Research Article

Pulse Propagation in Presence of Polarization Mode Dispersion and Chromatic Dispersion in Single Mode Fibers

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The presence of (first and second orders) polarization mode dispersion (PMD), chromatic dispersion, and initial chirp makes effects on the propagated pulses in single mode fiber. Nowadays, there is not an accurate mathematical formula that describes the pulse shape in the presence of these effects. In this work, a theoretical study is introduced to derive a generalized formula. This formula is exactly approached to mathematical relations used in their special cases. The presence of second-order PMD (SOPMD) will not affect the orthogonality property between the principal states of polarization. The simulation results explain that the interaction of the SOPMD components with the conventional effects (chromatic dispersion and chirp) will cause a broadening/narrowing and shape distortion. This changes depend on the specified values of SOPMD components as well as the present conventional parameters.

1. Introduction

As a pulse propagates through a light-wave transmission system with a polarization mode dispersion (PMD), the pulse is split into a fast and slow one and therefore becomes broadened. This kind of PMD is commonly known as first-order PMD (FOPMD). Under FOPMD, a pulse at the input of a fiber can be decomposed into two pulses with orthogonal states of polarization (SOP). Both pulses will arrive at the output of the fiber undistorted and polarized along different SOPs, the output SOPs being orthogonal [1]. Both the PSPs and the differential group delay (DGD) are assumed to be frequency independent when only first-order PMD is being considered [2].

PMD also has higher-order components. Second-order PMD (SOPMD) is formed due to dependence of DGD on wavelength, because of that PMD is random variable. Statistical nature of PMD makes compensation very difficult [3]. Statistical properties of first- and second-order PMD have been reported in [4–6]. In these approaches, mean DGD plays a central role. Further, analytical calculations of pulse broadening and PMD compensation have also been

reported [7, 8]. However, these analytical results make use of autocorrelation of the PMD vector [4].

Also, it is shown that FOPMD and SOPMD vectors are statistically dependent and tend to be mutually perpendicular. It is important to calculate the probability that a certain value of SOPMD happens, when the value of DGD is known in a transmission line [8, 9]. The frequency dependence of the DGD causes polarization-dependent chromatic dispersion (PCD), resulting in polarization-dependent pulse compression and broadening. While PCD is a simple mechanism, it is a relatively minor component of the SOPMD vector since the frequency dependence of PMD vector has a statistical tendency to point away from the PSP. The dominant impairment mechanism is due to the depolarization, which causes distortion of the pulse shape. Statistical analyses and numerical simulations indicate a strong interaction between CD and the SOPMD in general [10–13].

Chirp in the transmitted signal has a significant impact on second-order impairments. Impairments were found to be dependent on chirp and CD and to be highly dependent on launch SOP. Impairments due to SOPMD and higher-order PMD occur for larger signal bandwidth, particularly when

these PMD components combine with chromatic dispersion or signal chirp [1].

The chromatic dispersion and SOPMD can have a significant influence on pulse shape. Whereas FOPMD deals with two pulses delayed relative to each other, pulse distortions due to SOPMD can be visualized in terms of the interplay of six different pulse replicas. These mechanisms result in pulse overshoots and the generation of satellite pulses [2, 14].

Section 1 presents introduction of CD, FOPMD, SOPMD, and chirp effects. Section 2 discusses theoretical treatments prevalent in the area. Section 3 describes the suggested theory to find the reconstructed pulses under the effects of these effects. Section 4 gives special results, while Section 5 presents the results and discussion. Finally in Section 6, conclusions of the work done have been drawn.

2. Statement of the Problem

Consider a generic input field $\vec{A}_{in}(T) = A_{in}(T)|s\rangle$, where $A_{in}(T)$ represents the field complex amplitude and $|s\rangle$ represents the input Jones polarization vector, which depends on the azimuth θ and the ellipticity η of the input SOP. Mathematically, the SOP is defined as [15]

$$|s\rangle = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \eta - i \sin \theta \sin \eta \\ \cos \theta \sin \eta + i \cos \theta \cos \eta \end{bmatrix}. \quad (1)$$

The SOP and the pulse shape of the propagated pulses are affected by the FOPMD. The SOPMD induces other changes at the pulse properties. However, the SOPMD contains the PCD that may be taken as an additional CD. The origin CD is influenced by the chirp. These effects will be studied simultaneously in this paper to explain an accurate expressions of the reconstructed pulses.

The frequency response of a single mode fiber with CD and PMD has the form [2, 14]

$$H(w) = T_{PMD}(w) \exp \left[\frac{i\beta_2 L w^2}{2} \right], \quad (2)$$

where β_2 is the fiber's GVD parameter, L is the fiber length, and β_2 is the carrier frequency. The matrix $T_{PMD}(w)$ is responsible for PMD effects, which is a 2×2 unitary matrix that is defined as [16]

$$T_{PMD} = \exp \left[-\frac{iw}{2} \vec{\tau}(w) \cdot \vec{\sigma} \right], \quad (3)$$

where $\vec{\tau}(w)$ is the PMD vector, $|\vec{\tau}(w)|$ is the DGD between the PSPs, and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli spin matrices which can be defined as [1]

$$\begin{aligned} \sigma_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & \sigma_1 &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \\ \sigma_2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, & \sigma_3 &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \end{aligned} \quad (4)$$

Here $\vec{\tau} \cdot \vec{\sigma} = \tau_1 \sigma_1 + \tau_2 \sigma_2 + \tau_3 \sigma_3$ is a 2×2 matrix. Now, the output field is given by

$$\begin{aligned} \vec{A}_{out}(w) &= \exp \left[\frac{i\beta_2 L w^2}{2} \right] A_{in}(w) T_{PMD}(w) |s\rangle \\ &= A_{out}(w) |t(w)\rangle, \end{aligned} \quad (5)$$

where $A_{in}(w)$ is the Fourier transform of $A_{in}(T)$ and $|t(w)\rangle = T_{PMD}(w)|s\rangle$ is the output SOP. Note that all the fiber birefringence effects are included in the frequency-dependent vector $|t(w)\rangle$, while the CD effects and pulse characteristics are accounted for inside the output field $A_{out}(w)$.

In the three-dimensional Poincare representation, PMD is described by the dispersion vector $\vec{\tau} = \Delta\tau \hat{\tau}$, where $\Delta\tau = |\vec{\tau}|$ and $\hat{\tau}$ is the direction of one of the two orthogonal PSP's of $T_{PMD}(w)$. In a first-order approximation both $\Delta\tau$ and $\hat{\tau}$ are frequency independent; that is, $\vec{\tau} = \Delta\tau_0 \hat{\tau}_0$, where $\Delta\tau_0$ is the DGD between two input signals polarized along the two PSP's of the fiber. The Taylor series of $\vec{\tau}$ vector up to second order is defined as [12, 13]

$$\vec{\tau} = \Delta\tau_0 \hat{\tau}_0 + (\Delta\tau' \hat{\tau}_0 + \Delta\tau_0 \hat{\tau}_w) w, \quad (6)$$

where $\hat{\tau}_0 = \hat{\tau}|_{w=w_0}$, $\Delta\tau_0 = \Delta\tau|_{w=w_0}$, $\Delta\tau' = \partial\Delta\tau/\partial w|_{w=w_0}$, $\hat{\tau}_w = \partial\hat{\tau}/\partial w|_{w=w_0}$, w_0 is the central frequency, and w is the respective deviation to the central frequency w_0 .

It is important to note that $\hat{\tau}_w$ is not unit vector and the vectors $\hat{\tau}_0$ and $\hat{\tau}_w$ are orthogonal in Stokes space. The value of $\Delta\tau'$ is responsible for PCD, while the factor $\hat{\tau}_w$ represents the depolarization.

3. Theoretical Representation

In this section, we will explain the effects of PMD, chromatic dispersion, and initial chirp on the degree of polarization, state of polarization, and pulse shape. Substituting (6) into (3) and using the notation $\hat{\tau}_w = 2k\hat{m}$, one may obtain

$$T_{PMD} = \exp \left[-\frac{i}{2} (\varphi_1 \hat{\tau}_0 + \varphi_2 \hat{m}) \cdot \vec{\sigma} \right], \quad (7)$$

where $\varphi_1 = w(\Delta\tau_0 + \Delta\tau' w)$ and $\varphi_2 = w^2 \Delta\tau_0 2k$. Under the assumption that the combined effect of PMD may be represented as a concatenation of FOPMD and SOPMD processes, (7) may be rewritten as follows:

$$T_{PMD} = (C_1 \sigma_0 - i \hat{\tau}_0 \cdot \vec{\sigma} S_1) (C_2 \sigma_0 - i \hat{m} \cdot \vec{\sigma} S_2), \quad (8)$$

where we have used the abbreviations C_1 , C_2 , S_1 , and S_2 to represent $\cos(\varphi_1/2)$, $\cos(\varphi_2/2)$, $\sin(\varphi_1/2)$, and $\sin(\varphi_2/2)$, respectively. Using the property $\hat{\tau}_0 \perp \hat{m}$ and the identity $(\hat{a} \cdot \vec{\sigma})(\hat{b} \cdot \vec{\sigma}) = \hat{a} \cdot \hat{b} + i \hat{a} \times \hat{b} \cdot \vec{\sigma}$, (8) will be

$$T_{PMD} = C_1 C_2 \sigma_0 - i (S_1 C_2 \hat{\tau}_0 + S_2 C_1 \hat{m} + S_1 S_2 \hat{n}) \cdot \vec{\sigma}, \quad (9)$$

where $\hat{n} = \hat{\tau}_0 \times \hat{m}$. Note that the vectors $\hat{\tau}_0$, $\hat{m}$, and $\hat{n}$ constituted the orthogonal basis of Stokes space. However, to advance analysis, let

$$\vec{Q} = \begin{bmatrix} S_1 C_2 \tau_1 + S_2 C_1 m_1 + S_1 S_2 n_1 \\ S_1 C_2 \tau_2 + S_2 C_1 m_2 + S_1 S_2 n_2 \\ S_1 C_2 \tau_3 + S_2 C_1 m_3 + S_1 S_2 n_3 \end{bmatrix}, \quad (10)$$

such that (9) will be

$$T_{\text{PMD}} = C_1 C_2 \sigma_0 - i \vec{Q} \cdot \vec{\sigma}. \quad (11)$$

Using (11), it may be found that

$$T_{\text{PMD}}^\dagger T_{\text{PMD}} = C_1^2 C_2^2 \sigma_0 + |\vec{Q}|^2 \sigma_0. \quad (12)$$

Also, using (10), it is easy to get

$$|\vec{Q}|^2 = S_1^2 C_2^2 + S_2^2 C_1^2 + S_3^2 S_2^2. \quad (13)$$

Substituting (13) into (12) will make $T_{\text{PMD}}^\dagger T_{\text{PMD}} = \sigma_0$. The eigenvalues of the matrix T_{PMD} are

$$\lambda_{1,2} = C_1 C_2 \mp \sqrt{C_1^2 C_2^2 - 1}. \quad (14)$$

In presence of PMD, we have $C_1^2 C_2^2 < 1$, such that the eigenvalues are different and complex and their product is $\lambda_1 \lambda_2 = 1$. Using these facts and the analysis presented in [16], we concluded a very important conclusion, which is the principle states of polarization (PSPs) vectors stay orthogonal but the orientation of antiparallel PSP's vector may be changed. That is, the orthogonality property will not be affected by SOPMD.

Return back to (11), which can be written as

$$T_{\text{PMD}} = \begin{bmatrix} C_1 C_2 - i q_1 & -i(q_2 - i q_3) \\ -i(q_2 + i q_3) & C_1 C_2 + i q_1 \end{bmatrix}. \quad (15)$$

Using (11) and the definition of $|s\rangle$ from (1), the output SOP may be

$$|t(w)\rangle = T_{\text{PMD}} |s\rangle = \begin{bmatrix} a(C_1 C_2 - i q_1) - i b(q_2 - i q_3) \\ -i a(q_2 + i q_3) + a(C_1 C_2 + i q_1) \end{bmatrix}. \quad (16)$$

Using (10) into (16) yields

$$|t(w)\rangle = C_1 C_2 \begin{bmatrix} a \\ b \end{bmatrix} + S_1 C_2 \begin{bmatrix} \theta_1 \\ \theta_4 \end{bmatrix} + S_2 C_1 \begin{bmatrix} \theta_2 \\ \theta_5 \end{bmatrix} + S_1 S_2 \begin{bmatrix} \theta_3 \\ \theta_6 \end{bmatrix}, \quad (17)$$

where

$$\begin{aligned} \begin{bmatrix} \theta_1 \\ \theta_4 \end{bmatrix} &= -i \hat{\tau}_0 \cdot \vec{\sigma} |s\rangle, \\ \begin{bmatrix} \theta_2 \\ \theta_5 \end{bmatrix} &= -i \hat{m} \cdot \vec{\sigma} |s\rangle, \\ \begin{bmatrix} \theta_3 \\ \theta_6 \end{bmatrix} &= -i \hat{n} \cdot \vec{\sigma} |s\rangle. \end{aligned} \quad (18)$$

In order to explain the effects of PMD on the signal DOP, it is interesting to rewrite (16) as follows:

$$|t(w)\rangle = \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} u_1 a + u_2 b \\ -u_2^* a + u_1^* b \end{bmatrix}, \quad (19)$$

where $u_1 = C_1 C_2 - i q_1$ and $u_2 = -i(q_2 - i q_3)$.

The corresponding Stokes vector is

$$\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} |t_x|^2 - |t_y|^2 \\ t_x t_y^* + t_y t_x^* \\ i(t_x t_y^* - t_y t_x^*) \end{bmatrix}. \quad (20)$$

Since different frequency components of the input broadband signal will undergo different polarizations evaluation when they propagate through the fiber, the signal DOP will degrade. Consequently, the weighted average of the Stokes vector is necessary, which is calculated by multiplication the normalized spectral intensity function $I = |A_{\text{in}}(w)|^2$ and integrating over w . The DOP will be [14]

$$\text{DOP} = \frac{\sqrt{|\int_{-\infty}^{\infty} t_1 I dw|^2 + |\int_{-\infty}^{\infty} t_2 I dw|^2 + |\int_{-\infty}^{\infty} t_3 I dw|^2}}{\int_{-\infty}^{\infty} I dw}. \quad (21)$$

Note that the parameters t_1 , t_2 , and t_3 are functions of $\Delta\tau_0$, $\Delta\tau'$, and k . That is, the signal DOP is a function of FOPMD and SOPMD. The integrations in (21) are very hard to solve analytically because the parameters t_1 , t_2 , and t_3 are complicated functions of frequency. However, determining the analytic form of the signal DOP is beyond this paper.

Now, the trigonometric functions in (17) will be simplified as follows:

$$C_1 C_2 = \frac{1}{4} [\psi_1 + \psi_1^* + \psi_2 + \psi_2^*], \quad (22a)$$

$$S_1 C_2 = \frac{-i}{4} [\psi_1 - \psi_1^* + \psi_2 - \psi_2^*], \quad (22b)$$

$$S_2 C_1 = \frac{-i}{4} [\psi_1 - \psi_1^* - \psi_2 + \psi_2^*], \quad (22c)$$

$$S_1 S_2 = \frac{1}{4} [-\psi_1 - \psi_1^* + \psi_2 + \psi_2^*], \quad (22d)$$

where

$$\psi_q = \exp \left[\frac{i}{2} (\Delta\tau_0 w + \alpha_q w^2) \right], \quad q = 1, 2, \quad (23)$$

$\alpha_1 = \Delta\tau' + 2k\Delta\tau_0$, and $\alpha_2 = \Delta\tau' - 2k\Delta\tau_0$. Substituting (18) and (22a)–(22d) into (17) will get

$$|t(w)\rangle = \frac{1}{4} [\psi_1 |s_1\rangle + \psi_1^* |s_2\rangle + \psi_2 |s_3\rangle + \psi_2^* |s_4\rangle], \quad (24)$$

where

$$|s_1\rangle = [I + (i\hat{n} - (\hat{\tau} + \hat{m})) \cdot \vec{\sigma}] |s\rangle, \quad (25a)$$

$$|s_2\rangle = [I + (i\hat{n} + (\hat{\tau} + \hat{m})) \cdot \vec{\sigma}] |s\rangle, \quad (25b)$$

$$|s_3\rangle = [I + (-i\hat{n} - (\hat{\tau} - \hat{m})) \cdot \vec{\sigma}] |s\rangle, \quad (25c)$$

$$|s_4\rangle = [I + (-i\hat{n} + (\hat{\tau} - \hat{m})) \cdot \vec{\sigma}] |s\rangle. \quad (25d)$$

Assuming that the input pulse is a normalized Gaussian pulse, this is defined as [1]

$$A(T) = \exp\left(-\frac{1+iC}{2} \frac{T^2}{T_0^2}\right), \quad (26)$$

where C and T_0 are the initial chirp and pulse width. Substituting (24) and the Fourier transform of (26) into (5) yields

$$\vec{A}_{\text{out}}(L, \omega) = \frac{1}{4} [F_1^+ |s_1\rangle + F_1^- |s_2\rangle + F_2^+ |s_3\rangle + F_2^- |s_4\rangle], \quad (27)$$

where

$$F_q^\pm = \sqrt{\frac{2\pi T_0^2}{1+iC}} \exp\left[\frac{\pm i\omega\Delta\tau_0}{2} - \frac{(T_0^2 + \alpha_q^\pm C - i\alpha_q^\pm) \omega^2}{2(1+iC)}\right]. \quad (28)$$

Also

$$\bar{\alpha}_1 = \beta_2 L + \alpha_1 = \beta_2 L + (\Delta\tau' + 2k\Delta\tau_0), \quad (29a)$$

$$\bar{\alpha}_2 = \beta_2 L - \alpha_1 = \beta_2 L - (\Delta\tau' + 2k\Delta\tau_0), \quad (29b)$$

$$\bar{\alpha}_3 = \beta_2 L + \alpha_2 = \beta_2 L + (\Delta\tau' - 2k\Delta\tau_0), \quad (29c)$$

$$\bar{\alpha}_4 = \beta_2 L - \alpha_2 = \beta_2 L - (\Delta\tau' - 2k\Delta\tau_0). \quad (29d)$$

The inverse Fourier transform on (27) will obtain

$$\vec{A}_{\text{out}}(L, T) = \frac{1}{4} [A_1 |s_1\rangle + A_2 |s_2\rangle + A_3 |s_3\rangle + A_4 |s_4\rangle], \quad (30)$$

where

$$A_q = M_q \exp(i\varphi_q) \exp\left[-\frac{(T + (-1)^{q+1} \Delta\tau_0/2)^2}{2T_q^2}\right], \quad (31a)$$

$$T_q = T_0 \sqrt{1 + \frac{2\bar{\alpha}_q C}{T_0^2} + \frac{\bar{\alpha}_q^2 (C^2 + 1)}{T_0^4}}, \quad (31b)$$

$$\varphi_q = \frac{1}{2} \left[\tan^{-1} \left(\frac{\bar{\alpha}_q}{T_0^2 + \bar{\alpha}_q C} \right) - \beta \right], \quad (31c)$$

$$M_q = \sqrt{\frac{T_0}{T_q}}, \quad q = 1, 2, 3, 4, \quad (31d)$$

$$\beta = \frac{(T + (-1)^{q+1} \Delta\tau_0/2)^2 [CT_0^2 + \bar{\alpha}_q (C^2 + 1)]}{(T_0^2 + \bar{\alpha}_q C)^2 + \bar{\alpha}_q^2}. \quad (31e)$$

Here, T_q , φ_q , and M_q represent the reconstructed pulse width, generated phase, and amplitude, respectively. Using the vectors in (25a)–(25d) into (30) and conducting a routine

simplification, the output power equation takes the following formula:

$$\begin{aligned} P_{\text{out}}(L, T) = & \frac{1}{4} [|A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2] \\ & + \frac{\hat{\tau}_0 \cdot \hat{s}}{2} \text{Re}(A_2 A_4^* - A_1 A_3^*) \\ & + \frac{\hat{m} \cdot \hat{s}}{4} [|A_2|^2 + |A_3|^2 - |A_1|^2 - |A_4|^2] \\ & - \frac{\hat{n} \cdot \hat{s}}{2} \text{Im}(A_2 A_4^* - A_1 A_3^*), \end{aligned} \quad (32)$$

where Re and Im represent real and imaginary parts, respectively. Equation (32) represents the most important achievement in this paper as it contains the effects of CD, chirp, FOPMD, and SOPMD. It is clear from this equation that the second term results from the FOPMD effects and the third term results because of the SOPMD effects, while the fourth term results from the interaction between FOPMD and SOPMD. The first term has nothing to do with the SOP or PSPs, so it represents the CD effects in the absence of other effects. By the adoption of Table 1, (32) will be reduced to special formulas according to the following effects that accompany the propagated pulse.

4. Simplified Analytic Forms

In this section, we will address the mathematical analysis for special cases resulting from (32) to highlight theoretically the individual effects of the phenomena under study as well as to check the validity of the evidence contained in the previous section.

(a) *CD Only*. In this simple case, (32) will be reduced to

$$P_{\text{out}}(L, T) = \frac{T_0}{T_1} \exp \frac{-T^2}{T_1^2}, \quad (33)$$

where T_1 is computed from (31b) and Table 1. Here there is no PSPs because the CD has nothing to do with SOP, but this case is causing broadening or compressing of the output pulse depending on $\beta_2 C$ sign. Note that this formula is well known in the scientific researches.

(b) *FOPMD and CD*. In this case, (32) will be

$$P_{\text{out}}(L, T) = \frac{T_0}{T_1} \left[R_- \exp \frac{-T_-^2}{T_1^2} + R_+ \exp \frac{-T_+^2}{T_1^2} \right], \quad (34)$$

where $R_\pm = (1 \mp \hat{\tau}_0 \cdot \hat{s})/2$, T_1 is as in the previous case, and $T_\pm = T \pm \Delta\tau_0/2$. We note here that the PSP is exactly $\pm\hat{\tau}_0$, and the CD will be broadened or compressed each of the two orthogonal components. Neglecting the CD effects will make the orthogonal components not suffer from any

TABLE 1: Parameters specifications for different effects.

Case	Effects	Parameters specification
1	CD only	$\bar{\alpha}_1 = \bar{\alpha}_2 = \bar{\alpha}_3 = \bar{\alpha}_4 = \beta_2 L,$ $T_1 = T_2 = T_3 = T_4,$ $\hat{\tau}_0 = \hat{m} = \hat{n} = 0,$ $\Delta\tau_0 = \Delta\tau' = 2k = 0$
2	FOPMD and CD	$\bar{\alpha}_1 = \bar{\alpha}_2 = \bar{\alpha}_3 = \bar{\alpha}_4 = \beta_2 L,$ $T_1 = T_2 = T_3 = T_4,$ $\hat{\tau}_0 \neq 0, \hat{m} = \hat{n} = 0$
3	SOPMD and CD	$\bar{\alpha}_1 = -\bar{\alpha}_2 = \Delta\tau' + 2k\Delta\tau_0$ $\bar{\alpha}_3 = -\bar{\alpha}_4 = \Delta\tau' - 2k\Delta\tau_0$ $T_1 \neq T_2 \neq T_3 \neq T_4$ If $C \neq 0; T_1 = T_2; T_3 = T_4$ If $C = 0, \hat{\tau}_0 = \hat{n} = 0, \hat{m} \neq 0$
4	Orthogonal SOPMD and CD. $2k \neq 0,$ $\Delta\tau' = 0$	$\bar{\alpha}_1 = \bar{\alpha}_4 = \beta_2 L + 2k\Delta\tau_0,$ $\bar{\alpha}_2 = \bar{\alpha}_3 = \beta_2 L - 2k\Delta\tau_0,$ $T_1 = T_4; T_2 = T_3$ $\hat{\tau}_0 = \hat{n} = 0, \hat{m} \neq 0$
5	Parallel SOPMD and CD, $2k = 0,$ $\Delta\tau' \neq 0$	$\bar{\alpha}_1 = \bar{\alpha}_3 = \beta_2 L + \Delta\tau',$ $\bar{\alpha}_2 = \bar{\alpha}_4 = \beta_2 L - \Delta\tau',$ $T_1 = T_3; T_2 = T_4; \hat{\tau}_0 = \hat{n} = 0, \hat{m} \neq 0$
6	SOPMD and FOPMD (general)	$\bar{\alpha}_1 = -\bar{\alpha}_2 = \Delta\tau' + 2k\Delta\tau_0,$ $\bar{\alpha}_3 = -\bar{\alpha}_4 = \Delta\tau' - 2k\Delta\tau_0$ If $C \neq 0$ then $T_1 \neq T_2 \neq T_3 \neq T_4$ If $C = 0$ then $T_1 = T_2; T_3 = T_4,$ $\hat{\tau}_0 \perp \hat{m} \perp \hat{n} \neq 0$

change expect the shifting to right or left by $\Delta\tau_0/2$. However, the resulted pulse will be broadened by $\Delta\tau_0$.

(c) *SOPMD and CD*. Assuming that the SOPMD will be compensated, then

$$P_{\text{out}}(L, T) = R_- [|A_1|^2 + |A_4|^2] + R_+ [|A_2|^2 + |A_3|^2]. \quad (35)$$

Here $R_{\mp} = (1 \mp \hat{m} \cdot \hat{s})/2$. In this case, the PSPs are $\pm\hat{m}$, where there are two orthogonal components and each one contains a sum of two pulses that are shifted to right and left.

(d) *Orthogonal SOPMD and CD*. Suppose that $\Delta\tau' = 0$; thus

$$P_{\text{out}}(L, T) = T_0 \left[R_- \left[\exp \frac{-T_+^2}{T_1^2} + \exp \frac{-T_-^2}{T_1^2} \right] / T_1 + R_+ \left[\exp \frac{-T_+^2}{T_2^2} + \exp \frac{-T_-^2}{T_2^2} \right] / T_2 \right], \quad (36)$$

where $R_{\mp}$ is as in the previous case and $T_{1,2}$ are computed from (31b) and Table 1. The PSPs are to be $\pm\hat{m}$ here, where there are two orthogonal components and each one has different amplitudes and widths. This means that the orthogonal SOPMD will create the new PSPs, which are $\pm\hat{m}$.

(e) *Parallel SOPMD and CD*. Suppose that $2k = 0$; thus

$$P_{\text{out}}(L, T) = \frac{T_0}{2} \left[\frac{1}{T_1} \exp \frac{-T_+^2}{T_1^2} + \frac{1}{T_2} \exp \frac{-T_-^2}{T_2^2} \right], \quad (37)$$

where $T_{1,2}$ are computed from (31b) and Table 1. There is no relationship here with the SOP or the PSPs. This confirms that the changes will be to the pulse width, where each component will be broadened or compressed, and the result is equivalent to a polarization-dependent CD. This result is physically similar to that of case (a).

(f) *FOPMD and SOPMD*. This case involved general where all terms of (32) will be used in creating the resulting pulse. In this case, PSPs will not be $\pm\hat{\tau}_0$, but they are created from both $\pm\hat{\tau}_0$ and $\pm\hat{m}$; that is, the presence of SOPMD will cause a change in the fiber optic PSPs and this is what is called depolarization.

5. Results and Discussion

Equation (32) has been used for having simulated the behavior of propagated pulse that the input pulse has been considered as a normalized Gaussian one (one can see (23)). The orthogonal vectors $\hat{\tau}_0$, $\hat{m}$, and $\hat{n}$ are randomly generated. The direction of $\hat{s}$ vector for these orthogonal ones is important such that it will determine the output pulse shape. If $\hat{s}$ is parallel to $\hat{\tau}_0$, the effects of SOPMD will not appear, while the case that $\hat{s}$ is parallel to $\hat{m}$ will prevent the effects of FOPMD. But as $\hat{s}$ is parallel to $\hat{n}$, the individual effects of FOPMD and SOPMD will be discarded save the last term of (32) which represents the interaction between two kinds of PMD. Since these vectors are random ones,

FIGURE 1: The resulted pulses with $\Delta t_w = 0 \text{ ps}^2$ and $\beta_2 = 0 \text{ ps}^2/\text{km}$ for different values of chirp and k .

the pulse shape differ from simulation process to another one. To determine the actual behavior, it should calculate the statistical average. However, this leads to not clarifying individual effects for parameters in research. So, the pulse's behavior is proposed by using a specific simulation to point out the effects of the parameters β_2 , C , Δt_w , and k . We have not focused on the investigation of Δt_0 as FOPMD is a well-covered in the scientific literatures. Some parameters are considered constant within a simulation process as $L = 50 \text{ km}$, $T_0 = 10 \text{ ps}$, and $\Delta t_0 = 5 \text{ ps}$, and the others parameters β_2 , Δt_w , C , and k are changed within simulation. All figures are labeled by colors: blue, red, and green stand for the cases $k = 0, 5, 10 \text{ ps}$, respectively.

Figure 1 illustrates the output pulse at $\Delta t_w = 0$ and $\beta_2 = 0$ with various values of k and C . If $C = 0$, the effects are only concerned to attenuation that increases by increasing k . This is in a good agreement with applied physics because k stands for normal component of SOPMD. The effects of k are shown in the form of broadening/compression and distortion in output pulses by increasing C . The calculated variables in Figure 2 are the same as those in Figure 1 except that $\beta_2 = 1 \text{ ps}^2/\text{km}$. As is seen that pulses are tended to broaden because the chromatic dispersion. The amount of broadening will raise by increasing the chirp. At higher value of k , the SOPMD tends to balance chromatic dispersion effect. So, the output pulses are shown less broadening at a higher value of k . As the pulse with high values of k is affected by the parallel component of SOPMD and chromatic dispersion, then it tends to be distorted.

The calculated variables in Figure 3 are the same as those in Figure 1 except that $\Delta t_w = 30 \text{ ps}^2$. As is seen the action of the two components of SOPMD is given to rise broadening/compression and distortion which increase their rates with increasing C . For high values of k the pulses tend for broadening and vice versa. This is corresponding to applications as long the components of SOPMD are orthogonal. The broadening is attributed to the interaction

FIGURE 2: The resulted pulses with $\Delta t_w = 0 \text{ ps}^2$ and $\beta_2 = 1 \text{ ps}^2/\text{km}$ for different values of chirp and k .

FIGURE 3: The resulted pulses with $\Delta t_w = 30 \text{ ps}^2$ and $\beta_2 = 0 \text{ ps}^2/\text{km}$ for different values of chirp and k .

among C , Δt_w , and k . The calculated variables in Figure 4 are the same as those in Figure 3 except that $\beta_2 = 1 \text{ ps}^2/\text{km}$. Note that the effects become in contrast to these in Figure 3 because the presence of chromatic dispersion is about to rebalance among those parameters mentioned above so as to be projected with existing high values of C .

The results are implicitly similar with those in Figure 1 in [12], where the power pulses are exceeded "1" as PSPs are rotated at a high value of C . Overshoot in power may happen by increasing the SOPMD effects, especially, with higher values of C .

FIGURE 4: The resulted pulses with $\Delta t_w = 30 \text{ ps}^2$ and $\beta_2 = 1 \text{ ps}^2/\text{km}$ for different values of chirp and k .

6. Conclusions

The above equations and results illustrated the following conclusions. The short pulses are more affected by the initial chirp and vice versa. The SOPMD will add a new complexity that is presented by PCD and depolarization. The interaction between the FOPMD and SOPMD will cause a more degradation on the resulted pulses. The compensation of FOPMD will make the pulse under the affection of SOPMD only, which may destroy the pulse because of the presence of $\Delta\tau_w$ and k . That is, the omitting or compensation of FOPMD may set serious variations. The increasing amount of k will make a more distortion on the propagated signal, while the variation of $\Delta\tau_w$ presents a new type of CD.

The resulted pulses are affected, too, by $\beta_2 C$, $\beta_2 \Delta\tau'$, and $C\Delta\tau'$. The presence of CD only broadens/narrows the resulted pulses. The mixing of CD and PMD pictures will alter the known behavior, where the values of $\Delta\tau_w$ and k are very important which enhanced/complicated the certain problem. However, the problem is very small for $C = 0$, while the variations of pulse shape will be large for $C \neq 0$.

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