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Research Article

Analytical Model for Optical Scattering of Infrared Laser by Nonspherical Raindrops

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Analytic model was developed to investigate the interaction of infrared laser with nonspherical raindrops. The method based on Fraunhofer diffraction and geometrical optics was presented to obtain the analytic solution for single-scattering properties by approximate ellipsoid raindrops. Light scattering patterns were obtained for different drop sizes and shapes. Computational results demonstrate that the scattering of raindrops was contributed by both Fraunhofer diffraction and geometric scattering, and the results obtained from analytic formulas were consistent with the conclusions of using Monte Carlo ray tracing approach.

1. Introduction

Rainfall is one of the most common types of precipitation. As pulsed laser propagates in rain, the interaction of light with raindrops, such as absorption and scattering, will take place, so the received signal is attenuated and distorted which leads to the degradation of laser ranging performance. As a consequence, the rain attenuation greatly affects the application of laser detection in rain. In recent several decades, a large amount of researches about rain attenuation of infrared laser has been carried out [1–6], in which the scattering properties of raindrops play an important role. For the scattering of raindrops on laser beam, most previous researches were restricted to spherical raindrops [7–10]. For nonspherical raindrops, the electromagnetic scattering properties of ellipsoidal particles can be strictly solved by spheroidal wave function based on Mie theory. But the presentation of this spheroidal wave function is very complex, which brings many difficulties to the theoretical derivation and numerical calculation, having especially more difficulty handling the boundary conditions. Therefore, with the improvement of scattering calculation for nonspherical particles, researchers began to use point matching method [11], T-matrix approach [12, 13], ray tracing method [14–17], and other more accurate methods to study the problem of nonspherical particle scattering. Of these methods, the ray tracing method has been developed extensively in the last two decades for a huge variety of applications. However, the results obtained from ray tracing method were statistically counted by Monte Carlo approach and unable to get the analytical solution. Therefore, in the current study we are interested in using approximate ellipsoid substituting the actual raindrop to present the analytical formula for optical scattering of infrared laser by nonspherical raindrops based on Fraunhofer diffraction and geometric optics method.

2. Model of Nonspherical Raindrop Shape

It is well known that the shape of raindrops is important to the calculation of rain attenuation. The raindrop size is within 0.5~7 mm in diameter and generally not more than 5.5 mm, for the raindrop larger than 5.5 mm is unstable and will rupture. Following the photography, the raindrops less than 1 mm in radius are generally spherical, and as the drop size increases, they tend towards an oblate spheroid shape with a groove at the bottom and the rotation axis is almost vertical [18].

There is no analytical solution to compute the scattering of distorted raindrops, so the nonspherical raindrops are generally approximated as the area-equivalent ellipsoids [14, 19]. The vertical section of actual raindrop is symmetrical about x-axis and asymmetrical about y-axis. And the upper part of the section is larger than the lower half. Suppose the major axis of the elliptical section is a and minor axis is b. The elliptical section along the horizontal axis and vertical axis is completely symmetrical, so the minor axis b of approximate ellipsoid is half of the vertical axis value of the B-C model. The vertical section area of the B-C model drops can be calculated from the following:

$$\begin{aligned} A &= \frac{1}{2}\oint_C r^2(\theta)\,d\theta = a^2\int_0^{\pi}\left[1+\sum_{n=0}^{10}c_n\cos(n\theta)\right]^2 \\ &= \pi a^2\left[1+2c_0+c_0^2+\sum_{n=1}^{10}\frac{c_n^2}{2}\right]. \end{aligned} \tag{1}$$

Let the section area of the ellipsoid model equal B-C model, and then the major axis can be solved. Withal, the horizontal section of B-C model is circular, so another axis of ellipsoid $c = a$. Reference [14] gives the results of comparison between the approximate ellipsoid model and the actual drop shapes; that is, the approximate ellipsoid model is close to the actual raindrops. Therefore, the optical scattering of nonspherical raindrop is studied based on the approximate ellipsoid model.

3. Optical Scattering of Nonspherical Raindrop

3.1. Diffraction from Ellipsoid Raindrop. The diameter of raindrops is much longer than laser wavelength. The investigation indicates that when the particle size is much longer than laser wavelength, the scattering light mostly centralizes in the forward angle range, and the forward optical scattering can be calculated by the theory of Fraunhofer diffraction [20]. Therefore, the forward scattering of particles could be approximated by Fraunhofer diffraction.

The optical scattering of any three-dimensional particles can be regarded as being generated by the equiphase surface (i.e., the equiphase two-dimensional scattering section), and this equiphase surface can be as the feature face of the particle [21]. For the three-dimensional ellipsoid, we only need to study the cross section vertical with the incident light (i.e., elliptical surface). According to Babinet's principle in physics, in a certain area, the diffraction intensity of the raindrop can be obtained from Fraunhofer diffraction of an elliptical hole.

Suppose that the major and minor axis length of elliptical cross section are a and b, and $\beta = b/a$; when $\beta = 1$, it turns to be a spherical particle. Obviously, compared with the sphere, this model is more extensive. The radius of equivalent area $r = \sqrt{ab}$, size parameter $\alpha = kr$, wave number $k = 2\pi/\lambda$, and λ is the incident wavelength. In the plane $\varepsilon o \eta$, the transmittance function is

$$g(\varepsilon,\eta) = \begin{cases} 1 & \dfrac{\varepsilon^2}{b^2}+\dfrac{\eta^2}{a^2} < 1 \\ 0 & \text{others.} \end{cases} \tag{2}$$

Then in the Fourier plane $\varepsilon' o' \eta'$ (receiving plane), the diffraction field distribution yields to

$$u(\varepsilon',\eta') = c_0\iint_{\infty} g(\varepsilon,\eta)\exp\left[-ik(p\varepsilon+q\eta)\right]d\varepsilon\,d\eta, \tag{3}$$

where f_m is the distance between the center of particle and the receiving plane, and equations $p = \varepsilon'/f_m$ and $q = \eta'/f_m$ are obtained when the incident direction is positive. We establish the curvilinear coordinates in the receiving plane, and the relations between the point coordinates (ρ,φ) and (ε',η') are

$$\begin{aligned} \varepsilon' &= \rho\cos\varphi \\ \eta' &= \left(\frac{1}{\beta}\right)\rho\sin\varphi. \end{aligned} \tag{4}$$

By integral of Bessel function and its recurrence relations, we can obtain the following from (4):

$$u(\rho,\varphi) = c\frac{2J_1(kr\mu\theta)}{kr\mu\theta}, \tag{5}$$

where θ is scattering angle and $\mu = \sqrt{1+(\beta^2-1)\sin^2\varphi}$. The complex amplitude function $u(\rho,\varphi)$ is relevant to the size of scattering particles and azimuth angle φ, regardless of the impact of refractive index. Thus, when the azimuth angle is confirmed, the diffraction intensity at the distance r from the particle is

$$I_d(\theta) = \frac{u^2(\theta)\,I_0}{r^2}. \tag{6}$$

The optical scattering method based on diffraction theory to measure the scattering intensity of raindrops is restricted to a narrow range of forward angles. Considering the computing speed of scattering intensity, the scattering energy in a larger range of angles should not be ignored. When the particle is put in a parallel light field, the light incidents are reflected and refracted which are called geometric optical scattering. Here, the scattering intensity in wide-angle range can be accurately and fast computed.

Figure 1 shows the geometric optical scattering by elliptical section. Light incident on the first interface of particle would be reflected (ray 0) and refracted. The Stokes parameters of reflection and refraction as well as the refraction path can be obtained from Fresnel equations and Snell's law. The refraction ray incident on the sphere may exit from the second interface after twofold refraction (ray 1) or may be reduced by twofold reflection within the particle (ray 2). The ray will repeat the behavior until they exit the sphere (ray 3 and above) or are absorbed by particles.

The refractive index of raindrop is n and the rays propagate parallel to the x-axis from negative x toward the drop. Suppose that the angle between the incident ray and spherical surface is τ and the angle between the internal refraction ray and spherical surface is τ', the intensity of incident light is I_0, and the azimuth angle perpendicular to the paper is φ. We

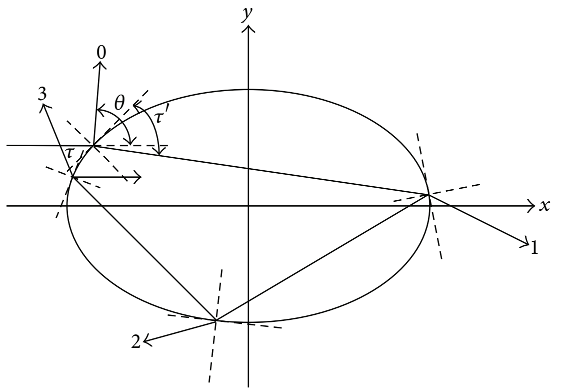


Figure 1: Geometric optical scattering by elliptical section.

utilize $d\tau$ and $d\varphi$ to ensure the direction of incident beam, and the incident energy flow is

$$d\Phi = I_0 \frac{\sin\tau}{\cos\tau} \frac{b^2}{(\tan\tau/\beta)^2 + 1} d\tau\, d\varphi. \quad (7)$$

The incident ray is reflected and refracted several times and number p denotes the emitted ray p. The angle between the emitted ray and positive x-axis is θ and the corresponding solid angle of emitted ray is $\sin\theta d\theta d\varphi$; thus the emitted light intensity with the distance r from the center of ellipsoidal raindrop is

$$\begin{aligned} I_j^{(p)} &= \frac{\varepsilon_j^{(p)} I_0 (\sin\tau/\cos\tau)\left(b^2/\left((\tan\tau/\beta)^2+1\right)\right) d\tau\, d\varphi}{r^2 \sin\theta d\,\theta d\varphi} \\ &= \frac{I_0}{r^2}\varepsilon_j^{(p)} G^{(p)} \quad (j = 1,2), \end{aligned} \quad (8)$$

where subscript $j = 1$ for the perpendicular and $j = 2$ for the parallel polarization component of light. r_1 and r_2 are the Fresnel reflection coefficients. ε_j and G are separately specified by

$$\begin{aligned} \varepsilon_j^{(0)} &= r_j^2 \quad p = 0 \\ \varepsilon_j^{(p)} &= \left[\left(1 - r_j^2\right)\left(1 - r_j\right)^{p-1}\right]^2 \quad p = 1,2,3,\ldots, \end{aligned} \quad (9)$$

$$G = \frac{b^2}{\left[\tan\tau (1/\beta)^2 + \cot\tau\right] \sin\theta \left|d\theta/d\tau\right|}. \quad (10)$$

We introduce at this point the dimensionless intensity $i_j^{(p)}(a,b,\tau) = \varepsilon_j^{(p)} G^{(p)}$ $(j = 1,2)$ which is the intensity function of one reflection or refraction based on the incident angle τ, so the total intensity function of all reflections and infractions for the scattering angle θ is defined as

$$i_j(a,b,\theta) = \sum_{\theta_p(\tau)=\theta} i_j^{(p)}(a,b,\tau) = \sum_{\theta_p(\tau)=\theta} \varepsilon_j^{(p)} G^{(p)} \quad (j = 1,2). \quad (11)$$

From van de Hulst, for nonabsorptive particles, upward of 99.5% of the total forward scattered light for both polarizations emerges from the first interface after simple reflection $p = 0$ and from the second interface after twofold refraction $p = 1$ [20], and the fraction increases for absorptive drops. Therefore, it is sufficient to justify the rays $p \geq 2$ being neglected and consider only the contributions of $p = 0$ and $p = 1$ rays to the total intensity function given as

$$i_j(a,b,\theta) = i_j^{(0)}(a,b,\theta) + i_j^{(1)}(a,b,\theta) \quad (j = 1,2). \quad (12)$$

Take (12) into (8), and the angular scattering intensity of particles for incident plane wave can be obtained:

$$I_g(\theta) = \frac{I_0}{r^2(i_1 + i_2)}. \quad (13)$$

As known from the above analysis, the diffraction scattering is only related to the particles size without considering the impact of refractive index, and the geometric scattering is affected not only by the drop size but also by the refractive index of drops. The optical scattering distribution of raindrops contributed from both Fraunhofer diffraction and geometric scattering is consistent with the conclusions of Mie theory. Therefore, the optical scattering distribution of raindrops can be given in detail by

$$I(\theta) = I_d(\theta) + I_g(\theta) = \frac{I_0}{r^2\left[u^2(\theta) + (i_1 + i_2)\right]}. \quad (14)$$

We adopt the scattering model to compute the optical scattering of laser radiation at 1.064 μm by raindrops. Following the above analysis, the complex amplitude function $u(\rho,\varphi)$ of diffraction from elliptical raindrops is affected by the drop size, shape, and the azimuth angle φ. Figure 2 shows the comparison of complex amplitude function for the same shape ($\beta = 0.8$) with different sizes, and Figure 3 shows the results for 3 mm raindrops with different shapes.

As shown in Figure 2, the diffraction pattern is dominated by a narrow diffraction peak in the central forward direction. In a narrow range of forward angle mainly concentrated in 0~1°, the value of complex amplitude function is large and the diffraction scattering is centralized. When the scattering angle increases, the complex amplitude varies in the maximum and minimum. The results indicate that the raindrop size greatly affects the diffraction, the complex amplitude function increases with the increase of raindrop size, and the central maximum for large drop is much sharper and slightly higher than that for the smaller drop. But the complex amplitude function in scattering angle and amplitude faintly varies for different raindrop shapes that can be seen from Figure 3.

Theoretical researches indicate that the scattering of raindrops is contributed by both Fraunhofer diffraction and geometric scattering. In a narrow range of angles, the diffraction intensity is great, and with the scattering angle increasing, the incident light emerges from reflection and refraction on the raindrop interface. Figure 4 gives the results of incident light being reflected and refracted by raindrops on the parallel and perpendicular polarization directions,

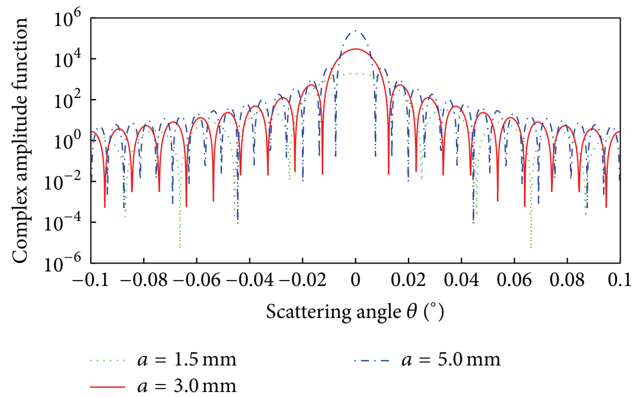


Figure 2: Diffraction of elliptical section obtained by $\beta = 0.8$ for different sizes.

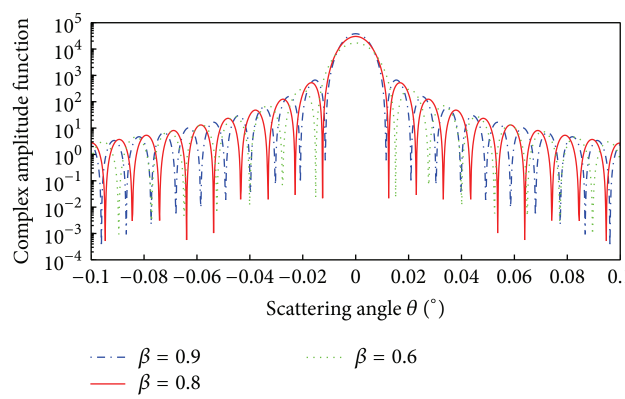


Figure 3: Diffraction of elliptical section obtained by $a = 3$ mm for different shapes.

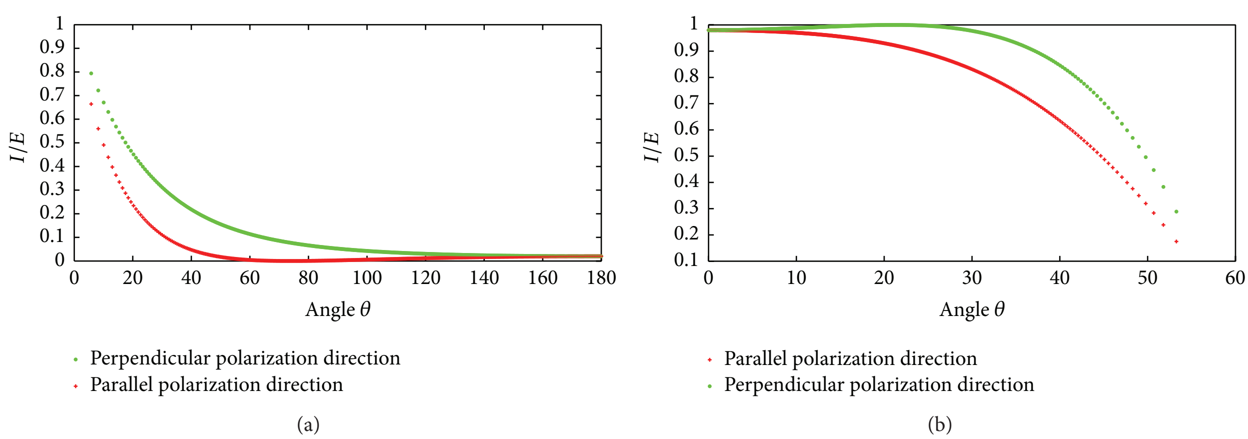
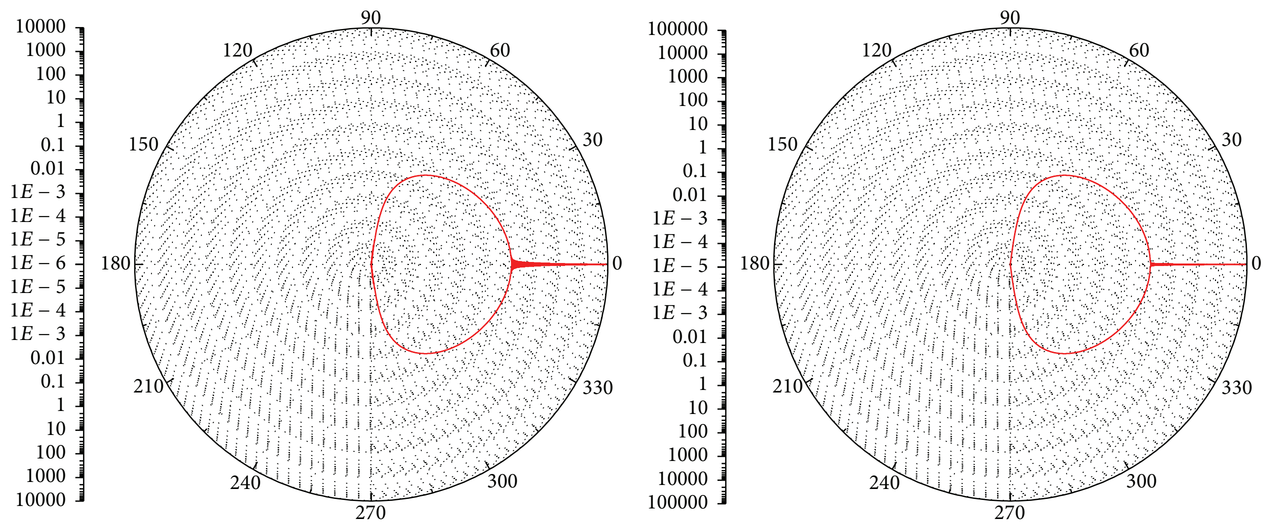


Figure 4: Geometric scattering of raindrops on the two polarization directions ((a) simple reflection; (b) twofold refraction).

where $a = 1.5$ mm and $\beta = 0.8$. From Figure 4, the simple reflection (Figure 4(a)) on the parallel polarization direction is mainly centralized in the forward 0~30° and less in other scattering angles. And the reflection on the perpendicular polarization direction is mainly centralized in the forward 0~60° and larger than that on the parallel polarization direction. The twofold refraction (Figure 4(b)) of incident light on the raindrop interface is only in the range of forward 0~60° and larger on the parallel polarization direction than the perpendicular polarization.

Consequently, according to the analysis, the optical scattering patterns calculated by (13) are shown in Figure 5, where the raindrop size in Figure 5(a) is $a = 1.5$ mm and $\beta = 0.8$ and in Figure 5(b) the raindrop size is $a = 3.0$ mm and $\beta = 0.8$. As can be seen, the larger the raindrop size, the stronger the forward scattering intensity and the narrower the



(a) (b)

Figure 5: Optical scattering distribution by raindrops ((a) $a = 1.5$ mm, $\beta = 0.8$; (b) $a = 3.0$ mm, $\beta = 0.8$).

diffraction scattering angle. The analytical results obtained from (13) are consistent with the conclusions of using Monte Carlo approach in [14].

4. Conclusions

The models of nonspherical raindrop shape were analyzed and the approximate elliptical model was adopted to substitute for the actual raindrops. Then the analytic formula to calculate the scattering of elliptical raindrops was presented; this was achieved through a combination of Fraunhofer diffraction and geometric optics. The investigation and computing results indicate that, in a narrow range of angle, the diffraction intensity is great, and with the scattering angle increasing, the geometric scattering is obvious. The diffraction scattering is relevant to the particle size and azimuth angle, without considering the impact of refractive index and the central maximum for large drop is much sharper and slightly higher than that for the smaller drop. The geometric scattering of raindrop is mainly centralized in forward $0{\sim}60^\circ$ and quite weak at other angles. The results obtained from analytic formulas are well consistent with the conclusions of using Monte Carlo ray tracing method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] S.-Q. Zhang, "The analysis of scattering effect for the transmission of 1.06 μm laser in lower altitude atmosphere," *International Journal of Infrared and Millimeter Waves*, vol. 28, no. 6, pp. 491–497, 2007.

[2] P. Singh and M. L. Singh, "Experimental determination and comparison of rain attenuation in free space optic link operating at 532 nm and 655 nm wavelength," *Optik*, vol. 125, no. 17, pp. 4599–4602, 2014.

[3] J. Wojtanowski, M. Zygmunt, M. Kaszczuk, Z. Mierczyk, and M. Muzal, "Comparison of 905 nm and 1550 nm semiconductor laser rangefinders' performance deterioration due to adverse environmental conditions," *Opto-Electronics Review*, vol. 22, no. 3, pp. 183–190, 2014.

[4] S. Dhawan and M. Singh, "To study the effect of fog, snow and rain attenuation on FSO link," *CT International Journal of Information & Communication Technology*, vol. 2, no. 1, pp. 16–22, 2014.

[5] S.-Q. Zhang, "The analysis of scattering effect for the transmission of 1.06 μm laser in lower altitude atmosphere," *International Journal of Infrared and Millimeter Waves*, vol. 28, no. 6, pp. 491–497, 2007.

[6] P. Singh and M. L. Singh, "Experimental determination and comparison of rain attenuation in free space optic link operating at 532 nm and 655 nm wavelength," *Optik*, vol. 125, no. 17, pp. 4599–4602, 2014.

[7] G.-Z. Sun, Q. Sun, and Z.-B. Ren, "Analysis of the radius of microsphere particles based on Mie scattering theory," *Infrared and Laser Engineering*, vol. 34, no. 4, pp. 495–498, 2005.

[8] M. Wang, W.-Q. Liu, Y.-H. Lu et al., "Study on the measurement of the atmospheric extinction of fog and rain by forward-scattering near infrared spectroscopy," *Spectroscopy and Spectral Analysis*, vol. 28, no. 8, pp. 1776–1780, 2008.

[9] Q. Fu and W. B. Sun, "Mie theory for light scattering by a spherical particle in an absorbing medium," *Applied Optics*, vol. 40, no. 9, pp. 1354–1361, 2001.

[10] F. Xu, X. S. Cai, and J. Q. Shen, "Geometric approximation of light scattering in arbitrary diffraction regime for absorbing

particles: application in laser particle sizing," *Acta Optica Sinica*, vol. 23, no. 12, pp. 1467–1469, 2003.

[11] M. O. Ajewole, L. B. Kolawole, and G. O. Ajayi, "Theoretical study of the effect of different types of tropical rainfall on microwave and millimeter-wave propagation," *Radio Science*, vol. 34, no. 5, pp. 1103–1124, 1999.

[12] J. Hellmers and T. Wriedt, "T-Matrix light scattering simulation of rough, non-symmetrical spherical particles," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 106, no. 1 3, pp. 90–103, 2007.

[13] T. Wriedt, "Light scattering theories and computer codes," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 110, no. 11, pp. 833–843, 2009.

[14] O. N. Ross and S. G. Bradley, "Model for optical forward scattering by nonspherical raindrops," *Applied Optics*, vol. 41, no. 24, pp. 5130–5141, 2002.

[15] A. Macke and M. Großklaus, "Light scattering by nonspherical raindrops: implications for lidar remote sensing of rainrates," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 60, no. 3, pp. 355–363, 1998.

[16] M. Mikrenska and P. Koulev, "Simulation of light scattering by large particles with randomly distributed spherical or cubic inclusions," *Journal of Quantitative Spectroscopy and Radiative Transfer*, vol. 110, no. 14–16, pp. 1411–1417, 2009.

[17] M. Mikrenska and P. Koulev, "Monte Carlo simulations of multiple light scattering by large non-spherical particles," *Electromagnetic & Light Scattering XII*, vol. 162, 2010.

[18] M. Szakáll, S. K. Mitra, K. Diehl, and S. Borrmann, "Shapes and oscillations of falling raindrops—a review," *Atmospheric Research*, vol. 97, no. 4, pp. 416–425, 2010.

[19] L. Liu, H. Li, and T. C. Gao, "The approximate ellipsoid model for raindrop and its near-infrared scattering characteristics," *Scientia Meteorologica Sinica*, vol. 28, no. 3, pp. 271–275, 2008.

[20] H. C. van de Hulst, *Light Scattering by Small Particles*, Dover, New York, NY, USA, 1981.

[21] B. Dai, X.-D. Luo, and Y.-W. Wang, "Multiple light scattering of non-spherical particles with elliptical cross section," *Acta Physica Sinica*, vol. 58, no. 6, pp. 3864–3869, 2009.