Research Article

Irradiance Scintillation Index for a Gaussian Beam Based on the Generalized Modified Atmospheric Spectrum with Aperture Averaged

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Received 18 August 2015; Revised 5 January 2016; Accepted 21 January 2016

Academic Editor: Fortunato Tito Arecchi

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This paper investigates the aperture-averaged irradiance scintillation index of a Gaussian beam propagating through a horizontal path in weak non-Kolmogorov turbulence. Mathematical expressions are obtained based on the generalized modified atmospheric spectrum, which includes the spectral power law value of non-Kolmogorov turbulence, the finite inner and outer scales of turbulence, and other optical parameters of the Gaussian beam. The numerical results are conducted to analyze the influences of optical parameters on the aperture-averaged irradiance scintillation index for different Gaussian beams. This paper also examines the effects of the irradiance scintillation on the performance of the point-to-point optical wireless communication system with intensity modulation/direct detection scheme.

1. Introduction

Optical wireless communication technology has drawn much attention for its significant technological challenges and prospective applications. It uses beams of laser propagating through the atmosphere to transmit data wirelessly at high speed [1, 2]. However, the atmosphere is full of numerous turbulence eddies, which has great degrading impacts on the performance of the communication system. Irradiance scintillation, one typical effect of atmospheric turbulence, results from stochastic redistribution of the optical energy within a cross section of the laser beam [3, 4]. The degree of irradiance scintillation for optical propagation on the receiving antenna can be characterized statistically by the irradiance scintillation index.

In the past few decades, various power spectrum models of refractive index have been proposed to analyze the irradiance scintillation index for different situations [5–15]. Generally speaking, these models can be classified into two categories: Kolmogorov models and non-Kolmogorov models. The former have a fixed power law value of $11/3$

whereas the latter allow the power law value to vary in the range from 3 to 4. Practically, most non-Kolmogorov models can be generalized from corresponding Kolmogorov models [16]. Among the non-Kolmogorov models, the generalized modified atmospheric spectrum not only considers the finite inner and outer scales of turbulence, but also features the small rise at a high wave number, which is clearly seen in temperature data recorded by sensors [17, 18]. These properties make the generalized modified atmospheric spectrum useful in the investigation of the irradiance scintillation index along the radial and longitudinal propagation [10, 11, 13].

In this study, the generalized modified atmospheric spectrum is used to investigate the aperture-averaged irradiance scintillation index of a Gaussian beam in weak non-Kolmogorov turbulence along a horizontal path. The Gaussian beam, whose transverse electric field and intensity are normally distributed, is a typical kind of electromagnetic wave. The rest of the paper is organized as follows. Section 2 introduces the generalized modified atmospheric spectrum and the irradiance scintillation index of a Gaussian beam in weak turbulence. Section 3 presents a detailed expression

reduction. The influences of the inner and outer scales of turbulence on the irradiance scintillation index of a Gaussian beam are analyzed in Section 4, followed by conclusions in Section 5.

2. Theoretical Models

2.1. Generalized Modified Atmospheric Spectrum. The generalized modified atmospheric spectrum takes the form [17]

$$\Phi_n(\kappa, \alpha) = A(\alpha) C_n^2 \kappa^{-\alpha} f(\kappa, \alpha, l_0, L_0), \quad (1)$$

where $\kappa \in [0, +\infty)$ is the angular wavenumber of the turbulence scale, $\alpha \in (3, 4)$ is the spectral power law value, C_n^2 is the generalized atmospheric structure parameter, and l_0 and L_0 are the inner and outer scales of turbulence. $A(\alpha)$ is a function related to α :

$$A(\alpha) = \frac{\Gamma(\alpha - 1)}{4\pi^2} \sin \frac{\pi(\alpha - 3)}{2}, \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function.

Let

$$\begin{aligned} \kappa_0 &= \frac{4\pi}{L_0}, \\ \kappa_l &= \frac{(\pi A(\alpha) C_n^2)^{1/(\alpha-5)}}{l_0}, \end{aligned} \quad (3)$$

where

$$\begin{aligned} C_\alpha &= \frac{3-\alpha}{3} \Gamma\left(\frac{3-\alpha}{2}\right) + a \frac{4-\alpha}{3} \Gamma\left(\frac{4-\alpha}{2}\right) \\ &\quad - b \frac{3+\beta-\alpha}{3} \Gamma\left(\frac{3+\beta-\alpha}{2}\right), \end{aligned} \quad (4)$$

and the constant coefficients a , b , and β in (4) are usually set as $a = 1.802$, $b = 0.254$, and $\beta = 7/6$. The term $f(\kappa, \alpha, l_0, L_0)$ in (1) is given by

$$\begin{aligned} f(\kappa, \alpha, l_0, L_0) &= \left(1 - \exp\left(-\frac{\kappa^2}{\kappa_0^2}\right)\right) \exp\left(-\frac{\kappa^2}{\kappa_l^2}\right) \\ &\quad \cdot \left(1 + a\left(\frac{\kappa}{\kappa_l}\right) - b\left(\frac{\kappa}{\kappa_l}\right)^\beta\right). \end{aligned} \quad (5)$$

For the convenience of mathematical analysis, (5) is rewritten as

$$f(\kappa, \alpha, l_0, L_0) = \sum_{i=1}^3 \sum_{j=1}^2 (-1)^{j-1} c_i \kappa^{p_i} \exp(-d_j^2 \kappa^2), \quad (6)$$

where the coefficients are $c_1 = 1$, $c_2 = a/\kappa_l$, $c_3 = -b/\kappa_l^\beta$, $p_1 = 0$, $p_2 = 1$, $p_3 = \beta$, $d_1 = \sqrt{1/\kappa_0^2}$, and $d_2 = \sqrt{1/\kappa_l^2 + 1/\kappa_0^2}$.

It must be pointed that the values of these coefficients a , b , and β are based on the experimental data for the classic Kolmogorov turbulence. Besides, the generalized modified atmospheric spectrum will turn into the generalized exponential spectrum if $a = b = 0$ or the generalized Kolmogorov turbulence if $a = b = l_0 = 0$ and $L_0 \rightarrow +\infty$.

2.2. Aperture-Averaged Irradiance Scintillation Index of a Gaussian Beam. The mathematical model of a Gaussian beam depends on the radius W_0 and the phase front radius R_0 at the transmitter. Let L be the propagation path length, and let k be the angular wavenumber of the Gaussian wave

$$k = \frac{2\pi}{\lambda}, \quad (7)$$

where λ is the wavelength of the Gaussian wave. Thus, the curvature parameter of the Gaussian wave at the transmitter Θ_0 and the Fresnel ratio of the Gaussian wave at the transmitter Λ_0 are defined as [19]

$$\begin{aligned} \Theta_0 &= 1 - \frac{L}{R_0}, \\ \Lambda_0 &= \frac{2L}{kW_0^2}. \end{aligned} \quad (8)$$

Θ , $\bar{\Theta}$, and Λ are nondimensional parameters of the Gaussian wave at the receiver:

$$\begin{aligned} \Theta &= \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2}, \\ \bar{\Theta} &= 1 - \Theta, \\ \Lambda &= \frac{\Lambda_0}{\Theta_0^2 + \Lambda_0^2}. \end{aligned} \quad (9)$$

The aperture-averaged irradiance scintillation index of a Gaussian beam propagating through weak atmospheric turbulence in the absence of beam wander takes the form [8]

$$\begin{aligned} \sigma_I^2(D_G) &= 8\pi^2 k^2 L \int_0^1 d\xi \int_0^{+\infty} d\kappa \times \kappa \Phi_n(\kappa, \alpha) \\ &\quad \cdot \exp\left(-\frac{L\kappa^2}{k(\Omega_G + \Lambda)} \left((1 - \bar{\Theta}\xi)^2 + \Lambda\Omega_G\xi^2\right)\right) \\ &\quad \cdot \left(1 - \cos\left(\frac{L\kappa^2}{k} \left(\frac{\Omega_G - \Lambda}{\Omega_G + \Lambda}\right) \xi(1 - \bar{\Theta}\xi)\right)\right), \end{aligned} \quad (10)$$

where ξ is the normalized path coordinate

$$\xi = 1 - \frac{z}{L}, \quad (11)$$

z is the coordinate along the propagation direction, Ω_G is a nondimensional parameter characterizing the relative radius of the collecting lens

$$\Omega_G = \frac{16L}{kD_G^2} > \Lambda, \quad (12)$$

and D_G is the collecting lens diameter.

2.3. Optical Wireless Communication Link Performance. The aperture-averaged irradiance scintillation index is relevant to the point-to-point optical wireless communication system with intensity modulation/direct detection scheme.

The probability of fade, the mean signal-to-noise ratio, and the mean bit-error rate are often used to judge the performance of the link [15].

The probability of fade describes the percentage of time; the irradiance of the beam at the receiver is below certain threshold value I_T . For weak turbulence, the probability of fade is defined by

$$P_I(I < I_T) = \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{0.5\sigma_I^2(D_G) - 0.23F_T}{\sqrt{2\sigma_I^2}} \right) \right), \quad (13)$$

where $\operatorname{erf}(\cdot)$ is the error function and F_T is the fade threshold parameter:

$$F_T = 10 \log_{10} \left(\frac{\langle I \rangle}{I_T} \right). \quad (14)$$

The fade parameter stands for the mean irradiance in decibels below the threshold value I_T .

The mean signal-to-noise ratio is a measure which compares the level of the received beam to the level of background radiance. For weak turbulence, the mean signal-to-noise ratio is defined by

$$\langle \text{SNR} \rangle = \frac{\text{SNR}_0}{\sqrt{1 + \sigma_I^2(D_G) \cdot \text{SNR}_0^2}}, \quad (15)$$

where SNR_0 is the signal-to-noise ratio in the absence of turbulence.

The mean bit-error rate is the number of bit errors in unit time. For weak turbulence, the mean bit-error rate is defined by

$$\langle \text{BER} \rangle = \frac{1}{2} \int_0^{+\infty} p_I(u) \operatorname{erfc} \left(\frac{\langle \text{SNR} \rangle u}{2\sqrt{2}} \right) du, \quad (16)$$

where $p_I(u)$ is the probability density function of lognormal distribution

$$p_I(u) = \frac{1}{u\sqrt{2\pi\sigma_I^2(D_G)}} \exp \left(-\frac{(\ln u + (1/2)\sigma_I^2(D_G))^2}{2\sigma_I^2(D_G)} \right) \quad (17)$$

and $\operatorname{erfc}(\cdot)$ is the complementary error function.

3. Expression Reduction

This section mainly discusses the reduction of (10).

Without loss of generality, we define

$$P(\xi) = \frac{L}{k(\Omega_G + \Lambda)} \left((1 - \bar{\Theta}\xi)^2 + \Lambda\Omega_G\xi^2 \right) \quad (18)$$

$$Q(\xi) = \frac{L}{k} \left(\frac{\Omega_G - \Lambda}{\Omega_G + \Lambda} \right) \xi (1 - \bar{\Theta}\xi).$$

Based on Euler's formula, we can get [20, 21]

$$(1 - \cos(Q(\xi)\kappa^2)) = \operatorname{Re} (1 - \exp(i|Q(\xi)|\kappa^2)). \quad (19)$$

Thus, (10) can be rewritten as

$$\begin{aligned} \sigma_I^2(D_G) &= 8\pi^2 k^2 L \int_0^1 d\xi \int_0^{+\infty} \kappa \Phi_n(\kappa, \alpha) \\ &\cdot \exp(-P(\xi)\kappa^2) \\ &\cdot \operatorname{Re} (1 - \exp(-i|Q(\xi)|\kappa^2)) d\kappa. \end{aligned} \quad (20)$$

Substituting (1) into (20), it follows that

$$\begin{aligned} \sigma_I^2(D_G) &= 8\pi^2 k^2 L A(\alpha) C_n^2 \int_0^1 d\xi \\ &\cdot \int_0^{+\infty} \kappa^{1-\alpha} f(\kappa, \alpha, l_0, L_0) \exp(-P(\xi)\kappa^2) \\ &\cdot \operatorname{Re} (1 - \exp(-i|Q(\xi)|\kappa^2)) d\kappa. \end{aligned} \quad (21)$$

To reduce the iterated integral in (21), it is necessary to expand the integrand by (6):

$$\begin{aligned} &\kappa^{1-\alpha} f(\kappa, \alpha, l_0, L_0) \exp(-P(\xi)\kappa^2) \\ &\cdot \operatorname{Re} (1 - \exp(-i|Q(\xi)|\kappa^2)) = \sum_{i=1}^3 \sum_{j=1}^2 (-1)^{j-1} \\ &\cdot c_i \kappa^{1-\alpha+p_i} \operatorname{Re} [\exp(-(P(\xi) + d_j^2)\kappa^2) \\ &- \exp(-(P(\xi) + d_j^2 + i|Q(\xi)|)\kappa^2)]. \end{aligned} \quad (22)$$

Based on the equation for $p > -3$, $\operatorname{Re}(u) > 0$, and $\operatorname{Re}(v) > 0$ [21, 22]

$$\begin{aligned} &\int_0^{+\infty} x^p (\exp(-ux^2) - \exp(-vx^2)) dx \\ &= \frac{1}{p+1} (u^{-(p+1)/2} - v^{-(p+1)/2}) \Gamma\left(\frac{p+3}{2}\right), \end{aligned} \quad (23)$$

the improper integral of κ can be rewritten as

$$\begin{aligned} &\int_0^{+\infty} \kappa^{1-\alpha} f(\kappa, \alpha, l_0, L_0) \exp(-P(\xi)\kappa^2) \\ &\cdot \operatorname{Re} (1 - \exp(-i|Q(\xi)|\kappa^2)) d\kappa \\ &= \sum_{i=1}^3 \sum_{j=1}^2 (-1)^{j-1} c_i \frac{1}{2-\alpha+p_i} \Gamma\left(\frac{4-\alpha+p_i}{2}\right) \\ &\cdot \operatorname{Re} \left((P(\xi) + d_j^2)^{-(2-\alpha+p_i)/2} \right. \\ &\left. - (P(\xi) + d_j^2 + i|Q(\xi)|)^{-(2-\alpha+p_i)/2} \right). \end{aligned} \quad (24)$$

Based on Euler's formula and de Moivre's formula, we can get [20, 21]

$$\begin{aligned} & \text{Re} \left(P(\xi) + d_j^2 + i |Q(\xi)| \right)^{-(2-\alpha+p_i)/2} \\ &= \left((P(\xi) + d_j^2)^2 + Q^2(\xi) \right)^{-(2-\alpha+p_i)/4} \\ & \cdot \cos \left(\frac{2-\alpha+p_i}{2} \arctan \frac{Q(\xi)}{P(\xi) + d_j^2} \right). \end{aligned} \quad (25)$$

Thus, the iterated integral in (21) has been reduced to the definite integral of the real variable ξ in the bounded interval $[0, 1]$, which can be easily computed by methods of numerical integration with arbitrary precision.

4. Numerical Simulations

This section conducts numerical simulations to analyze the influences of α , l_0 , L_0 , W_0 , and λ on the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ of different Gaussian beams. The simulations performed in this paper, however, should be only considered as arbitrary examples to indicate certain trend of results. Unless specified otherwise, all the numerical simulations are conducted with the default settings: $\lambda = 1.55 \times 10^{-6}$ m, $k \approx 4.0537 \times 10^6$ rad/m, $L = 8000$ m, $C_n = 1.0 \times 10^{-14}$ m^{3- α} , $W_0 = 0.1$ m, and $\Lambda_0 = 0.0493$. Of course, other values which satisfy (12) can also be chosen.

Figure 1 depicts the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ for different Gaussian beams as a function of the spectral power law α for several pairs of the inner and outer scales l_0 and L_0 . As shown for the convergent beam ($\Theta_0 = 1$) in Figure 1(c), the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ first increases and then decreases with an increase in the spectral power law α when other parameters are fixed, which acts in accordance with that under the generalized non-Kolmogorov spectrum and the generalized scale-dependent anisotropic spectrum, as presented in [8, 15]. Similar phenomena can be also found for the focused beam ($\Theta_0 = 0$) in Figure 1(a), the convergent beam ($\Theta_0 = 0.5$) in Figure 1(b), and the divergent beam ($\Theta_0 = 1.5$) in Figure 1(d), respectively. According to Figure 1, it also shows that the aperture-averaged irradiance scintillation index σ_I^2 is more sensitive to the outer scale L_0 than to the inner scale l_0 , and an increase in the outer scale L_0 will lead to an increase in the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ when other optical parameters are fixed. These phenomena can be physically explained by the fact that the irradiance scintillation can be decomposed into radial and longitudinal components. The former is sensitive to the outer scale L_0 , while the latter is sensitive to the inner scale l_0 . The longitudinal component of the irradiance scintillation only exists at the beam center, and thus the radial component occupies the dominant position in the influences on the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$. As the outer scale of turbulence L_0 increases,

the Gaussian beam will meet more turbulence eddies along its propagation.

For further discussions and analyses, the inner and outer scales of turbulence are assigned to constant values $l_0 = 0.01$ m and $L_0 = 1$ m, respectively. Some typical values of wavelength in the near infrared region $\lambda = 0.85 \times 10^{-6}$ m, $\lambda = 1.06 \times 10^{-6}$ m, and $\lambda = 1.55 \times 10^{-6}$ m are investigated in this simulation. Figure 2 depicts the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ for different Gaussian beams as a function of the radius W_0 for several pairs of the spectral power law α and the wavelength λ . The radius W_0 along the abscissa axis uses the logarithmic (10-base) scale. To avoid the mutual interferences, $\alpha = 3.6 < 11/3$ and $\alpha = 3.7 > 11/3$ are used. As shown for the convergent beam ($\Theta_0 = 1$) in Figure 2(c), the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ first decreases and then increases with an increase in the radius W_0 when other parameters are fixed. Physically, these turbulence eddies act as lenses to change the focusing properties of transmission medium, and the small turbulence eddies with scales less than $\sqrt{L/k}$ play a significant role in the fluctuations of irradiance. When the radius W_0 is small enough, the Gaussian beam is hypersensitively affected by the small turbulence eddies. When the radius W_0 is larger, the transversal surface of the Gaussian beam contains numerous small turbulence eddies, which makes the fluctuations of irradiance greater. Figure 2(c) also shows that the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ decreases with an increase in λ for a Gaussian wave if other optical parameters are fixed. This phenomenon may be caused by the fact that the longer the beam wavelength, the more pronounced the diffraction. Thus, a laser beam with longer wavelength can be less affected by turbulence eddies. Similar phenomena can be also found for the focused beam ($\Theta_0 = 0$) in Figure 2(a), the convergent beam ($\Theta_0 = 0.5$) in Figure 2(b), and the divergent beam ($\Theta_0 = 1.5$) in Figure 2(d), respectively.

To analyze the influence of the irradiance scintillation on the performance of point-to-point optical wireless communication system with intensity modulation/direct detection scheme, Figure 3 depicts the probability of fade P_f as a function of the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ and the fade parameter F_T , and Figure 4 depicts the mean signal-to-noise ratio $\langle \text{SNR} \rangle$ as a function of the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ and the signal-to-noise ratio in the absence of turbulence SNR_0 , and Figure 5 depicts the mean bit-error rate $\langle \text{BER} \rangle$ as a function of the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ and the signal-to-noise ratio in the absence of turbulence SNR_0 . Both the aperture-averaged irradiance scintillation index $\sigma_I^2(D_G)$ and the signal-to-noise ratio in the absence of turbulence SNR_0 are presented on a logarithmic (10-base) scale. Figures 3–5 indicate that the atmospheric turbulence would produce much more negative effects on the wireless optical communication system with an increase in the irradiance scintillation. To improve the performance of the wireless optical communication system, an alternative method is to raise the signal-to-noise ratio in the absence of turbulence SNR_0 .

FIGURE 1: σ_I^2 of different Gaussian beams as a function of α for several pairs of l_0 and L_0 .

5. Conclusions

In this paper, the aperture-averaged irradiance scintillation derived for a Gaussian beam propagating through the weak

non-Kolmogorov atmospheric turbulence along a horizontal path is investigated. The generalized modified atmospheric spectrum is utilized to take the variable spectral power law value, finite inner and outer scales of turbulence, and

FIGURE 2: σ_I^2 of different Gaussian beams as a function of W_0 for several pairs of α and λ .

other important optical parameters of a Gaussian beam into account. The experimental results show the following:

- (1) The irradiance scintillation index first increases and then decreases with an increase in the spectral power law.

- (2) The aperture-averaged irradiance scintillation index is more sensitive to the outer scale than to the inner scale, and an increase in the outer scale will lead to an increase in the aperture-averaged irradiance scintillation index.

FIGURE 3: P_I as a function of σ_I^2 and F_T . FIGURE 5: $\langle \text{BER} \rangle$ as a function of σ_I^2 and SNR_0 . FIGURE 4: $\langle \text{SNR} \rangle$ as a function of σ_I^2 and SNR_0 .

- (3) The irradiance scintillation index first decreases and then increases with an increase in the radius.
- (4) A longer beam wavelength will lead to a smaller irradiance scintillation index.
- (5) The irradiance scintillation is harmful to the performance of the wireless optical communication system.

As mentioned above, the simulations performed in this paper should be only regarded as tentative examples to discover certain trend of results and mainly for the purpose of theoretical analyses. Due to the lack of adequate data and prior information about the non-Kolmogorov turbulence, the constant coefficients a , b , and β in (4) may fail to describe the actual situations of atmospheric turbulence. Once it is capable of determining the exact values of these coefficients

in the non-Kolmogorov turbulence, the expressions deduced in this paper could be generalized to the whole atmospheric layers. Besides, there are several literatures assuming the spectral power law value α to change in the range between 3 and 5, but in this paper the value of α is utilizing the smaller range between 3 and 4. For values of the spectral power law value α between 4 and 5, the condition in (23) is no more satisfied for the values of $1 - \alpha + p_i$ and thus the reduction fails. Therefore, the analysis is only valid for $\alpha \in (3, 4)$ in the derived model.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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