Research Article

Fischer-Clifford Matrices and Character Table of the Maximal Subgroup $(2^9 : (L_3(4)) : 2$ of $U_6(2) : 2$

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The automorphism group $U_6(2) : 2$ of the unitary group $U_6(2) \cong Fi_{21}$ has a maximal subgroup $\overline{G}$ of the form $(2^9 : (L_3(4)) : 2$ of order 20643840. In this paper, Fischer-Clifford theory is applied to the split extension group $\overline{G}$ to construct its character table. Also, class fusion from $\overline{G}$ into the parent group $U_6(2) : 2$ is determined.

1. Introduction

The unitary simple group $U = U_6(2)$ has outer automorphisms of orders 2, 3, and 6 and hence automorphism groups of the forms $U_1 = U_6(2) : 2$, $U_2 = U_6(2) : 3$, and $U_3 = U_6(2) : S_3$ exist for U (see the ATLAS [1]). The reader is referred to [2] for more information about the construction of matrix representations for the covering and automorphism groups of $U_6(2)$. Recently in [3], a 3-local identification is given for the group $PSU_6 \cong U_6(2)$ and its automorphism groups $PSU_6(2) : 3$, $PSU_6(2) : 2$, and $PSU_6(2) : S_3$. Also, we found in the ATLAS that one of the 16 maximal subgroups of U is a split extension group A of the type $2^9 : L_3(4)$ of index 891 and has order 10321920. The group A has automorphism groups of the form $A_1 = (2^9 : L_3(4)) : 2$, $A_2 = (2^9 : L_3(4)) : 3$, and $A_3 = (2^9 : L_3(4)) : S_3$ which sit maximally inside the groups U_1 , U_2 , and U_3 , respectively.

The character table of A is stored in the GAP Library [4], whereas the character table of A_1 and A_2 are not yet uploaded in GAP. In this paper, the unknown Fischer-Clifford matrices [5] of A_1 and its associated character table will be constructed. Readers are referred to [6] on a survey of Fischer-Clifford theory. It is interesting to note that our group

A_1 is the stabilizer $\text{Stab}_{U_1}(v_1)$ of a singular vector $v_1 \in V_{20}(2)$, where $V_{20}(2)$ is the irreducible module of dimension 20 over the field $GF(2)$ for $U_6(2) : 2$ (see [3]). The method of coset analysis as discussed in [7–9] will be used in the computation of the conjugacy classes of elements of the group A_1 . The Fischer-Clifford matrices and character table of A_3 were determined in [10].

2. Group $2^9 : (L_3(4) : 2)$

In this section, using a suitable permutation representation of $U_6(2) : 2$, we identify our group $A_1 = (2^9 : L_3(4)) : 2$ as a split extension 2^9 by $L_3(4) : 2$ with the aid of GAP [4] and MAGMA [11]. Then with the help of MAGMA we represent $L_3(4) : 2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. Since G acts absolutely irreducibly on its natural module 2^9 , a split extension S of the form $2^9 : (L_3(4) : 2)$ exists. Then we create S as a subgroup of $SL_{10}(2)$ and show with the help of MAGMA that A_1 is indeed an isomorphic copy of S .

We construct $U_1 = U_6(2) : 2$ within GAP, using its smallest permutation representation of degree 672 found in the online ATLAS of Finite Group Representations [12].

Next, we use the GAP commands “MS:= ConjugacyClassesMaximalSubgroups (U_1)”, “A1:=MS[4]”, and “Size(A1)” to represent $A_1 = (2^9 : (L_3(4)) : 2$ as a permutation group on 672 points and then use this permutation representation to construct A_1 within MAGMA. Using the sequence of MAGMA commands “a,b:=ChiefSeries(A1)”, “N:= a[3]”, “NormalSubgroups(A1)”, “IsNormal(A1,a[3])”, “IsElementaryAbelian(N)”, “C:= Complements(A1,N)”, “Order(C[1])”, “C[1] meet N”, and “IsIsomorphic(C[1], $L_3(4) : 2$)”, we verified that $A_1 = (2^9 : L_3(4)) : 2 \cong 2^9 : (L_3(4) : 2)$.

Having A_1 as a permutation group on 672 points, we use the MAGMA commands “M:=GModule(A1,N)” and “M:Maximal” to represent $L_3(4) : 2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. Thus we obtain the matrix group $G = \langle g_1, g_2 \rangle$ having 14 conjugacy classes of elements, where $o(g_1) = 4$ and $o(g_2) = 14$. The generators g_1 and g_2 of G are as follows:

$$g_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \quad (1)$$

$$g_2 = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

The MAGMA command “IsAbsolutelyIrreducible(G)” tells us that the action of the matrix group G on its natural module 2^9 is absolutely irreducible. Thus a split extension of the type $2^9 : (L_3(4) : 2)$ does exist. Hence we can construct $2^9 : (L_3(4) : 2)$ as a subgroup S of $SL_{10}(2)$ such that $S = \langle s_1, s_2, s_3 \rangle$ and $L_3(4) : 2 = \langle s_1, s_2 \rangle$, where $o(s_1) = 4$, $o(s_2) = 14$, and $o(s_3) = 2$. The generators of the matrix group S of degree 10 over $GF(2)$ are as follows:

$$s_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

$$s_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Since we can represent S and A_1 as a matrix and permutation group, respectively, we use the MAGMA command “IsIsomorphic (S, A_1)” to confirm that $S \cong A_1$. Hence we can regard $A_1 = (2^9 : L_3(4)) : 2$ as the split extension $S = 2^9 : (L_3(4) : 2)$.

3. The Conjugacy Classes of $\overline{G} = 2^9 : (L_3(4) : 2)$

In this section, the conjugacy classes of $\overline{G}$ are computed using the technique of coset analysis and readers are encouraged to consult [8, 9] for a sound theoretical background on this technique.

Throughout the remainder of this chapter, let $\overline{G} = 2^9 : (L_3(4) : 2)$ be a split extension of $N = 2^9$ by $L_3(4) :$

TABLE 1: The values of $\chi(L_3(4) : 2 \mid 2^9)$ on the different classes of $L_3(4) : 2$.

$[h]_{L_3(4):2}$	1A	2A	2B	3A	4A	4B	4C	5A	6A	7A	7B	8A	14A	14B
$\chi(L_3(4) : 3 \mid P_1)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(L_3(4) : 3 \mid P_2)$	21	7	5	3	1	1	3	1	1	0	0	1	0	0
$\chi(L_3(4) : 3 \mid P_3)$	210	28	18	3	2	2	4	0	1	0	0	0	0	0
$\chi(L_3(4) : 3 \mid P_4)$	280	28	8	1	4	4	0	0	1	0	0	2	0	0
k	512	64	32	8	8	8	8	2	4	1	1	4	1	1

2, where N is the vector space $V_9(2)$ of dimension 9 over $GF(2)$ on which the linear group G acts. Since $G = \langle g_1, g_2 \rangle$ is represented as a matrix group, we used the MAGMA commands “O:=Orbits(G)”, “#O”, “#O[1]”, “#O[2]”, “#O[3]”, and “#O[4]” to compute the orbit lengths of the action of G on N . We obtain 4 orbits of lengths 1, 21, 210, and 280 and using the MAGMA commands “P1:=Stabilizer(G,O[1,1])”, “P2:=Stabilizer(G,O[2,1])”, “P3:=Stabilizer(G,O[3,1])”, and “P4:=Stabilizer(G,O[4,1])”, we are able to compute the corresponding point stabilizers P_i , $i = 1, 2, 3, 4$, which are subgroups of G . With the aid of MAGMA and also checking the indices of the maximal subgroups of G in the ATLAS, the structures of the stabilizers P_i are identified as $P_1 = L_3(4) : 2$, $P_2 = 2^4 : S_5$, $P_3 = 2^4 : (2 \times S_3)$, and $P_4 = 3^2 : (Q_8.2)$, where P_2 and P_4 are maximal subgroups of P_1 . We should note here that the group $L_3(4) : 2$ has two nonconjugate isomorphic maximal subgroups $L_1 = P_2$ and L_2 , having the same structure $2^4 : S_5$. The stabilizer P_3 sits maximally in L_2 . Alternatively, we can use [13] to identify the structures of the groups P_i . Since the action of G on N does not fix any nontrivial subspace of 2^9 , we have that 2^9 is an irreducible module for G . We can readily verify this fact by using the MAGMA command “IsIrreducible(G)”.

Let $\chi(L_3(4) : 2 \mid 2^9)$ be the permutation character of $L_3(4) : 2$ on the classes of 2^9 . Then, from methods that were developed by Mpono [14], we obtain that $\chi(L_3(4) : 2 \mid 2^9) = I_{P_1}^{P_1} + I_{P_2}^{P_1} + I_{P_3}^{P_1} + I_{P_4}^{P_1} = 4 \times 1a + 4 \times 20a + 2 \times 35a + 45a + 45b + 2 \times 64a + 2 \times 70a$, where $I_{P_1}^{P_1}$, $I_{P_2}^{P_1}$, $I_{P_3}^{P_1}$, and $I_{P_4}^{P_1}$ are the identity characters of the point stabilizers P_i , $i = 1, 2, 3, 4$, induced to G . Note that the identity characters $I_{P_i}^{P_1}$ are identified with the permutation characters $\chi(L_3(4) : 2 \mid P_i)$ of $L_3(4) : 2$ acting on the classes of the point stabilizers P_i . We found that $I_{P_1}^{P_1} = \chi(L_3(4) : 3 \mid P_1) = 1a$, $I_{P_2}^{P_1} = \chi(L_3(4) : 2 \mid P_2) = 1a + 20a$, $I_{P_3}^{P_1} = \chi(L_3(4) : 2 \mid P_3) = 1a + 2 \times 20a + 35a + 64a + 70a$, and $I_{P_4}^{P_1} = \chi(L_3(4) : 3 \mid P_4) = 1a + 20a + 35a + 45a + 45b + 64a + 70a$. The permutation characters $\chi(L_3(4) : 2 \mid P_i)$ are written in terms of the ordinary irreducible characters of G . Since we have the generators g_1 and g_2 for G , we compute the character tables of G and the P_i 's directly in MAGMA and use these tables together with the fusion maps of the stabilizers into G , to compute $\chi(L_3(4) : 2 \mid P_i)$ and $\chi(L_3(4) : 2 \mid 2^9)$. The values of $\chi(L_3(4) : 2 \mid 2^9)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^9 . The values of k are listed in Table 1.

The values of k enabled us to determine the number f_j of orbits Q_i 's, $1 \leq i \leq k$, which have fused together under the action of $C_G(g)$, for each class representative $g \in G$, to form one orbit Δ_f . Mpono in [14] used the technique of coset analysis to develop Programmes A and B in CAYLEY [15] for the computation of the conjugacy classes of a split extension $\overline{G} = N : G$, where N is an elementary abelian p -group for a prime p on which a linear group G acts. Ali [16] adapted Programmes A and B to be used in MAGMA. Programme A computes the values of the f_j 's, whereas Programme B determines the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. We obtain that $\overline{G}$ has exactly 49 conjugacy classes. The parameters d_j and w are defined in [14]. The centralizer order of each class $[x]$ of $\overline{G}$ is computed using the formula $|C_{\overline{G}}(x)| = (k/f)|C_G(g)|$. All the information involving the conjugacy classes of $\overline{G}$ are listed in Table 2.

4. The Inertia Groups of $\overline{G}=2^9:(L_3(4):2)$

Since G has four orbits on N , then by Brauer's Theorem [17] G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be $1, r, s$, and t where $r + s + t = 511$, with corresponding point stabilizers H_1, H_2, H_3 , and H_4 as subgroups of G such that $[G : H_1] = 1$, $[G : H_2] = r$, $[G : H_3] = s$, and $[G : H_4] = t$. We generate G as a permutation group on a set of cardinality 672 within MAGMA. Then the maximal and submaximal subgroups of G are computed. Now, considering the indices of these subgroups in G , the number of the classes of these subgroups, and also the fact that $\overline{G}$ has 49 conjugacy classes, we deduce that the action of G on $Irr(N)$ has orbits of lengths $1, r = 21, s = 210$, and $t = 280$ with respective point stabilizers $H_1 = L_3(4) : 2$, $H_2 = 2^4 : S_5$, $H_3 = 2^4 : (2 \times S_3)$, and $H_4 = 3^2 : (Q_8 : 2)$. Thus we obtain four inertia groups $\overline{H}_i = 2^9 : H_i$, $i \in \{1, 2, 3, 4\}$, for $2^9 : (L_3(4) : 2)$ on $Irr(N)$. Alternatively, we can also determine the inertia factor groups if we let T be the matrix group of dimension 9 over $GF(2)$ formed by the transpose of the generators of G . Then the action of T on the classes of $N = 2^9$ is the equivalent of G acting on $Irr(N)$. Then with the help of MAGMA or GAP, we can easily verify that the action of T on N has orbits of lengths $1, 21, 210$, and 280 with corresponding point stabilizers $T, 2^4 : S_5, 2^4 : (2 \times S_3)$, and $3^2 : (Q_8 : 2)$. The structures of H_2 and H_4 have been identified by checking the indices of the maximal subgroups of $L_3(4) : 2 \cong L_3(4).2_2$ in the ATLAS. The structure of H_3 was determined by direct computations

TABLE 2: The conjugacy classes of elements of $\overline{G} = 2^9 : (L_3(4) : 2)$.

$[g]_G$	k	f_i	d_i	w	$[x]_{\overline{G}}$	$ \overline{C}_{\overline{G}}(x) $	$ C_{\overline{G}}(x) $
1A	512	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	1A	1	20643840
		$f_2 = 21$	(0, 0, 0, 0, 1, 1, 0, 1, 1)	(0, 0, 0, 0, 1, 1, 0, 1, 1)	2A	21	983040
		$f_3 = 210$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(1, 0, 0, 0, 0, 0, 0, 0, 0)	2B	210	98304
		$f_4 = 280$	(1, 0, 1, 0, 1, 1, 1, 0, 1)	(1, 0, 1, 0, 1, 1, 1, 0, 1)	2C	280	73728
2A	64	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	2D	960	21504
		$f_2 = 7$	(0, 0, 0, 0, 0, 1, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	2E	6720	3072
		$f_3 = 7$	(0, 0, 0, 0, 0, 0, 0, 0, 1)	(0, 1, 1, 1, 1, 0, 0, 1, 0)	4A	6720	3072
		$f_4 = 21$	(1, 0, 0, 0, 1, 1, 1, 0, 0)	(0, 0, 0, 0, 1, 1, 1, 0, 0)	4B	20160	1024
		$f_5 = 28$	(0, 0, 0, 0, 0, 0, 0, 0, 1)	(0, 1, 1, 1, 0, 1, 1, 1, 0)	4C	26880	768
2B	32	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	2F	5040	4096
		$f_2 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	2G	5040	4096
		$f_3 = 1$	(1, 1, 1, 1, 1, 1, 1, 1, 1)	(0, 1, 1, 1, 1, 0, 0, 0, 1)	4D	5040	4096
		$f_4 = 1$	(1, 1, 0, 0, 0, 0, 0, 0, 1)	(0, 1, 1, 1, 1, 0, 0, 0, 1)	4E	5040	4096
		$f_5 = 4$	(1, 1, 1, 0, 1, 0, 0, 0, 1)	(0, 0, 1, 0, 0, 1, 1, 0, 0)	4F	20160	1024
		$f_6 = 8$	(0, 1, 1, 1, 0, 1, 0, 1, 0)	(0, 0, 1, 1, 1, 1, 1, 1, 1)	4G	40320	512
		$f_7 = 8$	(1, 1, 1, 1, 1, 0, 1, 1, 1)	(0, 0, 0, 0, 1, 1, 1, 1, 0)	4H	40320	512
		$f_8 = 8$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 1, 1, 0, 0, 0, 0, 1)	4I	40320	512
3A	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	3A	143360	144
		$f_2 = 1$	(1, 0, 0, 1, 0, 1, 1, 1, 1)	(1, 0, 0, 1, 0, 1, 1, 1, 1)	6A	143360	144
		$f_3 = 3$	(1, 1, 1, 1, 0, 1, 0, 0, 1)	(1, 1, 1, 1, 0, 1, 0, 0, 1)	6B	430080	48
		$f_4 = 3$	(0, 0, 0, 0, 0, 0, 0, 0, 1)	(1, 0, 0, 1, 1, 0, 1, 1, 0)	6C	430080	48
4A	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4J	80640	256
		$f_2 = 1$	(0, 1, 0, 0, 1, 0, 0, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4K	80640	256
		$f_3 = 2$	(0, 0, 1, 0, 0, 0, 0, 0, 0)	(0, 1, 1, 1, 1, 0, 0, 0, 1)	8A	161280	128
		$f_4 = 4$	(1, 0, 1, 0, 1, 0, 1, 1, 1)	(0, 1, 0, 1, 1, 1, 1, 0, 1)	8B	322560	64
4B	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4L	161280	128
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 1)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4M	161280	128
		$f_3 = 2$	(1, 0, 0, 0, 1, 1, 1, 1, 1)	(0, 0, 1, 1, 0, 0, 0, 1, 0)	8C	322560	64
		$f_4 = 4$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(1, 1, 0, 1, 1, 0, 0, 1, 0)	8D	645120	32
4C	8	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4N	161280	128
		$f_2 = 1$	(1, 0, 1, 1, 1, 0, 1, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	4O	161280	128
		$f_3 = 1$	(1, 0, 0, 0, 1, 1, 0, 1, 0)	(1, 0, 1, 0, 1, 1, 1, 1, 0)	8E	161280	128
		$f_4 = 1$	(0, 1, 1, 1, 1, 1, 1, 1, 0)	(1, 0, 1, 0, 1, 1, 1, 1, 0)	8F	161280	128
		$f_5 = 2$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(1, 1, 0, 1, 1, 1, 0, 0, 1)	8G	322560	64
		$f_6 = 2$	(1, 0, 1, 0, 0, 1, 0, 0, 1)	(0, 1, 1, 1, 0, 0, 1, 1, 1)	8H	322560	64
5A	2	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	5A	2064384	10
		$f_2 = 1$	(1, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 1, 1, 0, 0, 1)	10A	2064384	10
6A	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	6D	860160	24
		$f_2 = 1$	(1, 0, 0, 0, 1, 0, 1, 1, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	6E	860160	24
		$f_3 = 1$	(1, 0, 0, 1, 1, 0, 1, 1, 1)	(1, 0, 0, 1, 0, 1, 1, 1, 0)	12A	860160	24
		$f_4 = 1$	(1, 1, 1, 0, 1, 0, 1, 1, 1)	(1, 0, 0, 1, 0, 1, 1, 1, 0)	12B	860160	24
7A	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	7A	1474560	14
7B	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	7B	1474560	14
8A	4	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	8I	645120	32
		$f_2 = 1$	(1, 0, 0, 1, 1, 1, 1, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	8J	645120	32
		$f_3 = 1$	(1, 1, 1, 0, 1, 0, 1, 1, 1)	(0, 0, 0, 0, 0, 0, 0, 1, 1)	16A	645120	32
		$f_4 = 1$	(1, 1, 1, 1, 1, 1, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 1, 1)	16B	645120	32
14A	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	14A	1474560	14
14B	1	$f_1 = 1$	(0, 0, 0, 0, 0, 0, 0, 0, 0)	(0, 0, 0, 0, 0, 0, 0, 0, 0)	14B	1474560	14

in MAGMA. The groups H_2 , H_3 , and H_4 are constructed from elements within G and the generators are as follows:

(i) $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 3A$, and $\alpha_2 \in 6A$ where

$$\alpha_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix},$$

$$\alpha_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \quad (3)$$

(ii) $H_3 = \langle \beta_1, \beta_2 \rangle$, $\beta_1 \in 4C$, and $\beta_2 \in 6A$ where

$$\beta_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix},$$

$$\beta_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \quad (4)$$

(iii) $H_4 = \langle \gamma_1, \gamma_2 \rangle$, $\gamma_1 \in 2A$, and $\gamma_2 \in 8A$ where

$$\gamma_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix},$$

$$\gamma_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

For the purpose of constructing the character table of $\overline{G}$, we use the above generators of the H_i 's to compute their character tables.

5. The Fusion of H_2 , H_3 , and H_4 into G

We obtain the fusions of the inertia factors H_2 , H_3 , and H_4 into G by using direct matrix conjugation in G and their permutation characters in G of degrees 21, 210, and 280, respectively. MAGMA was used for the various computations. The fusion maps of H_2 , H_3 , and H_4 into G are shown in Tables 3, 4, and 5.

6. The Fischer-Clifford Matrices of $2^9:(L_3(4):2)$

Having obtained the fusions of the inertia factors into $L_3(4):2$ and the conjugacy classes of $L_3(4):2$ displayed in the format of Table 2, we can proceed to use the theory and properties discussed in [9] or [14] to help us in the construction of the Fischer-Clifford matrices of $2^9:(L_3(4):2)$. Note that all the relations hold since 2^9 is an elementary abelian group.

For example, consider the conjugacy class $2A$ of $L_3(4):2$. Then we obtain that $M(2A)$ has the following form with corresponding weights attached to the rows and columns:

TABLE 3: The fusion of H_2 into $L_3(4) : 2$.

$[h]_{H_2} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2} \longrightarrow$	$[g]_{L_3(4):2}$
1A	1A	2C	2B	4B	4B	5A	5A
2A	2B	3A	3A	4C	4C	6A	6A
2B	2A	4A	4A	4D	4C	8A	8A

TABLE 4: The fusion of H_3 into $L_3(4) : 2$.

$[h]_{H_3} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3} \longrightarrow$	$[g]_{L_3(4):2}$
1A	1A	2D	2B	4A	4C	4E	4B
2A	2B	2E	2B	4B	4C	6A	6A
2B	2A	2F	2A	4C	4A		
2C	2B	3A	3A	4D	4C		

$$\begin{aligned}
 M(2A) &= \begin{matrix} & |C_{\overline{G}}(2D)| & |C_{\overline{G}}(2E)| & |C_{\overline{G}}(4A)| & |C_{\overline{G}}(4B)| & |C_{\overline{G}}(4C)| \\ \begin{matrix} |C_G(2A)| \\ |C_{H_2}(2B)| \\ |C_{H_3}(2B)| \\ |C_{H_3}(2F)| \\ |C_{H_4}(2B)| \end{matrix} & \begin{pmatrix} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{pmatrix} \\ & \begin{matrix} m_1 & m_2 & m_3 & m_4 & m_5 \end{matrix} \end{matrix} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 & \begin{matrix} 21504 & 3072 & 3072 & 1024 & 768 \\ 336 & & & & \\ 48 & & & & \\ 48 & & & & \\ 16 & & & & \\ 12 & & & & \end{matrix} \begin{pmatrix} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{pmatrix} \\
 & \begin{matrix} 8 & 56 & 56 & 168 & 224 \end{matrix}
 \end{aligned}$$

We have $a = f = k = p = u = 1$, $b = c = 7$, $d = 21$, and $e = 28$ by using Theorem 5.2.4 and property (e) of the Fischer-Clifford matrix $M(g)$ (both found in [14]). Thus we obtain the following form:

$M(2A)$

$$\begin{aligned}
 & \begin{matrix} 21504 & 3072 & 3072 & 1024 & 768 \\ 336 & & & & \\ 48 & & & & \\ 48 & & & & \\ 16 & & & & \\ 12 & & & & \end{matrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & g & l & q & v \\ 7 & h & m & r & w \\ 21 & i & n & s & x \\ 28 & j & o & t & y \\ 8 & 56 & 56 & 168 & 224 \end{pmatrix} \quad (7)
 \end{aligned}$$

By the orthogonality relations for columns and rows and remaining properties of the matrix $M(g)$ found in Chapter 5

of [14], we obtain the desired Fischer-Clifford matrix $M(2A)$ of $\overline{G}$ given below:

$$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}. \quad (8)$$

For each class representative $g \in L_3(4) : 2$, we construct a Fischer-Clifford matrix $M(g)$. These are listed in Table 6.

7. Character Table of $2^9:(L_3(4):2)$

Having obtained the Fischer-Clifford matrices, the fusion maps of the H_i 's into $L_3(4) : 2$, and the character tables of the inertia factors H_i , we construct the character table

of $2^9 : (L_3(4) : 2)$ following the methodology discussed in Section 5.2 of [14]. For example, we calculate the partial character table of $2^9 : (L_3(4) : 2)$ corresponding to the coset of $2A \in L_3(4) : 2$. From the Fischer-Clifford matrix $M(2A)$ we obtain that

$$\begin{aligned} M_1(2A) &= (1 \ 1 \ 1 \ 1 \ 1), \\ M_2(2A) &= (7 \ -1 \ -5 \ 3 \ -1), \\ M_3(2A) &= \begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix} \end{aligned} \quad (9)$$

$$\text{and } M_4(2A) = (28 \ -4 \ -4 \ -4 \ 4).$$

Let $C_1(2A)$, $C_2(2A)$, $C_3(2A)$, and $C_4(2A)$ be the partial character tables of the inertia factors for the classes which fuse to $2A \in L_3(4) : 2$. Then the partial character table of $2^9 : (L_3(4) : 2)$ on the classes $\{2D, 2E, 4A, 4B, 4C\}$ is given by

$$C_1(2A)M_1(2A) = \begin{pmatrix} 1 \\ -1 \\ -6 \\ 6 \\ -7 \\ 7 \\ -3 \\ -3 \\ 3 \\ 3 \\ 8 \\ -8 \\ 0 \\ 0 \end{pmatrix} (1 \ 1 \ 1 \ 1 \ 1)$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -6 & -6 & -6 & -6 & -6 \\ 6 & 6 & 6 & 6 & 6 \\ -7 & -7 & -7 & -7 & -7 \\ 7 & 7 & 7 & 7 & 7 \\ -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 8 & 8 & 8 & 8 & 8 \\ -8 & -8 & -8 & -8 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_2(2A)M_2(2A) = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \\ -1 \\ 1 \\ 0 \\ -3 \\ -3 \\ 3 \\ 3 \\ 0 \end{pmatrix} (7 \ -1 \ -5 \ 3 \ -1)$$

$$= \begin{pmatrix} 7 & -1 & -5 & 3 & -1 \\ -7 & 1 & 5 & -3 & 1 \\ 14 & -2 & -10 & 6 & -2 \\ -14 & 2 & 10 & -6 & 2 \\ -7 & 1 & 5 & -3 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ -21 & 3 & 15 & -9 & 3 \\ -21 & 3 & 15 & -9 & 3 \\ 21 & -3 & -15 & 9 & -3 \\ 21 & -3 & -15 & 9 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_3(2A)M_3(2A)$$

$$= \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ 3 & -1 \\ -3 & -1 \\ 3 & 1 \\ -3 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix}$$

$$\begin{aligned}
&= \begin{pmatrix} 28 & 4 & 8 & 0 & -4 \\ -28 & -4 & -8 & 0 & 4 \\ -14 & 10 & -10 & -2 & 2 \\ 14 & -10 & 10 & 2 & -2 \\ -14 & -14 & 2 & 2 & 2 \\ 14 & 14 & -2 & -2 & -2 \\ 0 & 24 & -12 & -4 & 0 \\ -42 & -18 & -6 & 2 & 6 \\ 42 & 18 & 6 & -2 & -6 \\ 0 & -24 & 12 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -42 & 6 & -18 & -2 & 6 \\ 42 & -6 & 18 & 2 & -6 \end{pmatrix} \\
&C_4(2A)M_4(2A) = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \end{pmatrix} (28 \ -4 \ -4 \ -4 \ 4) \\
&= \begin{pmatrix} 28 & -4 & -4 & -4 & 4 \\ -28 & 4 & 4 & 4 & -4 \\ -28 & 4 & 4 & 4 & -4 \\ 28 & -4 & -4 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -56 & 8 & 8 & 8 & -8 \\ 56 & -8 & -8 & -8 & 8 \end{pmatrix}
\end{aligned} \tag{10}$$

Similarly, the partial character table associated with each coset Ng is computed. If necessary, we will restrict some characters of $\text{Irr}(U_6(2) : 2)$ to $\bar{G}$, to ensure that each partial character table corresponding to a coset Ng will give rise to the desired set $\text{Irr}(\bar{G})$.

The character table of $\bar{G}$ will be partitioned row-wise into 4 blocks $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 where each block corresponds to an inertia group $\bar{H}_i = 2^9 : H_i$. Therefore $\text{Irr}(2^9 : (L_3(4) : 2)) = \bigcup_{i=1}^4 \Delta_i$, where $\Delta_1 = \{\chi_j \mid 1 \leq j \leq 14\}$, $\Delta_2 = \{\chi_j \mid 15 \leq j \leq 26\}$, $\Delta_3 = \{\chi_j \mid 27 \leq j \leq 40\}$, and $\Delta_4 = \{\chi_j \mid 41 \leq$

TABLE 5: The fusion of H_4 into $L_3(4) : 2$.

$[h]_{H_4} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_4} \longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_4} \longrightarrow$	$[g]_{L_3(4):2}$
1A	1A	3A	3A	6A	6A
2A	2B	4A	4A	8A	8A
2B	2A	4B	4B	8B	8A

$j \leq 49\}$. The character table of $2^9 : (L_3(4) : 2)$ is shown in Table 7. The consistency and accuracy of the character table of $2^9 : (L_3(4) : 2)$ have been tested by using the GAP code labelled as Programme E in [18].

The information about the conjugacy classes found in Table 2 can be used to compute the power maps for the elements of $\bar{G}$ and then with the aid of Programme E in [18] we can verify that we obtained the unique p -power maps listed in Table 8 for our Table 7.

8. The Fusion of $2^9 : (L_3(4) : 2)$ into $U_6(2) : 2$

Since $\bar{G}$ is a maximal subgroup of $U_6(2) : 2$ of index 891, then the action of $U_6(2) : 2$ on the cosets of $\bar{G}$ gives rise to a permutation character $\chi(U_6(2) : 2 \mid \bar{G})$ of degree 891. We deduce from the character table of $U_6(2) : 2$ found in GAP that $\chi(U_6(2) : 2 \mid \bar{G}) = 1a + 22a + 252a + 616a$, where $1a$, $22a$, $252a$, and $616a$ are irreducible characters of $U_6(2) : 2$ of degrees 1, 22, 252, and 616, respectively.

We are able to obtain the partial fusion of $\bar{G}$ into $U_6(2) : 2$, using the information provided by the values of $\chi(U_6(2) : 2 \mid \bar{G})$ on the classes of $\bar{G}$ and the power maps of $\bar{G}$ and $U_6(2) : 2$. Then, the technique of set intersections for characters (see [9, 14, 19]) is applied to restrict some ordinary irreducible characters of $U_6(2) : 2$ of small degrees to $\bar{G}$, to determine fully the fusion of the classes of $\bar{G}$ into $U_6(2) : 2$.

Let ζ be the character afforded by the regular representation of $L_3(4) : 2$. We obtain that $\zeta = \sum_{i=1}^{14} \alpha_i \Phi_i$, where $\Phi_i \in \text{Irr}(L_3(4) : 2)$ and $\alpha_i = \deg(\Phi_i)$. Then ζ can be regarded as a character of $2^9 : (L_3(4) : 2)$ which contains 2^9 in its kernel such that

$$\zeta(x) = \begin{cases} |L_3(4) : 2| & \text{if } x \in 2^9 \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

If ϕ is a character of $U_6(2) : 2$ then we have that

$$\begin{aligned}
\langle \zeta, \phi \rangle_{\bar{G}} &= \frac{1}{|2^9 : (L_3(4) : 2)|} \{ \zeta(1A) \phi(1A) \\
&+ 21\zeta(2A) \phi(2A) + 210\zeta(2B) \phi(2B) + 280\zeta(2C) \\
&\cdot \phi(2C) \} = \frac{1}{|2^9 : (L_3(4) : 2)|} \{ |L_3(4) : 2| \\
&\cdot (\phi(1A) + 21\phi(2A) + 210\phi(2B) + 280\phi(2C)) \} \\
&= \frac{1}{512} \{ \phi(1A) + 21\phi(2A) + 210\phi(2B) \\
&+ 280\phi(2C) \} = \langle \phi_{2^9}, 1_{2^9} \rangle.
\end{aligned} \tag{12}$$

TABLE 6: The Fischer-Clifford matrices of $2^9 : (L_3(4) : 2)$.

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 21 & -11 & 5 & -3 \\ 210 & 50 & 2 & -6 \\ 280 & -40 & -8 & 8 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & 0 & -2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & -2 \\ 4 & -4 & 4 & -4 & 0 & 2 & -2 & 0 \\ 4 & 4 & 4 & 4 & -4 & 0 & 0 & 0 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(7A) = (1)$
$M(7B) = (1)$	$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(14A) = (1)$	$M(14B) = (1)$

Here 1_{2^9} is the identity character of 2^9 and ϕ_{2^9} is the restriction of ϕ to 2^9 . We obtain that

$$\phi_{2^9} = a_1\theta_1 + a_2\theta_2 + a_3\theta_3 + a_4\theta_4, \quad (13)$$

where $a_i \in \mathbb{N} \cup \{0\}$ and θ_i are the sums of the irreducible characters of 2^9 which are in the same orbit under the action

of $L_3(4) : 2$ on $\text{Irr}(2^9)$, for $i \in \{1, 2, 3, 4\}$. Let $\varphi_j \in \text{Irr}(2^9)$, where $j \in \{1, 2, 3, \dots, 512\}$. Then we obtain that

$$\theta_1 = \varphi_1, \quad \deg(\theta_1) = 1$$

$$\theta_2 = \sum_{j=2}^{22} \varphi_j, \quad \deg(\theta_2) = 21$$

TABLE 7: The character table of $2^9 : (L_3(4) : 2)$.

	1A			2A				2B				
	1A	2A	2B	2C	2D	2E	4A	4B	4C	2F	2G	4D
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	1	1	1
χ_3	20	20	20	20	-6	-6	-6	-6	-6	4	4	4
χ_4	20	20	20	20	6	6	6	6	6	4	4	4
χ_5	35	35	35	35	-7	-7	-7	-7	-7	3	3	3
χ_6	35	35	35	35	7	7	7	7	7	3	3	3
χ_7	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_8	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_9	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{10}	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{11}	64	64	64	64	8	8	8	8	8	0	0	0
χ_{12}	64	64	64	64	-8	-8	-8	-8	-8	0	0	0
χ_{13}	70	70	70	70	0	0	0	0	0	6	6	6
χ_{14}	126	126	126	126	0	0	0	0	0	-2	-2	-2
χ_{15}	21	-11	5	-3	7	-1	-5	3	-1	5	-3	5
χ_{16}	21	-11	5	-3	-7	1	5	-3	1	5	-3	5
χ_{17}	84	-44	20	-12	14	-2	-10	6	-2	4	4	4
χ_{18}	84	-44	20	-12	-14	2	10	-6	2	4	4	4
χ_{19}	105	-55	25	-15	-7	1	5	-3	1	9	1	9
χ_{20}	105	-55	25	-15	7	-1	-5	3	-1	9	1	9
χ_{21}	126	-66	30	-18	0	0	0	0	0	-2	14	-2
χ_{22}	315	-165	75	-45	-21	3	15	-9	3	11	-13	11
χ_{23}	315	-165	75	-45	-21	3	15	-9	3	-5	3	-5
χ_{24}	315	-165	75	-45	21	-3	-15	9	-3	11	-13	11
χ_{25}	315	-165	75	-45	21	-3	-15	9	-3	-5	3	-5
χ_{26}	630	-330	150	-90	0	0	0	0	0	-10	6	-10
χ_{27}	210	50	2	-6	28	4	8	0	-4	18	10	2
χ_{28}	210	50	2	-6	-28	-4	-8	0	4	18	10	2
χ_{29}	210	50	2	-6	-14	10	-10	-2	2	2	-6	18
χ_{30}	210	50	2	-6	14	-10	10	2	2	2	-6	18
χ_{31}	420	100	4	-12	-14	-14	2	2	-2	20	4	20
χ_{32}	420	100	4	-12	14	14	-2	-2	-2	20	4	20
χ_{33}	630	150	6	-18	0	24	-12	-4	0	-10	-2	6
χ_{34}	630	150	6	-18	-42	-18	-6	2	6	6	14	-10
χ_{35}	630	150	6	-18	42	18	6	-2	-6	6	14	-10
χ_{36}	630	150	6	-18	0	-24	12	4	0	-10	-2	6
χ_{37}	1260	300	12	-36	0	0	0	0	0	-20	-4	12
χ_{38}	1260	300	12	-36	0	0	0	0	0	12	28	-20
χ_{39}	1260	300	12	-36	-42	6	-18	-2	6	-4	-20	-4
χ_{40}	1260	300	12	-36	42	-6	18	2	-6	-4	-20	-4
χ_{41}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{42}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{43}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{44}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{45}	560	-80	-16	16	0	0	0	0	0	16	-16	-16
χ_{46}	560	-80	-16	16	0	0	0	0	0	-16	16	16

TABLE 7: Continued.

	2B				3A				4A			
	4E	4F	4G	4H	4I	3A	6A	6B	6C	4J	4K	8A
χ_{47}	560	-80	-16	16	0	0	0	0	0	-16	16	16
χ_{48}	2240	-320	-64	64	-56	8	8	8	-8	0	0	0
χ_{49}	2240	-320	-64	64	56	-8	-8	-8	8	0	0	0
	2B				3A				4A			
	4E	4F	4G	4H	4I	3A	6A	6B	6C	4J	4K	8A
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	4	4	4	4	4	2	2	2	2	0	0	0
χ_4	4	4	4	4	4	2	2	2	2	0	0	0
χ_5	3	3	3	3	3	-1	-1	-1	-1	3	3	3
χ_6	3	3	3	3	3	-1	-1	-1	-1	3	3	3
χ_7	-3	-3	-3	-3	-3	0	0	0	0	1	1	1
χ_8	-3	-3	-3	-3	-3	0	0	0	0	1	1	1
χ_9	-3	-3	-3	-3	-3	0	0	0	0	1	1	1
χ_{10}	-3	-3	-3	-3	-3	0	0	0	0	1	1	1
χ_{11}	0	0	0	0	0	1	1	1	1	0	0	0
χ_{12}	0	0	0	0	0	1	1	1	1	0	0	0
χ_{13}	6	6	6	6	6	-2	-2	-2	-2	-2	-2	-2
χ_{14}	-2	-2	-2	-2	-2	0	0	0	0	-2	-2	-2
χ_{15}	-3	1	-3	1	1	3	-3	1	-1	1	1	1
χ_{16}	-3	1	-3	1	1	3	-3	1	-1	1	1	1
χ_{17}	4	4	-4	-4	4	3	-3	1	-1	0	0	0
χ_{18}	4	4	-4	-4	4	3	-3	1	-1	0	0	0
χ_{19}	1	5	-7	-3	5	-3	3	-1	1	1	1	1
χ_{20}	1	5	-7	-3	5	-3	3	-1	1	1	1	1
χ_{21}	14	6	-2	-10	6	0	0	0	0	-2	-2	2
χ_{22}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	1
χ_{23}	3	-1	3	-1	-1	0	0	0	0	3	3	-3
χ_{24}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	1
χ_{25}	3	-1	3	-1	-1	0	0	0	0	3	3	-3
χ_{26}	6	-2	6	-2	-2	0	0	0	0	-2	-2	2
χ_{27}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	0
χ_{28}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	0
χ_{29}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	0
χ_{30}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	0
χ_{31}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0
χ_{32}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0
χ_{33}	14	10	-2	2	-6	0	0	0	0	-2	-2	0
χ_{34}	-2	10	-2	2	-6	0	0	0	0	2	2	0
χ_{35}	-2	10	-2	2	-6	0	0	0	0	2	2	0
χ_{36}	14	10	-2	2	-6	0	0	0	0	-2	-2	0
χ_{37}	28	-12	-4	4	4	0	0	0	0	4	4	0
χ_{38}	-4	-12	-4	4	4	0	0	0	0	-4	-4	0
χ_{39}	-20	4	4	-4	4	0	0	0	0	0	0	0
χ_{40}	-20	4	4	-4	4	0	0	0	0	0	0	0

TABLE 7: Continued.

	4B						4C				5A			
	4L	4M	8C	8D	4N	4O	8E	8F	8G	8H	5A	5A	10A	
χ_{41}	8	0	0	0	0	1	-1	-1	1	1	4	-4	0	
χ_{42}	8	0	0	0	0	1	-1	-1	1	1	4	-4	0	
χ_{43}	8	0	0	0	0	1	-1	-1	1	1	4	-4	0	
χ_{44}	8	0	0	0	0	1	-1	-1	1	1	4	-4	0	
χ_{45}	16	0	0	0	0	2	-2	-2	2	2	-8	8	0	
χ_{46}	-16	0	0	0	0	2	-2	-2	2	2	0	0	0	
χ_{47}	-16	0	0	0	0	2	-2	-2	2	2	0	0	0	
χ_{48}	0	0	0	0	0	-1	1	1	-1	-1	0	0	0	
χ_{49}	0	0	0	0	0	-1	1	1	-1	-1	0	0	0	
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	
χ_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	
χ_3	0	0	0	0	-2	-2	-2	-2	-2	-2	0	0	0	
χ_4	0	0	0	0	2	2	2	2	2	2	0	0	0	
χ_5	-1	-1	-1	-1	1	1	1	1	1	1	0	0	0	
χ_6	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	0	
χ_7	1	1	1	1	1	1	1	1	1	1	0	0	0	
χ_8	1	1	1	1	1	1	1	1	1	1	0	0	0	
χ_9	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	
χ_{10}	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	
χ_{11}	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	
χ_{12}	0	0	0	0	0	0	0	0	0	0	-1	-1	-1	
χ_{13}	2	2	2	2	0	0	0	0	0	0	0	0	0	
χ_{14}	-2	-2	-2	-2	0	0	0	0	0	0	1	1	1	
χ_{15}	1	1	1	-1	3	-1	1	-3	-1	1	1	1	-1	
χ_{16}	1	1	1	-1	-3	1	-1	3	1	-1	1	1	-1	
χ_{17}	0	0	0	0	-2	2	-2	-2	-2	2	-1	-1	1	
χ_{18}	0	0	0	0	-2	-2	2	2	2	-2	-1	-1	1	
χ_{19}	1	1	1	-1	1	-3	3	-1	1	-1	0	0	0	
χ_{20}	1	1	1	-1	-1	3	-3	1	-1	1	0	0	0	
χ_{21}	-2	-2	-2	2	0	0	0	0	0	0	1	1	-1	
χ_{22}	-1	-1	-1	1	-1	3	-3	1	-1	1	0	0	0	
χ_{23}	-1	-1	-1	1	3	-1	1	-3	-1	1	0	0	0	
χ_{24}	-1	-1	-1	1	1	-3	3	-1	1	-1	0	0	0	
χ_{25}	-1	-1	-1	1	-3	1	-1	3	1	-1	0	0	0	
χ_{26}	2	2	2	-2	0	0	0	0	0	0	0	0	0	
χ_{27}	2	2	-2	0	4	0	-2	2	0	-2	0	0	0	
χ_{28}	2	2	-2	0	-4	0	2	-2	0	2	0	0	0	
χ_{29}	-2	-2	2	0	-2	2	0	-4	2	0	0	0	0	
χ_{30}	-2	-2	2	0	2	-2	0	4	-2	0	0	0	0	
χ_{31}	0	0	0	0	-2	-2	2	2	-2	2	0	0	0	
χ_{32}	0	0	0	0	2	2	-2	-2	2	-2	0	0	0	
χ_{33}	2	2	-2	0	0	-4	-2	2	0	2	0	0	0	
χ_{34}	-2	-2	2	0	2	-2	-4	0	2	0	0	0	0	
χ_{35}	-2	-2	2	0	-2	2	4	0	-2	0	0	0	0	
χ_{36}	2	2	-2	0	0	4	2	-2	0	-2	0	0	0	

[illegible]

TABLE 7: Continued.

χ_{27}	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{30}	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{31}	1	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{32}	-1	-1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	1	-1	1	1	-1	0	0	2	-2	0	0	0	0	0	0	0
χ_{42}	-1	1	-1	-1	1	0	0	2	-2	0	0	0	0	0	0	0
χ_{43}	-1	1	-1	-1	1	0	0	-2	2	0	0	0	0	0	0	0
χ_{44}	1	-1	1	1	-1	0	0	-2	2	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	C	-C	0	0	0	0	0
χ_{48}	1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	-1	1	-1	-1	1	0	0	0	0	0	0	0	0	0	0	0

$A = (-1 - \sqrt{7}i)/2$, $B = (-1 + \sqrt{7}i)/2$, and $C = 2\sqrt{2}i$.

TABLE 8: The power maps of the elements of $2^9 : (L_3(4) : 2)$.

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7
1A	1A					2A	2D	1A			
	2A	1A					2E	1A			
	2B	1A					4A	2B			
	2C	1A					4B	2B			
							4C	2B			
2B	2F	1A				3A	3A		1A		
	2G	1A					6A	3A	2C		
	4D	2A					6B	3A	2A		
	4E	2A					6C	3A	2B		
	4F	2B									
	4G	2B									
	4H	2B									
	4I	2B									
4A	4J	2F				4B	4L	2F			
	4K	2F					4M	2F			
	8A	4E					8C	4E			
	8B	4F					8D	4I			
4C	4N	2F				5A	5A			1A	
	4O	2F					10A	5A		2A	
	8E	4F									
	8F	4F									
	8G	4D									
	8H	4F									
6A	6D	3A	2D			7A	7A			1A	
	6E	3A	2E								
	12A	6C	4C								
	12B	6C	4A								
7B	7B			1A	8A	8I	4J				
						8J	4J				
						16A	8A				
						16B	8A				
14A	14A	7A		2D	14B	14B	7B				2D

$$\theta_3 = \sum_{j=23}^{232} \varphi_j, \quad \deg(\theta_3) = 210$$

$$\theta_4 = \sum_{j=233}^{512} \varphi_j, \quad \deg(\theta_4) = 280. \quad (14)$$

Hence

$$\phi_{2^9} = a_1 \varphi_1 + a_2 \sum_{j=2}^{22} \varphi_j + a_3 \sum_{j=23}^{232} \varphi_j + a_4 \sum_{j=233}^{512} \varphi_j, \quad (15)$$

and therefore

$$\langle \phi_{2^9}, \phi_{2^9} \rangle = a_1^2 + 21a_2^2 + 210a_3^2 + 280a_4^2$$

$$= \frac{1}{512} \{ \phi(1A) \phi(1A) + 21\phi(2A) \phi(2A) + 210\phi(2B) \phi(2B) + 280\phi(2C) \phi(2C) \}, \quad (16)$$

where $a_1 = \langle \zeta, \phi \rangle_{2^9 : (L_3(4) : 2)}$.

We apply the above results to some of the irreducible characters of $U_6(2) : 2$ of small degrees, which in this case are $\phi_1 = 22a$, $\phi_2 = 22b$, $\phi_3 = 231a$, $\phi_4 = 231b$, $\phi_5 = 440a$, and $\phi_6 = 440b$. Their respective degrees are 22, 22, 231, 231, 440, and 440. For ϕ_1 we calculate that

$$\langle \zeta, \phi_1 \rangle_{2^9 : (L_3(4) : 2)} = \frac{1}{512} \{ 22 + 21(-10) + 210(6) + 280(-2) \} = 1. \quad (17)$$

Now $a_1 + 21a_2 + 210a_3 + 280a_4 = 22$, since $\deg \phi_1 = 22$. Since $a_1 = 1$, we must have that $a_2 = 1$ and $a_3 = a_4 = 0$. Note that $2^9 : (L_3(4) : 2)$ does not have irreducible characters of degree 22. We obtain that $(\phi_1)_{2^9 : (L_3(4) : 2)} = \chi_1 + \chi_{15}$ if the partial fusion of $2^9 : (L_3(4) : 2)$ into $U_6(2) : 2$ is taken into consideration. Similarly, for ϕ_3 and ϕ_5 we calculate that

$$\langle \zeta, \phi_3 \rangle_{2^9 : (L_3(4) : 2)} = \frac{1}{512} \{ 231 + 21(39) + 210(7) + 280(-9) \} = 0 \quad (18)$$

and

$$\langle \zeta, \phi_5 \rangle_{2^9 : (L_3(4) : 2)} = \frac{1}{512} \{ 440 + 21(120) + 210(24) + 280(8) \} = 20. \quad (19)$$

Since the respective degrees of ϕ_3 and ϕ_5 are 231 and 440, we have to solve the equations (i) $a_1 + 21a_2 + 210a_3 + 280a_4 = 231$ and (ii) $a_1 + 21a_2 + 210a_3 + 280a_4 = 440$, separately. If we are taking into account the fact that the set $\text{Irr}(\overline{G})$ (see Table 7) does not have any irreducible characters of degrees 231 and 440 and also that $\langle \zeta, \phi_3 \rangle_{2^9 : (L_3(4) : 2)} = 0$ and $\langle \zeta, \phi_5 \rangle_{2^9 : (L_3(4) : 2)} = 20$, we deduce that the two sets of values $\{a_1 = a_4 = 0, a_2 = a_3 = 1\}$ and $\{a_1 = 20, a_2 = 10, a_3 = 1, a_4 = 0\}$ are the only possibilities that satisfy equation (i) and (ii), respectively, hence we obtained that $(\phi_3)_{2^9 : (L_3(4) : 2)} = \chi_{16} + \chi_{27}$ and $(\phi_5)_{2^9 : (L_3(4) : 2)} = \chi_4 + \chi_{32}$. Similar computations were carried out to restrict the characters ϕ_2 , ϕ_4 , and ϕ_6 to $\overline{G}$ and we found that $(\phi_2)_{2^9 : (L_3(4) : 2)} = \chi_2 + \chi_{16}$, $(\phi_4)_{2^9 : (L_3(4) : 2)} = \chi_{15} + \chi_{28}$, and $(\phi_6)_{2^9 : (L_3(4) : 2)} = \chi_3 + \chi_{31}$.

By making use of the values of ϕ_1 , ϕ_2 , ϕ_3 , ϕ_4 , ϕ_5 , and ϕ_6 on the classes of $U_6(2) : 2$ and the values of $(\phi_1)_{2^9 : (L_3(4) : 2)}$, $(\phi_2)_{2^9 : (L_3(4) : 2)}$, $(\phi_3)_{2^9 : (L_3(4) : 2)}$, $(\phi_4)_{2^9 : (L_3(4) : 2)}$, $(\phi_5)_{2^9 : (L_3(4) : 2)}$, and $(\phi_6)_{2^9 : (L_3(4) : 2)}$ on the classes of $2^9 : (L_3(4) : 2)$ together with the partial fusion, the complete fusion map of $2^9 : (L_3(4) : 2)$ into $U_6(2) : 2$ is given in Table 9.

Data Availability

The data used to support the findings of this study are included within the article.

TABLE 9: The fusion of $2^9 : (L_3(4) : 2)$ into $U_6(2) : 2$.

$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)} \longrightarrow$	$[y]_{U_6(2):2}$	$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)} \longrightarrow$	$[y]_{U_6(2):2}$
1A	1A	1A	2A	2D	2D
	2A	2A		2E	2E
	2B	2B		4A	4G
	2C	2C		4B	4H
2B			3A	4C	4I
	2F	2B		3A	3C
	2G	2C		6A	6G
	4D	4A		6B	6E
	4E	4B		6C	6F
	4F	4C			
	4G	4E			
	4H	4F			
4A	4I	4D	4B		
	4J	4C		4L	4D
	4K	4F		4M	4F
	8A	8A		8C	8A
4B	8B	8B	5A	8D	8C
	4N	4H		5A	5A
	4O	4I		10A	10A
	8E	8F			
	8F	8G			
	8G	8D			
6A	8H	8E	7A		
	6D	6K		7A	7A
	6E	6L			
	12A	12K			
7B	12B	12J	8A		
	7B	7A		8I	8E
14A				8J	8F
				16A	16A
				16B	16B
	14A	14A	14B	14B	14A

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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