4, 4, 4, 4, and 4, respectively. The set containing the 30 projective characters is associated with the trivial factor set $\alpha_1^{-1} = 1$ of $M(G)$, and hence, they are the ordinary irreducible characters $\text{Irr}(H_2)$ of H_2 . Note in the above GAP codes that “Source(f)” is the full representation group $R = M(G).H_2$ of H_2 whereas “Kernel(f)” denotes the Schur multiplier $M(G)$.

From Remark 1, the irreducible projective characters of H_2 with a given factor set α_i^{-1} can be computed without using the full representation group $R = M(H_2).H_2$ of H_2 . Since all the nontrivial factor sets α_i^{-1} of $\text{IrrProj}(H_2, \alpha_i^{-1})$ are of order two, the aim is to find 15 double covers C_i of H_2 which contain the desired 15 sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ of projective characters of H_2 . The nontrivial maximal subgroups M_i of $M(H_2)$ are computed within GAP, and we found there are exactly 15 of them of order 8 and shape 2^3 . The 15 factor groups $C_i = R/M_i$, $i = 2, 3, 4, \dots, 16$, are the double covers $2_i \cdot H_2$ which contain the 15 sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ of desired irreducible projective characters of H_2 with factor sets α_i^{-1} such that $\alpha_i^2 \sim 1$. The following GAP code was used to “restrict” R to H_2 to obtain the sets $\text{IrrProj}(H_2, \alpha_i^{-1})$, $i = 2, 3, 4, \dots, 16$:

```
gap > t := CharacterTable("H2")
gap > 2t := CharacterTable("C_i")
gap > F := GetFusionMap(2t, t)
gap > map := ProjectionMap(F)
gap > projchars := List(2t, x -> {xmap})
```

From the sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ which contain 16 irreducible projective characters each, we obtain exactly 3 distinct irreducible projective character tables. Also, the sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ which contain 4 irreducible projective characters each, give rise to 3 distinct sets of irreducible

TABLE 2: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	4	0	4	0	0	0	0	0	0	0	1	1	B	B	0	0	D	D	4	0	0	0	1	1	B	B	0	0	D	D
χ_2	4	0	4	0	0	0	0	0	0	0	1	1	-B	-B	0	0	-D	-D	4	0	0	0	1	1	-B	-B	0	0	-D	-D
χ_3	4	0	4	0	0	0	0	0	0	0	A	A	B	B	0	0	E	-E	4	0	0	0	A	A	B	B	0	0	E	-E
χ_4	4	0	4	0	0	0	0	0	0	0	A	A	B	B	0	0	-E	E	4	0	0	0	A	A	B	B	0	0	-E	E
χ_5	4	0	4	0	0	0	0	0	0	0	A	A	-B	-B	0	0	-E	E	4	0	0	0	A	A	-B	-B	0	0	-E	E
χ_6	4	0	4	0	0	0	0	0	0	0	A	A	-B	-B	0	0	E	-E	4	0	0	0	A	A	-B	-B	0	0	E	-E
χ_7	4	0	4	0	0	0	0	0	0	0	1	1	-B	-B	0	0	-D	-D	-4	0	0	0	-1	-1	B	B	0	0	D	D
χ_8	4	0	4	0	0	0	0	0	0	0	1	1	B	B	0	0	D	D	-4	0	0	0	-1	-1	-B	-B	0	0	-D	-D
χ_9	4	0	4	0	0	0	0	0	0	0	A	A	-B	-B	0	0	-E	E	-4	0	0	0	-A	-A	B	B	0	0	E	-E
χ_{10}	4	0	4	0	0	0	0	0	0	0	A	A	-B	-B	0	0	E	-E	-4	0	0	0	-A	-A	B	B	0	0	-E	E
χ_{11}	4	0	4	0	0	0	0	0	0	0	A	A	B	B	0	0	E	-E	-4	0	0	0	-A	-A	-B	-B	0	0	-E	E
χ_{12}	4	0	4	0	0	0	0	0	0	0	A	A	B	B	0	0	-E	E	-4	0	0	0	-A	-A	-B	-B	0	0	-E	E
χ_{13}	12	0	-4	0	0	0	0	0	0	0	0	0	C	-B	0	0	0	0	0	-4	0	0	0	0	-C	B	0	0	0	0
χ_{14}	12	0	-4	0	0	0	0	0	0	0	0	0	-C	B	0	0	0	0	0	-4	0	0	0	0	C	-B	0	0	0	0
χ_{15}	12	0	-4	0	0	0	0	0	0	0	0	0	C	-B	0	0	0	0	0	4	0	0	0	0	C	-B	0	0	0	0
χ_{16}	12	0	-4	0	0	0	0	0	0	0	0	0	-C	B	0	0	0	0	0	4	0	0	0	0	-C	B	0	0	0	0

Note. $A = (-1 - \sqrt{3}i)/2$; $B = -2i$; $C = -6i$; $D = i$; $E = E(12)^{11}$.

TABLE 3: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	4	4	0	0	0	0	0	0	0	0	1	1	2	2	0	0	-1	-1	-2	-2	0	0	1	1	-4	0	0	0	-1	-1
χ_2	4	4	0	0	0	0	0	0	0	0	1	1	2	2	0	0	-1	-1	2	2	0	0	-1	-1	4	0	0	0	1	1
χ_3	4	4	0	0	0	0	0	0	0	0	1	1	-2	-2	0	0	1	1	2	2	0	0	-1	-1	-4	0	0	0	-1	-1
χ_4	4	4	0	0	0	0	0	0	0	0	1	1	-2	-2	0	0	1	1	-2	-2	0	0	1	1	4	0	0	0	1	1
χ_5	4	4	0	0	0	0	0	0	0	0	A	A	2	2	0	0	-A	-A	-2	-2	0	0	A	A	-4	0	0	0	-A	-A
χ_6	4	4	0	0	0	0	0	0	0	0	A	A	2	2	0	0	-A	-A	-2	-2	0	0	A	A	-4	0	0	0	-A	-A
χ_7	4	4	0	0	0	0	0	0	0	0	A	A	2	2	0	0	-A	-A	-2	-2	0	0	-A	-A	4	0	0	0	A	A
χ_8	4	4	0	0	0	0	0	0	0	0	A	A	2	2	0	0	-A	-A	-2	-2	0	0	-A	-A	4	0	0	0	A	A
χ_9	4	4	0	0	0	0	0	0	0	0	A	A	-2	-2	0	0	A	A	2	2	0	0	-A	-A	-4	0	0	0	-A	-A
χ_{10}	4	4	0	0	0	0	0	0	0	0	A	A	-2	-2	0	0	A	A	2	2	0	0	-A	-A	-4	0	0	0	-A	-A
χ_{11}	4	4	0	0	0	0	0	0	0	0	A	A	-2	-2	0	0	A	A	-2	-2	0	0	A	A	4	0	0	0	A	A
χ_{12}	4	4	0	0	0	0	0	0	0	0	A	A	-2	-2	0	0	A	A	-2	-2	0	0	A	A	4	0	0	0	A	A
χ_{13}	12	-4	0	0	0	0	0	0	0	0	0	0	-6	2	0	0	0	0	-6	2	0	0	0	0	0	4	0	0	0	0
χ_{14}	12	-4	0	0	0	0	0	0	0	0	0	0	-6	2	0	0	0	0	6	-2	0	0	0	0	0	-4	0	0	0	0
χ_{15}	12	-4	0	0	0	0	0	0	0	0	0	0	6	-2	0	0	0	0	-6	2	0	0	0	0	0	-4	0	0	0	0
χ_{16}	12	-4	0	0	0	0	0	0	0	0	0	0	6	-2	0	0	0	0	6	-2	0	0	0	0	0	4	0	0	0	0

Note. $A = (-1 - \sqrt{3}i)/2$.

TABLE 4: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	4	0	0	0	4	0	0	0	0	0	1	1	B	0	0	0	C	C	2	2	0	0	-1	-1	E	E	0	0	-C	-C
χ_2	4	0	0	0	4	0	0	0	0	0	1	1	-B	0	0	0	-C	-C	2	2	0	0	-1	-1	-E	-E	0	0	C	C
χ_3	4	0	0	0	4	0	0	0	0	0	A	/A	B	0	0	0	D	-/D	2	2	0	0	-A	-A	E	E	0	0	-D	/D
χ_4	4	0	0	0	4	0	0	0	0	0	/A	A	B	0	0	0	-/D	D	2	2	0	0	-/A	-A	E	E	0	0	/D	-D
χ_5	4	0	0	0	4	0	0	0	0	0	A	/A	-B	0	0	0	-D	/D	2	2	0	0	-A	-A	-E	-E	0	0	D	-/D
χ_6	4	0	0	0	4	0	0	0	0	0	/A	A	-B	0	0	0	/D	-D	2	2	0	0	-/A	-A	-E	-E	0	0	-/D	D
χ_7	4	0	0	0	4	0	0	0	0	0	1	1	B	0	0	0	C	C	-2	-2	0	0	1	1	-E	-E	0	0	C	C
χ_8	4	0	0	0	4	0	0	0	0	0	1	1	-B	0	0	0	-C	-C	-2	-2	0	0	1	1	E	E	0	0	-C	-C
χ_9	4	0	0	0	4	0	0	0	0	0	A	/A	B	0	0	0	D	-/D	-2	-2	0	0	A	/A	-E	-E	0	0	D	-/D
χ_{10}	4	0	0	0	4	0	0	0	0	0	/A	A	B	0	0	0	-/D	D	-2	-2	0	0	/A	A	-E	-E	0	0	-/D	D
χ_{11}	4	0	0	0	4	0	0	0	0	0	A	/A	-B	0	0	0	-D	/D	-2	-2	0	0	A	/A	E	E	0	0	-D	/D
χ_{12}	4	0	0	0	4	0	0	0	0	0	/A	A	-B	0	0	0	/D	-D	-2	-2	0	0	/A	A	E	E	0	0	/D	-D
χ_{13}	12	0	0	0	-4	0	0	0	0	0	0	0	0	-B	0	0	0	0	-6	2	0	0	0	0	F	-E	0	0	0	0
χ_{14}	12	0	0	0	-4	0	0	0	0	0	0	0	0	B	0	0	0	0	-6	2	0	0	0	0	-F	E	0	0	0	0
χ_{15}	12	0	0	0	-4	0	0	0	0	0	0	0	0	-B	0	0	0	0	6	-2	0	0	0	0	-F	E	0	0	0	0
χ_{16}	12	0	0	0	-4	0	0	0	0	0	0	0	0	B	0	0	0	0	6	-2	0	0	0	0	F	-E	0	0	0	0

Note. $A = (-1 - \sqrt{3}i)/2$; $B = 4i$; $C = i$; $D = E(12)^{11}$; $E = 2i$; $F = 6i$.

TABLE 5: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	8	0	8	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_2	8	0	8	0	0	0	0	0	0	0	A	/A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	8	0	8	0	0	0	0	0	0	0	/A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	24	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = -1 - i\sqrt{3}$.

TABLE 6: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	8	8	0	0	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_2	8	8	0	0	0	0	0	0	0	0	A	/A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	8	8	0	0	0	0	0	0	0	0	/A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	24	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = -1 - i\sqrt{3}$.

TABLE 7: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	32	32	12	12	192	64	32	32	12	12
χ_1	8	0	0	0	8	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_2	8	0	0	0	8	0	0	0	0	0	A	/A	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	8	0	0	0	8	0	0	0	0	0	/A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	24	0	0	0	-8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = -1 - i\sqrt{3}$.

TABLE 8: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	32	32	12	12	192	64	32	32	12	12
χ_1	4	4	4	4	4	4	0	0	0	0	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_2	4	4	4	4	4	4	0	0	0	0	A	/A	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	4	4	4	4	4	4	0	0	0	0	/A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	6	-2	-2	-2	-2	6	0	0	0	0	0	0	0	0	-2	-2	0	-B	0	0	0	0	B	B	0	0
χ_5	6	-2	-2	-2	-2	6	0	0	0	0	0	0	0	0	2	-2	0	-B	0	0	0	0	-B	-B	0	0
χ_6	6	-2	-2	-2	-2	6	0	0	0	0	0	0	0	0	2	2	0	B	0	0	0	0	-B	-B	0	0
χ_7	6	-2	-2	-2	-2	6	0	0	0	0	0	0	0	0	2	2	0	-B	0	0	0	0	B	B	0	0
χ_8	6	-2	-2	6	-2	-2	0	0	0	0	0	0	0	0	-2	2	0	B	0	0	0	0	-B	B	0	0
χ_9	6	-2	-2	6	-2	-2	0	0	0	0	0	0	0	0	-2	2	0	-B	0	0	0	0	B	-B	0	0
χ_{10}	6	-2	-2	6	-2	-2	0	0	0	0	0	0	0	0	2	-2	0	B	0	0	0	0	B	-B	0	0
χ_{11}	6	-2	-2	6	-2	-2	0	0	0	0	0	0	0	0	2	-2	0	-B	0	0	0	0	-B	B	0	0
χ_{12}	12	-4	12	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	12	12	-4	-4	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	12	-4	-4	-4	12	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A=1+i3$; $B=-2i$.

TABLE 9: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12
χ_1	2	2	2	2	2	2	2	2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_2	2	2	2	2	2	2	2	2	2	2	A	/A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_3	2	2	2	2	2	2	2	2	2	2	/A	A	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_4	6	6	6	6	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_5	6	-2	6	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0
χ_6	6	-2	6	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0
χ_7	6	-2	6	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	-4	0	0	0	0	0	0	0	0	0
χ_8	6	-2	6	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0
χ_9	6	6	-2	-2	-2	-2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	6	6	-2	-2	-2	-2	-2	-2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{11}	6	6	-2	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{12}	6	6	-2	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	6	-2	-2	-2	6	-2	2	2	2	2	0	0	0	0	0	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	6	-2	-2	-2	6	-2	2	2	2	2	0	0	0	0	0	-B	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	6	-2	-2	-2	6	-2	2	2	2	2	0	0	0	0	B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{16}	6	-2	-2	-2	6	-2	2	2	2	2	0	0	0	0	-B	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{17}	12	-4	-4	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{18}	12	-4	-4	-4	-4	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = -1 - i3$; $B = -4i$.

TABLE 10: Projective character table of H_2 with factor set α_i^{-1} .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	12	12	192	64	32	32	12	12
χ_1	2	2	2	2	2	2	0	0	0	0	-1	-1	-2	-2	0	0	0	0	-1	-1	-2	-2	0	0	1	1
χ_2	2	2	2	2	2	2	0	0	0	0	-1	-1	-2	-2	0	0	0	0	1	1	2	2	0	0	-1	-1
χ_3	2	2	2	2	2	2	0	0	0	0	-1	-1	2	2	0	0	0	0	1	1	-2	-2	0	0	1	1
χ_4	2	2	2	2	2	2	0	0	0	0	-1	-1	2	2	0	0	0	0	-1	-1	2	2	0	0	-1	-1
χ_5	2	2	2	2	2	2	0	0	0	0	A	/A	-2	-2	0	0	0	0	A	/A	-2	-2	0	0	-A	-A
χ_6	2	2	2	2	2	2	0	0	0	0	-A	-A	-2	-2	0	0	0	0	/A	/A	-2	-2	0	0	-A	-A
χ_7	2	2	2	2	2	2	0	0	0	0	-A	/A	-2	-2	0	0	0	0	-A	-A	2	2	0	0	A	/A
χ_8	2	2	2	2	2	2	0	0	0	0	/A	A	-2	-2	0	0	0	0	-A	-A	2	2	0	0	/A	/A
χ_9	2	2	2	2	2	2	0	0	0	0	A	/A	2	2	0	0	0	0	-A	-A	-2	-2	0	0	-A	-A
χ_{10}	2	2	2	2	2	2	0	0	0	0	/A	A	2	2	0	0	0	0	-A	-A	-2	-2	0	0	-A	-A
χ_{11}	2	2	2	2	2	2	0	0	0	0	A	/A	2	2	0	0	0	0	A	/A	2	2	0	0	A	/A
χ_{12}	2	2	2	2	2	2	0	0	0	0	/A	A	2	2	0	0	0	0	A	/A	2	2	0	0	A	/A
χ_{13}	6	-2	-2	-2	6	-2	0	0	0	0	0	0	-6	2	0	0	0	-B	0	0	0	0	-B	B	0	0
χ_{14}	6	-2	-2	-2	6	-2	0	0	0	0	0	0	-6	2	0	0	0	-B	0	0	0	0	B	-B	0	0
χ_{15}	6	-2	-2	-2	6	-2	0	0	0	0	0	0	6	-2	0	0	0	B	0	0	0	0	B	-B	0	0
χ_{16}	6	-2	-2	-2	6	-2	0	0	0	0	0	0	6	-2	0	0	0	-B	0	0	0	0	-B	B	0	0
χ_{17}	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0	B	B	0	B	0	0	6	-2	0	0	0	0
χ_{18}	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0	-B	-B	0	-B	0	0	6	-2	0	0	0	0
χ_{19}	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0	B	B	0	B	0	0	-6	2	0	0	0	0
χ_{20}	6	6	-2	-2	-2	-2	0	0	0	0	0	0	0	0	-B	-B	0	-B	0	0	-6	2	0	0	0	0
χ_{21}	6	-2	6	-2	-2	-2	0	0	0	0	0	0	0	0	B	-B	0	B	0	0	0	0	0	0	0	0
χ_{22}	6	-2	6	-2	-2	-2	0	0	0	0	0	0	0	0	-B	-B	0	-B	0	0	0	0	0	0	0	0
χ_{23}	6	-2	6	-2	-2	-2	0	0	0	0	0	0	0	0	B	-B	0	B	0	0	0	0	0	0	0	0
χ_{24}	6	-2	6	-2	-2	-2	0	0	0	0	0	0	0	0	-B	-B	0	-B	0	0	0	0	0	0	0	0
χ_{25}	12	-4	-4	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	12	-4	-4	-4	-4	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = (1 + \sqrt{3}i)/2$, $B = -2i$.

TABLE 11: Ordinary character table of H_2 .

$[h]_{H_2}$	1a	2a	2b	2c	2d	2e	2f	2g	2h	2i	3a	3b	2j	2k	4a	4b	6a	6b	2l	2m	4c	4d	6c	6d	2n	2o	4e	4f	6e	6f	
$ C_{H_2}(h) $	768	256	256	256	256	256	64	64	64	64	12	12	192	64	32	32	12	12	192	64	32	32	12	12	192	64	32	32	12	12	
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	
χ_3	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	
χ_4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
χ_5	1	1	1	1	1	1	1	1	1	1	A	/A	-1	-1	-1	-1	-A	-A	-1	-1	-1	-1	-A	-A	1	1	1	1	A	/A	
χ_6	1	1	1	1	1	1	1	1	1	1	/A	A	-1	-1	-1	-1	-A	-A	-1	-1	-1	-1	-A	-A	1	1	1	1	/A	A	
χ_7	1	1	1	1	1	1	1	1	1	1	A	/A	-1	-1	-1	-1	-A	-A	1	1	1	1	A	/A	-1	-1	-1	-1	-A	-A	
χ_8	1	1	1	1	1	1	1	1	1	1	/A	A	-1	-1	-1	-1	-A	-A	1	1	1	1	/A	A	-1	-1	-1	-1	-A	-A	
χ_9	1	1	1	1	1	1	1	1	1	1	A	/A	1	1	1	1	A	/A	-1	-1	-1	-1	-A	-A	-1	-1	-1	-1	-A	-A	
χ_{10}	1	1	1	1	1	1	1	1	1	1	/A	A	1	1	1	1	/A	A	-1	-1	-1	-1	-A	-A	-1	-1	-1	-1	-A	-A	
χ_{11}	1	1	1	1	1	1	1	1	1	1	A	/A	1	1	1	1	A	/A	1	1	1	1	A	/A	1	1	1	1	A	/A	
χ_{12}	1	1	1	1	1	1	1	1	1	1	/A	A	1	1	1	1	/A	A	1	1	1	1	/A	A	1	1	1	1	/A	A	
χ_{13}	3	3	3	3	3	3	-1	-1	-1	-1	0	0	-3	-3	1	1	0	0	-3	-3	1	1	0	0	3	3	-1	-1	0	0	0
χ_{14}	3	3	3	3	3	3	-1	-1	-1	-1	0	0	-3	-3	1	1	0	0	3	3	-1	-1	0	0	-3	-3	1	1	0	0	0
χ_{15}	3	3	3	3	3	3	-1	-1	-1	-1	0	0	3	3	-1	-1	0	0	-3	-3	1	1	0	0	-3	-3	1	1	0	0	0
χ_{16}	3	3	3	3	3	3	-1	-1	-1	-1	0	0	3	3	-1	-1	0	0	3	3	-1	-1	0	0	3	3	-1	-1	0	0	0
χ_{17}	6	-2	6	-2	-2	-2	-2	2	-2	2	0	0	0	0	0	0	0	0	-6	2	2	-2	0	0	0	0	0	0	0	0	0
χ_{18}	6	-2	6	-2	-2	-2	-2	2	-2	2	0	0	0	0	0	0	0	0	6	-2	-2	2	0	0	0	0	0	0	0	0	0
χ_{19}	6	-2	6	-2	-2	-2	2	-2	2	-2	0	0	0	0	0	0	0	0	-6	2	-2	2	0	0	0	0	0	0	0	0	0
χ_{20}	6	-2	6	-2	-2	-2	2	-2	2	-2	0	0	0	0	0	0	0	0	6	-2	-2	2	0	0	0	0	0	0	0	0	0
χ_{21}	6	6	-2	-2	-2	-2	-2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	2	2	-2	0	0	0
χ_{22}	6	6	-2	-2	-2	-2	-2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-2	-2	2	0	0	0
χ_{23}	6	6	-2	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6	2	-2	2	0	0	0
χ_{24}	6	6	-2	-2	-2	-2	2	2	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	-2	-2	2	0	0	0
χ_{25}	6	-2	-2	-2	6	-2	-2	2	2	-2	0	0	-6	2	2	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{26}	6	-2	-2	-2	6	-2	-2	2	2	-2	0	0	6	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	6	-2	-2	-2	6	-2	-2	2	2	-2	0	0	-6	2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	6	-2	-2	-2	6	-2	-2	2	2	-2	0	0	6	-2	-2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	12	-4	-4	12	-4	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	12	-4	-4	-4	-4	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note. $A = (-1 - \sqrt{3}i)/2$.

TABLE 12: Fischer–Clifford matrices of $\overline{G}$.

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$	$M(2A) = (1)$
$M(2B) = (1)$	$M(2C) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(2D) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(2E) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$
$M(3A) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$	$M(3B) = \begin{pmatrix} 1 & 1 \\ 15 & -1 \end{pmatrix}$
$M(3C) = (1)$	$M(3D) = (1)$
$M(3E) = (1)$	$M(4A) = (1)$
$M(4B) = (1)$	$M(5A) = (1)$
$M(5B) = (1)$	$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$
$M(6B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$	$M(6C) = (1)$
$M(6D) = (1)$	$M(6E) = (1)$
$M(10A) = (1)$	$M(10B) = (1)$
$M(15A) = (1)$	$M(15B) = (1)$
$M(15C) = (1)$	$M(15D) = (1)$

projective character tables. Hence, we obtain 9 distinct sets $\text{IrrProj}(H_2, \alpha_i^{-1})$ of irreducible projective characters for H_2 , and they are listed in Tables 2–10. The ordinary irreducible characters $\text{Irr}(H_2)$ of H_2 are found in Table 11. Note that the irreducible projective characters with associated factor set α_i^{-1} have values of zero on the α_i^{-1} -irregular classes of G . Hence, the number of irreducible projective characters equals the number of α_i^{-1} -regular classes of G in each of the below tables. The above results are summarized in the following theorem:

Theorem 1. *The inertia factor subgroup H_2 of the action of G on $\text{Irr}(N)$ has exactly 9 distinct sets of $\text{IrrProj}(H_2, \alpha_i^{-1})$ with associated factor sets α_i^{-1} such that $\alpha_i^2 \sim 1$.*

As mentioned in the introductory section, a set $\text{IrrProj}(H_2, \alpha_i^{-1})$ of 16 irreducible projective characters of H_2 with associated factor set α_i^{-1} is required in the construction of $\text{Irr}(\overline{G})$, using the technique of Fischer–Clifford matrices. But a choice is to be made amongst the 3 distinct projective characters tables of H_2 which contain 16 irreducible projective characters each. The desired set $\text{IrrProj}(H_2, \alpha_i^{-1})$ of 16 irreducible projective characters can be found in the inertia group $\overline{H}_2 = 2^4 H_2$ of $\overline{G}$ on $\text{Irr}(N)$.

Since we have a known permutation representation of $\overline{G}$, the group $\overline{H}_2$ is generated within GAP. The normal subgroups E_i of order 8 of $\overline{H}_2$ are computed. There are three such groups, E_1, E_2 , and E_3 , which are elementary abelian. The 3 quotient groups $D_i = \overline{H}_2/E_i$ are double covers for H_2 with each containing 46 ordinary irreducible characters. 30

TABLE 13: Classes of $\overline{G}$.

$[g]_G$	k	f_j	$[x]_{\overline{G}}$	$ C_{\overline{G}}(x) $
1A	16	$f_1 = 1$	1A	184320
		$f_2 = 15$	2A	12288
2A	16	$f_1 = 16$	2B	3840
2B	16	$f_1 = 16$	2C	768
2C	16	$f_1 = 8$	4A	512
		$f_2 = 8$	4B	512
		$f_1 = 1$	2D	768
		$f_2 = 1$	4C	768
2D	4	$f_3 = 1$	4D	768
		$f_4 = 1$	4E	768
		$f_1 = 1$	2E	256
		$f_2 = 1$	4F	256
2E	4	$f_3 = 1$	4G	256
		$f_4 = 1$	4H	256
3A	16	$f_1 = 1$	3A	2880
		$f_2 = 15$	6A	192
3B	16	$f_1 = 1$	3B	2880
		$f_2 = 15$	6B	192
3C	1	$f_1 = 1$	3C	36
3D	1	$f_1 = 1$	3D	36
3E	1	$f_1 = 1$	3E	36
4A	4	$f_1 = 4$	8A	32
4B	4	$f_1 = 4$	8B	32
5A	1	$f_1 = 1$	5A	60
5B	1	$f_1 = 1$	5B	60
		$f_1 = 1$	6C	48
		$f_2 = 1$	12A	48
6A	4	$f_3 = 1$	12B	48
		$f_4 = 1$	12C	48
		$f_1 = 1$	6D	48
		$f_2 = 1$	12D	48
6B	4	$f_3 = 1$	12E	48
		$f_4 = 1$	12F	48
6C	1	$f_1 = 1$	6E	12
6D	1	$f_1 = 1$	6F	12
6E	1	$f_1 = 1$	6G	12
10A	1	$f_1 = 1$	10A	20
10B	1	$f_1 = 1$	10B	20
15A	1	$f_1 = 1$	15A	15
15B	1	$f_1 = 1$	15B	15
15C	1	$f_1 = 1$	15C	15
15D	1	$f_1 = 1$	15D	15

characters from each of the sets $\text{Irr}(D_i)$ are the liftings of the ordinary irreducible characters of H_2 to D_i , while the remaining 16 ordinary characters of each D_i represent the set $\text{IrrProj}(H_2, \alpha_i^{-1})$ of irreducible projective characters of H_2 , which is of interest to us. Using the above GAP code as earlier, the desired set $\text{IrrProj}(H_2, \alpha_i^{-1})$ of projective characters of H_2 which is needed in the construction of the ordinary character table of $\overline{G}$ using Fischer–Clifford theory is identified as Table 3.

5. Fischer–Clifford Matrices and Conjugacy Classes of $\overline{G}$

For the readers who are interested in the construction of the ordinary character table of $\overline{G} = 2^4 \cdot (2^6 : (A_5 \times 3))$ using the technique of Fischer–Clifford matrices (see, for example,

[6, 9, 19–23]), the Fischer–Clifford matrices (Table 12) and the conjugacy classes (Table 13) of $\overline{G}$ are supplied. The conjugacy classes are arranged in a format obtained by the technique of coset analysis (see, for example, [6, 9, 21, 24]). The set of irreducible characters of $\overline{G}$ will be partitioned into 2 blocks $\Delta_1 = \{\chi_j \mid 1 \leq j \leq 26\}$ and $\Delta_2 = \{\chi_j \mid 27 \leq j \leq 42\}$ corresponding to the inertia factor groups H_1 and H_2 , respectively, where $\chi_j \in \text{Irr}(\overline{G})$.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

The content of this paper was presented as part of the author's short communications' presentation at the 2018 International Congress of Mathematicians in Rio de Janeiro, Brazil.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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