Research Article

Generation of Perfect Optical Vortices by Using a Transmission Liquid Crystal Spatial Light Modulator

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We have experimentally created perfect optical vortices by the Fourier transformation of holographic masks with combination of axicons and spiral functions, which are displayed on a transmission liquid crystal spatial light modulator. We showed theoretically that the size of the annular vortex in the Fourier plane is independent of the spiral phase topological charge but it is dependent on the axicon. We also studied numerically and experimentally the free space diffraction of a perfect optical vortex after the Fourier backplane and we found that the size of the intensity pattern of a perfect optical vortex depends on the topological charge and the propagation distance.

1. Introduction

As is well known, an optical vortex beam is an electromagnetic wave with a helical wavefront due to phase singularities [1]. These phase singularities are threads of darkness embedded within light fields in their spatial distribution, points for 2D, and lines for 3D [2, 3]. Allen and his collaborators proved that the complex amplitude of an optical vortex possessing an azimuthal phase factor \( \exp (i m \theta) \) carries an orbital angular momentum of \( m \hbar \), where \( m \) is the topological charge and \( \theta \) is the azimuthal angle [4]. The unique optical properties of the optical vortices have been widely used in applications such as optical tweezers [5–8], image processing [9–11], communication systems in free space [12–14], and optical fibers [15, 16]. Motivated by these applications several methods for generating the optical vortex beam have been proposed [17–27]; however the diameter of these optical vortices is related to their topological charges. This property causes difficulties to achieve a high spatial accuracy and high orbital angular momentum coupling optical vortices into a fiber.

To solve these requirements, Ostrovsk et al. have introduced the perfect optical vortex (POV) concept [28]. The perfect optical vortices are electromagnetic waves whose ring-width size and average ring-diameter (the arithmetic average of the inner and outer ring-diameters) are both independent of the topological charge. To experimentally generate the POVs or POV array a Gaussian beam (or a wave plane) is directed toward a special phase mask [28, 29], an axicon [30], or a phase mask by combining an axicon and a spiral phase function [31, 32]. The modulation of these phase masks is programmed onto a reflection liquid crystal spatial light modulator (RLC-SLM) working in phase only mode. The masks are created with the discretization of the phase into \( N \) levels depending on the characteristics of the RLC-SLM. The most of RLC-SLMs operate with an 8-bit dynamic range or 256 phase levels.

In this work, we present an experimental approach for generating perfect optical vortices by using of phase masks of three levels with shape of axicon and spiral phase functions, which are displayed in a transmission liquid crystal spatial light modulator. The intensity and vorticity of the POVs are measured with a CMOS camera and the far-field diffraction pattern through an equilateral triangular slit, respectively. We showed numerically and experimentally that the free space propagation diffraction patterns of the POVs are dependent on the distance and they are variant to ring-width size and to average ring-diameter.
2. Theory

As mentioned earlier, perfect optical vortices are beams whose diameter and width-ring are independent of its topological charge. POVs can be approximately generated by means of the Fourier transformation from Bessel-Gauss (BG) beams [31]. The complex field amplitude of a BG beam with amplitude unit can be described in cylindrical coordinates \((\rho, \theta)\), as [33]

\[
U(\rho, \theta) = J_m(k_r \rho) e^{im \theta} e^{-\rho^2 / w_o^2},
\]

where \(w_o\) is the waist width of the Gaussian beam, \(k_r\) is the radial wave vector, and \(J_m(\cdot)\) is an \(m\)th order Bessel function of first kind.

The field of a POV is obtained in the back focal plane of a converging lens by substituting (1) in the Fourier transform diffraction spectrum yielding the result

\[
U(r, \phi) = \frac{k}{2i \pi f} \int_0^\infty J_m(k_r \rho) e^{-\rho^2 / w_o^2} \cdot \rho d\rho \int_0^{2\pi} e^{-i k_p r \cos(\phi - \theta) / f} e^{i m \theta} d\theta,
\]

where \(f\) is the back focal length of the lens. By analytic solving of the integral in (2), we can obtain the Fourier diffraction spectrum for BG beams with different topological charges \(m\) at the back focal plane of a lens, which reads

\[
U(r, \phi) = i^{m-1} \frac{w_o}{w} J_m \left(2r \frac{r_o}{w} \right) e^{i m \phi} e^{-\left(r^2 + r r_o^2 / w^2 \right)}.
\]

with \(r_o\) and \(w\), the radius and half width-ring of the perfect optical vortices, respectively. \(J_m(\cdot)\) is an \(m\)th order modified Bessel function of first kind and it can be written as [34]

\[
J_m\left(Ze^{\pi i / 2}\right) = e^{-im/2} J_m\left(Ze^{\pi i / 2}\right).
\]

The radius \(r_o\) in (3) can be expressed approximately as

\[
r_o = \frac{k_r f}{k} \approx \frac{\lambda f}{P},
\]

where we had considered the experimental fact that \(\sqrt{P^2 + \lambda^2} = P\) (axicon period). It can see from (5) that the average radius of a POV is independent of the topological charge of the spiral phase and it basically governed Fourier transform of the axicon [35, 36]. The columns three and four of Figure 1 provide simulations of the theoretical results for the intensity distribution of POVs with topological charges \(m = 1, m = 2, m = 3, m = 5\), and \(m = 10\), obtained by Fourier transform diffraction of BG beams with axicon period \(P = 1.3\) [mm] and \(P = 0.7\) [mm], respectively. From Figure 1, we can observe that the diameters of BG beams increase with the topological charge (first column) however; the diameters of POVs apparently do not change (third and fourth column). Also in Figure 1, one can see that the radii of the POVs of the fourth column are bigger than the radii of the POVs of the third column. Last result can be easily explained by decrease of the axicon period in (5) because \(d\) is inverse to \(r_o\). The phase profiles of the BG beams on first column of Figure 1 are shown in second column of the same figure, while for the POVs on third and fourth column in Figure 1, the phase structures are equal and they are shown on fifth column of Figure 1. It is clear from each phase profile that the number of central dislocations is equal to the value of \(m\) for each used BG beam and each generated POV.

Figure 2(a) illustrates the line profiles through the center of the intensity distributions of POVs shown in third column of Figure 1, which were obtained with an axicon period of \(P = 1.3\) [mm]. In this graph, we can observe a shift in hundreds of the millimeters among the ring-diameters for the POVs with the topological charges \(m = 1, 2, 3, 5\) and \(m = 10\). This result can be explained by (3), in which the radial field amplitude of a POV depends on the combination of a Bessel-modified function \(I_m(\cdot)\) and a Gaussian function. Since the slope of the exponential function \(I_m(\cdot)\) decreases sligt as \(m\) increases, the ring-diameter shifts by a small value when \(I_m(\cdot)\) intersects the Gaussian function, as Figure 2(b) shows. However, the line profiles through the centers of the intensity distributions for the POVs with topological charges \(m = 1, 2, 3, 5\) and \(m = 10\) have spatial shifts smaller than hundreds of the millimeters, when the POVs are generated with \(P = 0.7\) [mm], as shown in Figure 2(c). This behavior can be explained by taking into account the fact that the Bessel-modified functions for the POVs with \(m = 1, 2, 3, 5\) and \(m = 10\) generated with \(P = 0.7\) [mm] have similar slopes, as shown in Figure 2(d). For this reason, taking this approximation on Fourier plane (\(w\) small), then for large \(r_o\), the function \(I_m(\cdot)\) can written asymptotically as [35]

\[
I_m\left(\frac{2rr_o}{w^2}\right) \approx e^{2rr_o / w^2}.
\]

Using (6), one finds easily the field distribution on the focal plane of the lens,

\[
U(r, \phi) = i^{m-1} \frac{w_o}{w} e^{i m \phi} e^{-\left(r^2 + r r_o^2 / w^2 \right)}.
\]

So the intensity distribution of a POV can be calculated as

\[
I(r, \phi) = \left(\frac{w_o}{w}\right)^2 e^{-2(r r_o^2 / w^2)}.
\]

This result explains the fact that intensity pattern of a POV does not depend upon the topological charge when \(r_o\) is large and the axicon period is small.

On the other hand, we also can verify the nature of the POVs through their far-field diffraction patterns by an equilateral triangular aperture. These far-field diffraction patterns can be obtained considering a POV as a centered incident beam on an equilateral triangular aperture located at plane \(z_0 = 0\). The size of the equilateral triangular aperture has been adjusted to POV size under consideration that the diameter of the bright ring does not depend on the topological charge \(m\). Then the diffracted field of a POV by an equilateral triangular aperture is proportional to the Fourier transform, when it
Figure 1: Simulations of the intensity patterns and the phase structures for Bessel-Gauss beams and perfect optical vortices. (a)–(e) Intensity distributions of the BG beams with topological charges $m = 1, 2, 3, 5$ and $m = 10$, respectively. (k)–(o) and (p)–(t) Intensity distributions of POV's after the Fourier transformation of (a)–(e) for $P = 1.3$ [mm] and $P = 0.7$ [mm], respectively. (f)–(j) and (u)–(y) Phase profiles for the BG beams and the POVs, respectively. In the phase structures the black and white colors are corresponding to 0 and $2\pi$. We used $\lambda = 632.8$ [nm] and $f = 380$ [mm] here and for theoretical and numerical solutions shown in Figures 2, 3, and 4.
By numerically solving the integral in (9), we can find the far-field intensity profile $|U(x', y', z_0)|^2$ for the POVs with different topological charges $m$. Figure 3 shows the POVs with topological charges, $m = 1, 2, 3, 5,$ and $10$, and their respective numerical far-field diffraction pattern. On the diffraction pattern of POVs the formation of a truncated triangular lattice can be seen, in which the number of the spots along an edge of the triangle minus one represents the topological charge of the incident POV [38]. Finally, it is possible to analyze the comportment of a POV after the Fourier plane if it considers the Fresnel formula for the diffraction in free space given by [39]

$$U(r, \phi, z) = \frac{k e^{i k z}}{i2\pi z} \int_0^\infty \int_0^{2\pi} U(r, \phi) e^{i(k(r_1^2 + r^2 - 2r_1 r \cos(\phi - \phi_1))/2z)} \cdot r_1 dr_1 d\phi_1.$$
After inserting the expression of the amplitude field of a POV in the back focal plane (see (3)) and evaluating analytically the integral, we obtain the complex amplitude of a POV in free space propagation [31]

\[
U(r_1, \phi_1, z) = \frac{m^{-1}}{w_{1}^{m}} (-1)^{m} I_m \left( \frac{2 r_1 r_{op} e^{i \psi}}{w_{1}} \right) \nonumber
\]

\[
\cdot e^{i(m \phi + k z + \psi)} e^{-i(k r_1^2 + r_{op}^2)/2R} e^{-i(z r_1^2 + z^2 r_{op}^2)/(z w_{1}^2)}
\]

\[
(12)
\]

with \(w_1\) and \(R\), the beam and curvature radii, respectively. \(\psi\) is the Gouy phase of the Gaussian beam. These parameters are related to propagation distance \(z\), waist size \(w\), and the Rayleigh range \(z_r\) by

\[
w_1 = w \sqrt{1 + \left( \frac{z}{z_r} \right)^2};
\]

\[
R = \frac{z^2 + z_r^2}{z}.
\]
modified Bessel functions. With distance, conclusion can be inferred from (12) due to field dependence. Variant diffraction properties and we cannot call that a POV. This doughnuts. Then, a POV in free space propagation has for other distances, the POV intensity distribution shows a ring-width, independent of the topological charge. However, the main POV characteristics: average ring-diameter and the intensity distribution changes when distance increases. Rayleigh range. As we can observe from each row in Figure 4, the POV intensity profile changes when distance increases. We can appreciate that only at \( z = 0, 2, 4, 6, 8 \) and \( z = 10 \) times the Rayleigh range. As we can observe from each row in Figure 4, the POV intensity profile changes when distance increases. We can appreciate that only at \( z = 0 \) the intensity distribution for topological charges \( m = 1, 5, 10 \) maintains the main POV characteristics: average ring-diameter and ring-width, independent of the topological charge. However, for other distances, the POV intensity distribution shows a drastic characteristics change; it develops other secondary doughnuts. Then, a POV in free space propagation has variant diffraction properties and we cannot call that a POV. This conclusion can be inferred from (12) due to field dependence with distance \( z \) through combination of the exponential and modified Bessel functions.

3. Experimental Setup

Figure 5 shows the experimental setup for generating perfect optical vortices. A random polarized light from a Helium-Neon laser (spectra physics, power 2.5 mW, and wavelength 632.8 [nm]) in the transverse electromagnetic ground state mode TEM\(_{00}\) is spatially filtered and collimated. Subsequently, the beam is directed toward an experimental configuration composed of two lineal polarizers, a transmission liquid crystal spatial light modulator (TLC-SLM, Sony model LCX038ARA spatial resolution: 1024(H) × 768(V) pixels), and two quarter-wave plates. In order to create phase masks with combination of axicon and spiral functions, the TLC-SLM has been placed in the arrangement A1 composed by linear polarizer (P1), quarter-wave plate 1 (QWP1), TLC-SLM, linear analyzer (P2), and quarter-wave plate 2 (QWP2). In this arrangement the orientation of the linear polarizer, analyzer, and the waveplates in front of and behind the TLC-SLM was obtained through calibration previously of TLC-SLM in phase only modulation [40, 41] for \( \lambda = 632.8 \) [nm]. Our TLC-SLM showed that the maximum phase modulation is \( \theta_{\max} = 1.3\pi \), which corresponds to \( N = 3 \) phase levels [42].

The BG beams with the same axicon period \( d \), but different topological charge \( m \), are generated using the phase computerized holograms. These computerized holograms displayed on the TLC-SLM are calculated through the following transmission function:

\[
T(x, y) = \begin{cases} 
\exp\{i2\pi X\} + P\sqrt{x^2 + y^2} & , 1.3\pi 
\end{cases},
\]

where \( t_0 \) is the modulation depth, \( X \) is the inverse of the spatial period, and \( T(x, y) = (m \tan^{-1}(y/x) + P\sqrt{x^2 + y^2}) \) denotes the combination of axicon and spiral functions. The phase distribution is added modulo 1.3\( \pi \) because this value is the maximum possible phase modulation for our TLC-SLM used. In addition, a linear phase shift (not shown in (14)) has been imposed onto incident laser beam by our TLC-SLM in order to separate the first order from the zero order beam (blazed hologram). The output BG beam after arrangement A1 is filtered from zero order using a pupil located just before the Fourier lens L. The filtered BG beam incident on Fourier lens (with 380 [mm] of focal length) allows obtaining the Fraunhofer diffraction pattern. Finally, a CCD camera is placed at the lens back focal plane to detect and save the intensity distribution pattern of the optical field diffracted onto a computer hard drive.

For verifying the nature of the perfect optical vortex generated from Fourier transform of a Bessel-Gauss beam we used an equilateral triangular aperture placed at the back focal plane of L1 within the experimental setup shown in Figure 6. The diffracted field by the equilateral triangular aperture incident onto the lens L2, which is located at distance \( f_1 \) from the aperture, focuses the image onto CCD camera. With the lens L2, the far-field diffraction pattern in the back focal plane of L1 is imaged onto the CCD camera that records and saves the experimental images of the POV diffraction pattern. To avoid the effect of the quadratic phase factor on the experimental Fourier transform plane located onto the equilateral triangular aperture and to achieve the best Fourier transform, we need to adjust the distance between TLC-SLM and the lens L1 to \( f \).

Figure 7 shows the experimental arrangement used for finding the POV diffraction intensity pattern in free space propagation after the back Fourier plane. Diffraction intensity
Figure 6: Schema of experimental setup for verifying the POV nature: spatial filter and collimator; TLC-SLM: transmission liquid crystal spatial light modulator; P1: polarizer; QWP1: quarter-wave plate 1; P2: analyzer; QWP2: quarter-wave plate 2; L1: Fourier lens 1; AP: triangular aperture; L2, Fourier lens 2 with \( f_1 = 80 \) [mm] and CCD camera. The TLC-SLM shows an example of computerized hologram used to generate a POV.

Figure 7: Schema of experimental setup for finding the POV diffraction intensity pattern after the Fourier plane: spatial filter and collimator; TLC-SLM: transmission liquid crystal spatial light modulator; P1: polarizer; QWP1: quarter-wave plate 1; P2: analyzer; QWP2: quarter-wave plate 2; L: Fourier lens; FP: Fourier plane; PL1: plane 1; PL2: plane 2; PL3: plane 3; \( z_r \), Rayleigh range; and \( f = 380 \) [mm]. The TLC-SLM shows an example of computerized hologram used to generate a POV.

Figure 8: Experimental POVs intensity pattern for topological charges \( m = 1, 2, 3, 5, \) and 10, respectively. (a)–(e) For axicon period of \( P = 1.3 \) [mm]. (f)–(j) For axicon period of \( P = 0.7 \) [mm].

patterns are recorded by a CCD camera placed after back focal plane of the Fourier lens L at positions PL1 \( (z = z_r) \), PL2 \( (z = 2z_r) \), and PL3 \( (z = 3z_r) \). We have adjusted the distance between TLC-SLM and the lens L1 to \( f \) for the purpose of canceling the quadratic phase factor in the Fourier spectrum.

4. Experimental Results and Discussion

Figure 8 shows the intensity distributions of the POV beams generated experimentally with topological charges \( m = 1, 2, 3, 5, \) and 10, for the phase masks with axicon period \( P = 1.3 \) [mm] (Figures 8(a)–8(e)) and \( P = 0.7 \) [mm] (Figures 8(f)–8(j)), using the experimental setup of Figure 6.

In this figure, one can see by naked eye that the intensity distributions are invariant with the topological charge. One can also observe that size of the intensity distributions changes with the axicon period according to theory; that is, \( P \) is inverse proportional to \( r_{op} \). All the images are considered in the same scale. Figures 9(f)–9(j) show experimental results of the vorticity verification of the POVs. In Figures 9(f)–9(j)
we can note that the pattern of the Fraunhofer diffraction intensity distribution of a POV by an equilateral triangular aperture exhibits a truncated triangular lattice. The size of the triangular array and the number of spots grow with the increase of the topological charge. In addition, we can observe that the number of brilliant spots in any external side minus one equals the topological charge value of the POV. These measurements have an excellent agreement with the theoretical results of Figure 2(b). For the topological charge $m = 10$, the magnitude of the vorticity in the respective POV does not match well because this value is the limit of the measurements using an equilateral triangular aperture [43].

Finally, each column of Figure 10 exhibits the diffraction patterns captured by CCD camera at positions $z_r$, $2z_r$, and $3z_r$. This figure shows the intensity patterns of POVs with topological charges $m = 1, 2, 3, 5$, and 10 for $P = 1.3$ [mm], respectively. (f)–(j) Truncated triangular lattices of respective POVs after Fourier transformation.
3\pi_r after the back Fourier plane of lens, when the POV with topological charges \( m = 1, 5, \) and 10 diffracts in free space, respectively. In these intensity patterns, we can see basically that the intensity distributions are size variant with the topological charge order. These experimental data clearly manifest the dependence between the free space diffraction pattern of a POV with a well-defined topological charge and their distance of propagation. In this figure, the POV in free space propagation covers the entire area of CCD camera at \( z = 3\pi_r \), which did not allow us to record them at higher distances.

5. Conclusions

We proved that an optical perfect vortex could be essentially created by the Fourier transformation of an adequate combination of an axicon function and a spiral function. We also showed that the size of the annular vortex on the back Fourier plane of the transforming lens is independent of topological charge of spiral phase function. Finally, numerical and experimental results for the free space diffraction propagation of perfect optical vortex, after the back Fourier plane, show that their intensity pattern depends on topological charge and propagation distance.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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