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Review Article

Phase Singularities to Polarization Singularities

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Polarization singularities are superpositions of orbital angular momentum (OAM) states in orthogonal circular polarization basis. The intrinsic OAM of light beams arises due to the helical wavefronts of phase singularities. In phase singularities, circulating phase gradients and, in polarization singularities, circulating ϕ_{12} Stokes phase gradients are present. At the phase and polarization singularities, undefined quantities are the phase and ϕ_{12} Stokes phase, respectively. Conversion of circulating phase gradient into circulating Stokes phase gradient reveals the connection between phase (scalar) and polarization (vector) singularities. We demonstrate this by theoretically and experimentally generating polarization singularities using phase singularities. Furthermore, the relation between scalar fields and Stokes fields and the singularities in each of them is discussed. This paper is written as a tutorial-cum-review-type article keeping in mind the beginners and researchers in other areas, yet many of the concepts are given novel explanations by adopting different approaches from the available literature on this subject.

1. Introduction

A singular point is characterized by an undefined physical parameter surrounded by a region of high gradient [1–3]. In electromagnetic fields, at a phase singularity, the phase and, at a polarization singularity, the polarization parameter azimuth is undefined. Singular optics is a new area which studies the singularity that occurs in any of the parameters that define optical fields. Azimuth refers to the angle the major axis of an ellipse of an elliptical polarization state makes with respect to a reference direction (say x–axis) [4–7]. The phase of the S_{12} Stokes field indicates the azimuth [8–10]. At the immediate neighborhood of a phase singularity, all phase values ranging from 0 to $2\pi m$ are present [11, 12], where m is the topological charge. Similarly, in the neighbourhood of a polarization singularity, the Stokes phase ϕ_{12} has values ranging from 0 to $2\pi\sigma_{12}$ [8, 9], where σ_{12} is the Stokes polarization singularity index. The phase gradient in a phase singularity and azimuth gradient in a polarization singularity circulate around the respective singularities. The phase and azimuth contours emanate from a phase and a polarization singularity, respectively.

While there are plethora of research articles available in literature on phase singularity [11, 13–30], the literature on polarization singularity is limited in number. Due to additional parameters associated with polarized light fields, the subject of polarization singularity has more complexity. In this article, some of the basics of polarization singularity and related useful tools in its understanding are presented. The paper is organized as follows: the article starts with a brief note on phase singularity in Section 2, followed by polarization singularity in Section 3; the subject of phase control to azimuth (ϕ_{12} Stokes phase) control and generation method for polarization singularities is presented in Section 4; polarization parameters and Stokes phases, useful in the study of polarization singularities, are presented in Sections 5 and 6, respectively; factors deciding the ellipticity and azimuth in different orthogonal decomposition schemes are also presented in Section 5; Stokes parameters and Poincaré sphere are also covered in Section 5 for completeness of the subject; Stokes phase distribution is explained in Section 6; converting phase distribution into Stokes phase distribution is presented in Section 7; we experimentally demonstrate the generation of polarization singularities such as bright C-points (lemons and stars), dark C-points, polarization flowers, and spider webs using Mach–Zehnder-type interferometer in Section 8; recently introduced topological spheres to represent ellipse and vector field singularities are presented in Section 9; in the last section, literature survey on phase and polarization singularities is presented that explains the current status of the field. This paper is written as a tutorial-cum-review-type article keeping in mind the beginners and researchers in other areas. Care is taken to avoid complications by adopting to simple explanations, yet many of the concepts are given novel explanations by adopting different approaches from the available literature on this subject.

2. Phase Singularity

The complex field of a phase singular beam is of the form given as follows:

$$\widetilde{U}(x, y) = f(r)\exp\{i\phi\} = f(r)\exp\{im\theta\}, \tag{1}$$

where $f(r)$ is the amplitude distribution. The phase singularity is also known as a phase vortex or a scalar vortex or an optical vortex in literature. The index m in equation (1) is called topological charge of the vortex and is defined as

$$m = \frac{1}{2\pi}\oint \nabla\phi \cdot \mathrm{d}l. \tag{2}$$

The wave fronts have helical shape [11, 14, 31, 32]. The topological charge can take positive and negative integer values [16, 32] depending on the handedness of the helical wavefront. The phase distribution ϕ is given by azimuthally varying function $m\theta$, where θ is the polar angle. The transverse phase gradient [18, 33–35] for this vortex is given by $\nabla\phi = (m/r)\widehat{\theta}$. This phase gradient is mainly circulating, and near the vortex core, its magnitude is high [11, 32, 36] and it also has some radial component [37–39]. The phase contours of different phase values terminate at the vortex (singular point), resulting in phase ambiguity, and therefore, the amplitude is zero at the vortex core. Figure 1 shows phase distributions, phase gradients, phase contours, and wavefronts of the scalar vortex beams with different topological charges. The amplitude distribution $f(r)$ in a phase singular beam can be that of Laguerre–Gaussian, or of the form r^m, and so on [40]. Some possible intensity distributions for a vortex beam of topological charge $m = 1$ are depicted in Figure 2. Note that, in all types of intensity variations, at the vortex core ($r = 0$), the amplitude is zero and is surrounded by a doughnut-type intensity distribution. At the singular point, the zero of real part as well as the zero of imaginary part of the wave function crosses each other.

Phase singularities have many applications [22, 41–53]. However, they are not useful in some situations [54–56]. Use of a fork grating [32] or a spiral phase plate [57] is common among many generation methods [58–69] reported in literature. Many reports exist in the arrays of phase singularities [19, 70–72]. Generation of phase singular beams has also been reported in nonlinear media [73–77]. Among the various detection methods of phase singular beams, interferometric [78, 79] and diffractive [80, 81] methods are commonly used due to their simplicity.

3. Polarization Singularity

Beams with slowly and spatially varying polarization distributions have attracted interest in recent years [8, 9, 82–89]. Polarization singularities occur in inhomogeneously polarized light fields that have spatially varying polarization distribution. Fields in which the distribution of state of polarization (SOP) is predominantly elliptical are called ellipse fields, and one such field is shown in Figure 3(a). On the contrary, fields in which the predominant SOP distribution is that of linearly polarized states are called vector fields. A vector field distribution is shown in Figure 3(b). Polarization singularities of ellipse fields are called C-points and that of vector fields are called V-points. In random fields, elliptical and linear polarization states can occur in different regions of the same field.

In an inhomogenously polarized ellipse field, an isolated polarization singular point is termed as C-point at which the SOP is circular. At circular polarization, the orientation of the major axis of polarization ellipse is undefined, and hence, it is a singularity in the azimuth distribution. In vector fields, the V-point singularity is an intensity null point at which the polarization azimuth is undefined. Basically, in both types of singularities, in the immediate neighborhood of the singularity, as one goes around the singular point in a positively oriented closed path (anticlockwise), the azimuths undergo rotation in a clockwise or anticlockwise sense. This rotation of the azimuths of neighbouring SOPs around a C-point and V-point singularity is depicted in Figures 4(a) and 4(b), respectively. The amount of rotation the azimuth undergoes around a polarization singularity can be found by evaluating the integral $(1/2\pi)\oint \nabla\gamma \cdot \mathrm{d}l$, where γ is the azimuth of the polarization ellipse. In case of linear polarization, the handedness is undefined. Linear polarization that normally occurs in ellipse fields along points on a curve is referred as a L-line. It segregates the regions of right and left handedness in an ellipse field distribution.

3.1. C-Points. For C-point singularities, C-point index I_C is defined by

$$I_C = \frac{1}{2\pi}\oint \nabla\gamma \cdot \mathrm{d}l. \tag{3}$$

For C-points, the attributes such as index, bright, dark, left-handed, and right-handed are decided by their OAM superpositions in the orthogonal circular basis states [90–94]. Generic C-points are lemon ($I_C = +(1/2)$), monstar ($I_C = +(1/2)$), and star ($I_C = -(1/2)$). The SOP distribution of a lemon and a star is shown in Figures 5(a) and 5(b), respectively. In these figures, filled ellipses in red color are right-handed (RH) and those drawn in blue color are left-handed (LH). C-points can be further classified based on handedness $h^{\pm}$. Thus, a C-point with a given C-point index can be right-handed (h^{+}) or left-handed (h^{-}) [91, 95–97]. This means, for example, in Figure 5(a), SOP distribution for a RH lemon is shown and a lemon which is LH is also possible. C-points can occur at any value of intensity. For example, C-points in Figures 5(a), 5(b), and 5(d) with

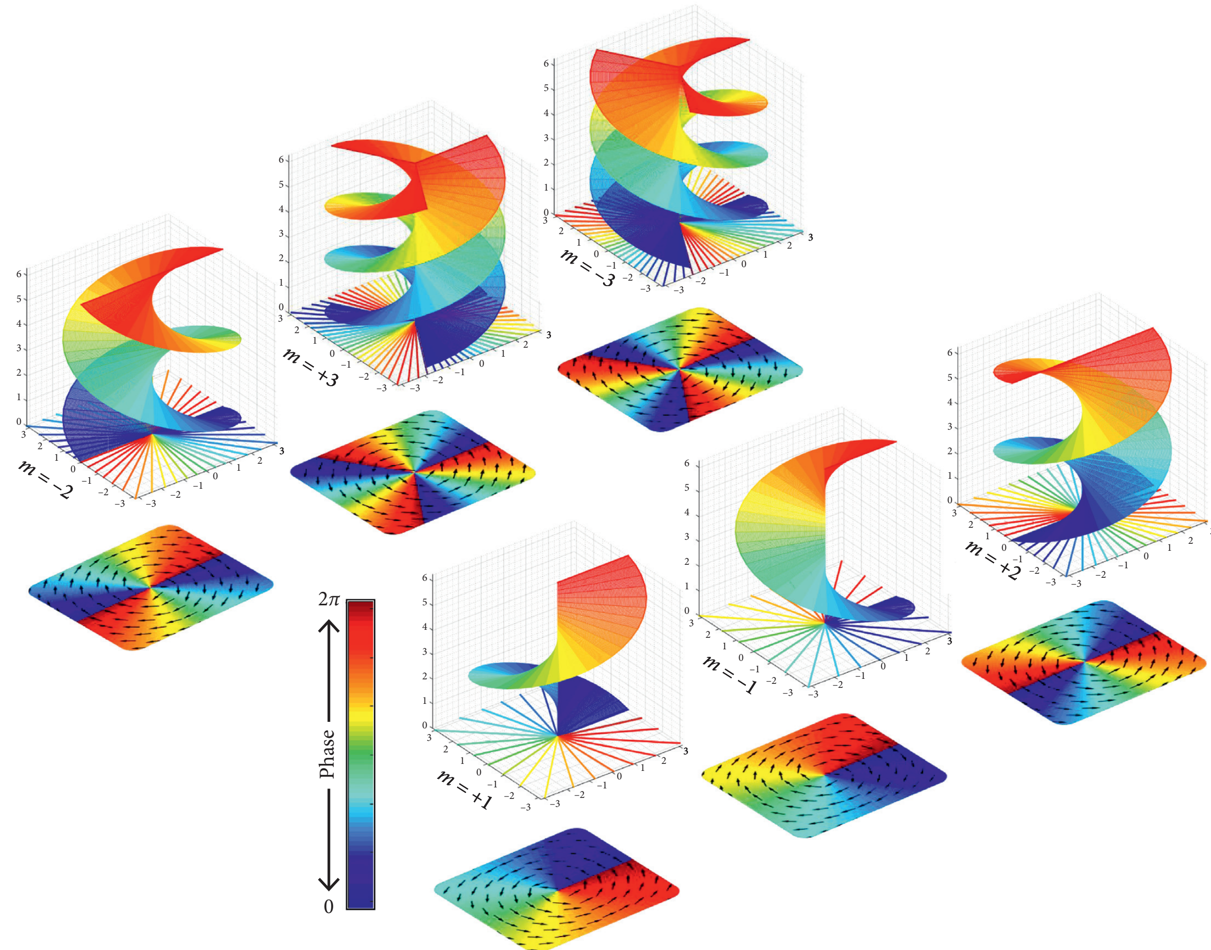

FIGURE 1: Phase distributions, phase gradients, and wavefronts of phase singular beams with different topological charges. Phase contours are shown below each wavefront.

respective indices $I_C = 1/2$, $I_C = -1/2$, and $I_C = -2$ are bright, whereas C-point ($I_C = -3/2$) in Figure 5(c) occurs at intermediate value of intensity. Two examples of dark C-points with index $I_C = -2$ and $I_C = 5/2$ are shown in Figures 5(e) and 5(f), respectively. In Figures 5(a)–5(f), the RH- and LH-handed regions are separated by a closed curve (L-line) on which SOPs are all linear. Anisotropic C-point is a monstar [98–101], and it has the index value $I_C = +(1/2)$. The C-point singularities are referred in recent literature as hybrid-order Poincaré sphere beams.

3.2. V-Points. V-point singularities are the polarization singularities occurring in linearly polarized light fields. They are characterized by Poincaré–Hopf index defined as

$$\eta = \frac{1}{2\pi} \oint \nabla\gamma \cdot \mathrm{d}l. \tag{4}$$

The handedness of V-points is undefined as they are made of linear states. V-points always occur at intensity null [102]. In Figures 5(g)–5(i), V-points with three different Poincaré–Hopf indices are shown. Even though V-points are devoid of handedness, during dissociation, they disintegrate into equal number of left- and right-handed C-points [103]. When diffracted through fork grating [104], a V-point segregates into C-points of one handedness in positive diffraction orders and of opposite handedness in negative diffraction orders. Helicity conservation has also been observed in diffraction scattering [105]. In recent literature, the V-point polarization singularities are referred as higher order Poincaré sphere beams. Spirally polarized beams are superposition states of radially and azimuthally polarized beams. The critical points in spirally polarized beams are characterized using streamline morphologies equivalent to the stability theory of autonomous systems of ordinary differential equations [106].

4. Phase Gradient to Azimuth Gradient

A comparison of equation (2) with equations (3) and (4) reveals the similarity in the form of line integrals. The term $\nabla\phi$ in equation (2) is replaced by $\nabla\gamma$ in equations (3) and (4). To have a polarization singularity, conversion of phase variation into azimuth variation is required. In the next section, we make a comparative study of the conventional linear decomposition of polarization state with circular decomposition. It is shown that, in circular decomposition, the phase difference between the right and left circular

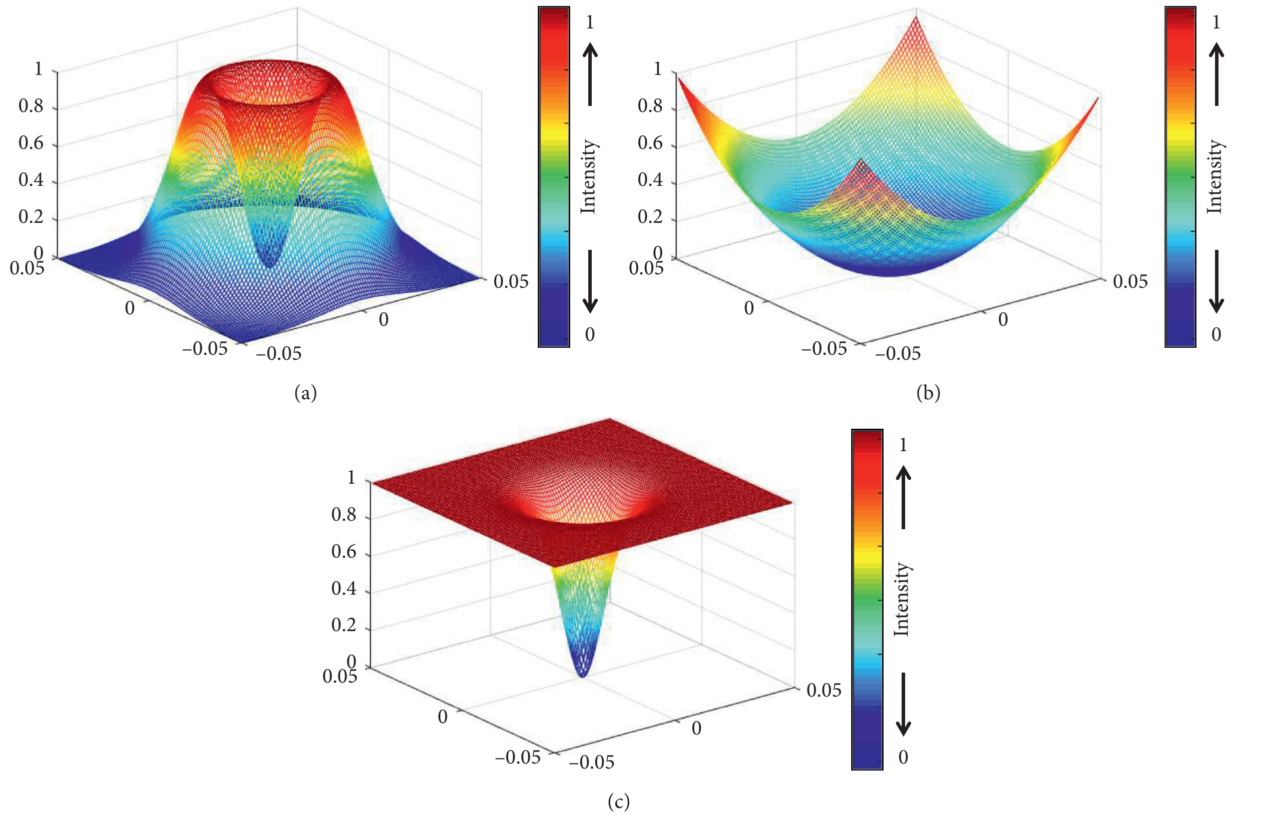

Figure 2: Possible intensity distributions for an optical vortex beam of topological charge $m = 1$. Note that amplitude is zero at the vortex core ($r = 0$) and is surrounded by a doughnut-type intensity distribution. (a) LG variation; (b) r variation; (c) tanh variation.

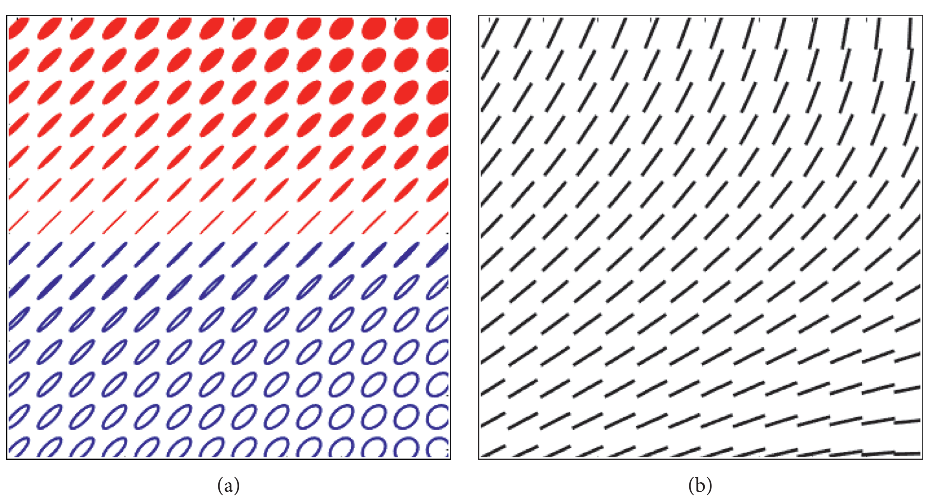

Figure 3: Inhomogeneous polarization distributions: (a) an ellipse field distribution; (b) a vector field distribution.

polarization components of a given SOP can be varied to achieve the azimuth variation. In superpositions, the phase difference between two fields which are in same SOP leads to intensity modulation, whereas the phase difference between orthogonal polarization basis states modulates the SOP of the light. This phase difference between orthogonal states is termed as the Stokes phase. Stokes phase and its importance are explained in subsequent sections. The phase singularities in the Stokes phase distributions are called Stokes singularities. This entails a revisit to the basics of polarization, which is presented in the next section.

5. Polarization

Polarization of light [4–7] refers to the study of temporal variation in electric field vector of light. We consider here

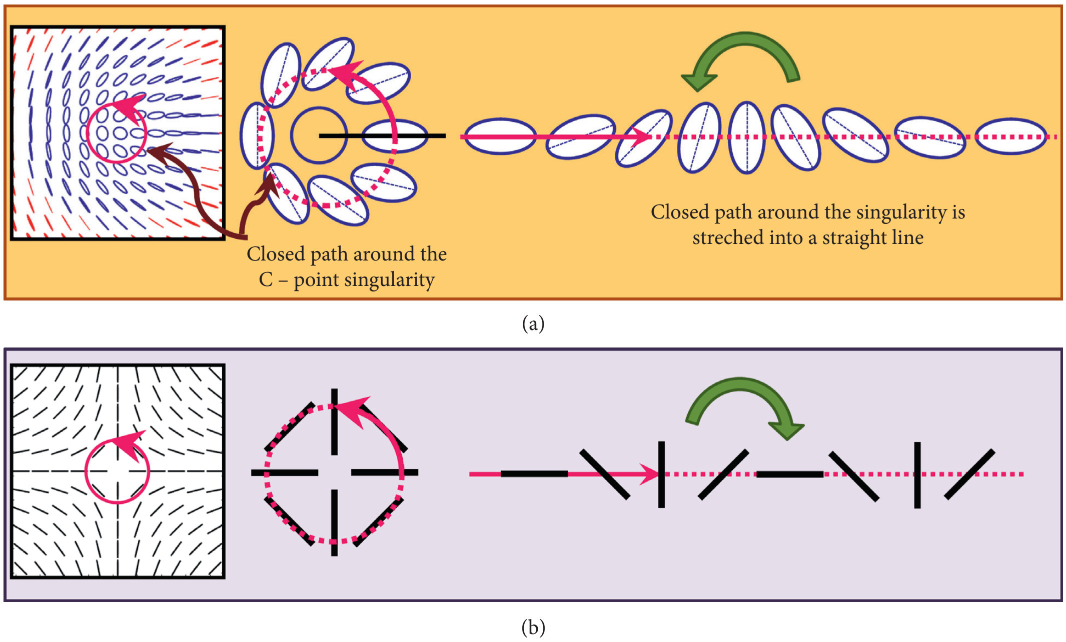

Figure 4: Rotation of the azimuths of the surrounding SOP states around a singular point: (a) a left-handed positive index C-point with $I_C = 1/2$; (b) a negative index V-point with $\eta = -1$. The pink arrow denotes the sense of traversal of a closed path around the singularity, i.e., anticlockwise. The green arrow depicts the sense of rotation of the major axes of ellipses (plane of polarization of linearly polarized state in case of a V-point) around the singularity.

two-dimensional paraxial fields, which means that the orientations of polarization ellipses in three-dimensional space are safely projected on to a two-dimensional plane. The amplitude and phase of the component oscillations of a given SOP during orthogonal decomposition are crucial. In this section, a brief note on the polarization parameters of interest to us is given.

5.1. Azimuth and Ellipticity. Even though there are various parameters available to describe the SOP of light, we first introduce the following two parameters that are crucial in the understanding of polarization singularities. These two parameters are ellipticity χ and azimuth γ. For a fully polarized light, ellipticity is defined as $\tan\chi = \pm b/a$, where a and b are the major and minor axis of the polarization ellipse, positive sign is for right-handed ellipse, and the negative sign is for the left-handed ellipse. It varies between $-\pi/4$ and $+\pi/4$. The azimuth γ is the orientation angle of the major axis of the ellipse with respect to a reference direction (usually x axis). It varies between $-\pi/2$ and $+\pi/2$. In Figure 6, polarization ellipses with different azimuths and/or ellipticities are depicted. SOPs with constant ellipticity but different azimuths are shown in first row in Figure 6. In the second row, polarization ellipses with constant azimuth but different ellipticities are shown. The polarization ellipses with varying both azimuth and ellipticity are depicted in the third row of Figure 6.

5.2. Decomposition. The azimuth and ellipticity of a given SOP can be given in terms of component polarization states. We present here two types of decompositions, namely, linear and circular decompositions.

5.2.1. Linear Decomposition. Any SOP of light can be decomposed into two linearly polarized orthogonal states. In other words, any SOP can be represented as a superposition of two linearly polarized states. Consider the following superposition

$$\hat{n} = \left\{a_x e^{i\phi_x}\hat{x} + a_y e^{i\phi_y}\hat{y}\right\}, \tag{5}$$

where a_x and a_y are the component amplitudes of the simple harmonic oscillations (for a coherent monochromatic light) occurring in xz and yz planes and ϕ_x and ϕ_y are the corresponding component phases. Circularly polarized light occurs when $a_x = a_y$ and $\Delta\phi_{\mathrm{LD}} = \phi_y - \phi_x = (2n+1)(\pi/2)$. Linear polarization occurs when $\Delta\phi_{\mathrm{LD}} = n\pi$ irrespective of the values of a_x and a_y. Elliptical polarization occurs when linear and circular polarization conditions are not met. Linear decomposition is explained in most common text books on polarization [4–7]. In linear decomposition, the azimuth is decided by the ratio of component amplitudes, and ellipticity is decided by the difference between the component phases. To have a nonzero ellipticity, the condition on phase difference between the two component oscillations is $\Delta\phi_{\mathrm{LD}} \neq n\pi$. But there is also another way of decomposition, namely, circular decomposition, which is very useful in the study of polarization singularities and is given below.

5.2.2. Circular Decomposition. Light in any SOP can be decomposed into two circularly polarized orthogonal states [9, 91, 92, 107]. In this case, the two component oscillations are clockwise (left) and counterclockwise (right) rotating circularly polarized light with amplitudes a_{L} and a_{R}, respectively. The superposition state here is given by

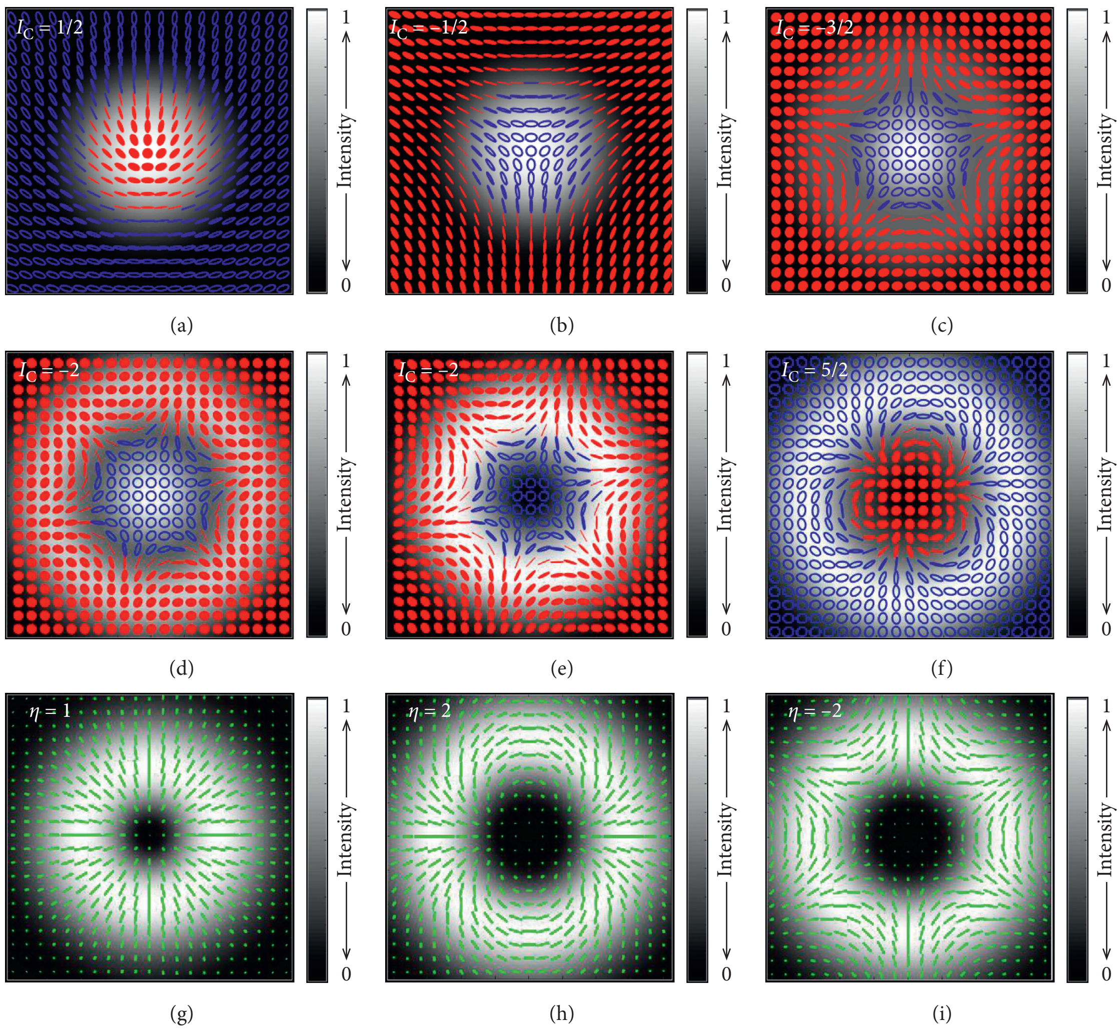

FIGURE 5: Polarization distributions containing (a–f) C-points and (g–i) V-points. Indices of C-points and V-points are mentioned above each figure. (a) and (b) are generic C-points and are commonly known as lemon and star, respectively. (a–d) Bright C-points; (e-f) dark C-points. Note that, in (d) and (e), a C-point with the same index can be dark or bright. V-points always occur at zero intensity. (g) A generic V-point known as radially polarized; (h) a polarization flower; (i) a spider web.

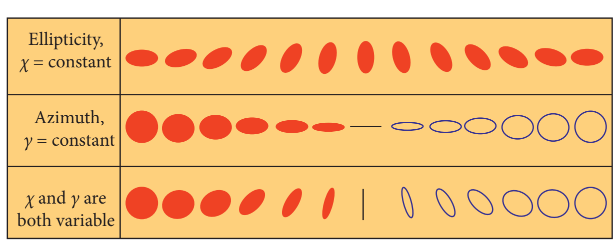

FIGURE 6: Polarization ellipses with different azimuths and/or ellipticities.

$$\hat{n} = \left\{ a_R e^{i\phi_R} \hat{R} + a_L e^{i\phi_L} \hat{L} \right\}, \tag{6}$$

where ϕ_R and ϕ_L are the phases of the component oscillations. In this decomposition, the linear states occur when the component oscillations have the same amplitude, i.e., $a_R = a_L$, and elliptical states occur when the component oscillations are such that $a_R \neq a_L$. The orientation of the plane of polarization or the orientation of the major axis of the ellipse is decided by the phase difference between two circular components $\Delta\phi_{CD} = \phi_L - \phi_R$.

Between the two types of decompositions, in achieving the azimuth and ellipticity controls, the conditions on component amplitudes and phases are reversed. Figure 7 summarizes the variation in azimuth and ellipticity in linear and circular decompositions. Note that, in linear decomposition, as one moves from left to right, change in

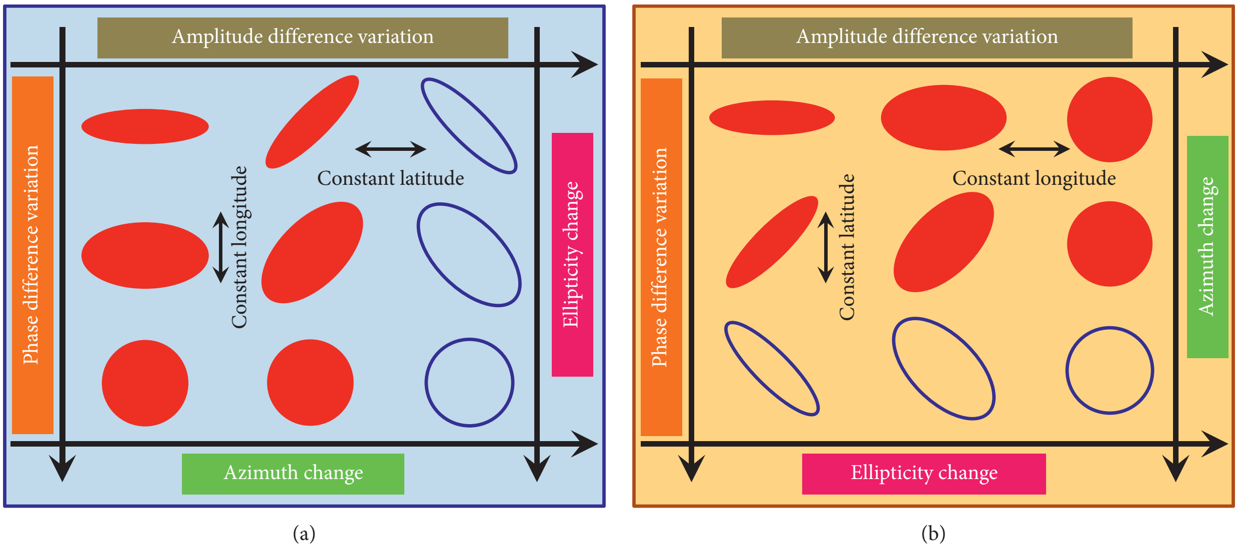

FIGURE 7: Role of amplitude and phase difference variation in azimuth and ellipticity in (a) linear and (b) circular basis.

amplitude changes the azimuth of polarization state, but ellipticity remains constant. Similarly, as one moves from top to bottom, change in phase difference between two orthogonal linear states leads to ellipticity change and azimuth remains constant. On the contrary, in circular decomposition, as one moves from left to right (top to bottom), change in amplitude (phase) between two orthogonal circular polarization components produces change in ellipticity (azimuth).

5.3. Stokes Parameters. The state of polarization of light can be described using Stokes parameters [4–7]. Using intensity measurements, these parameters can be found as

$$\begin{aligned} S_0 &= I_x + I_y, \\ S_1 &= I_x - I_y, \\ S_2 &= I_{45^\circ} - I_{-45^\circ}, \\ S_3 &= I_{RCP} - I_{LCP}, \end{aligned} \tag{7}$$

where I_x, I_y, I_{+45°, I_{-45°, I_{LCP}, and I_{RCP} are the component intensities when a given SOP is decomposed into linear states oriented along $\hat{x}$, $\hat{y}$, $(+45^\circ)$, (-45°), and circular states: left circularly polarized (LCP) and right circularly polarized (RCP) states, respectively. For a fully polarized light, $S_0^2 = S_1^2 + S_2^2 + S_3^2$. For a linearly polarized light, the Stokes parameter $S_3 = 0$ and $S_1^2 + S_2^2 = S_0^2$; for circularly polarized light, both $S_1 = 0$ and $S_2 = 0$ and $S_3 = \pm S_0$. Unlike homogeneous polarization distribution, for inhomogeneous polarization distribution, all the Stokes parameters are functions of position coordinates. This means the Stokes parameters $S_i(x, y)$, $i = 0$ to 3, are functions of two variables.

5.4. Geometric Representation of Polarization. Any general SOP of light can be represented by a point on the surface of a Poincaré sphere. Poincaré sphere is a unit radius sphere constructed using three normalized Stokes parameters (S_1, S_2, and S_3) as three coordinate axes. The poles on the sphere represent orthogonal circular polarization states, and points on the equator represent linear polarization states. All other points in Northern and Southern hemisphere correspond to right- and left-handed elliptical polarization states, respectively. A Poincaré sphere with various SOPs is shown in Figure 8(a). The parameters azimuth (γ) and ellipticity (χ) of a polarization ellipse are related to Stokes parameters as follows:

$$\begin{aligned} \gamma &= \frac{1}{2}\tan^{-1}\left(\frac{S_2}{S_1}\right), \\ \chi &= \frac{1}{2}\sin^{-1}\left(\frac{S_3}{S_0}\right). \end{aligned} \tag{8}$$

The SOPs on a particular latitude maintain constant ellipticity, whereas SOPs on a longitude maintain constant azimuth. This is depicted in Figure 8(b).

Stokes parameters when defined in terms of the orthogonal polarization field components take the form, as mentioned in Table 1. The subscripts p and q correspond to $\hat{x}$ and $\hat{y}$ in linear decomposition, $+45^\circ$, -45° in linear diagonal decomposition, and $\hat{R}$ and $\hat{L}$ in circular decomposition. The Stokes parameter S_0 is always the sum of orthogonal component intensities in respective decomposition, whereas the expressions of Stokes parameters (S_1, S_2, and S_3) change cyclically in three different field decompositions.

5.5. Helicity and Spin. Helicity and spin are different [108], and treating them as synonymous to each other is a common mistake committed by many. Photons are bosons, integer spin (spin = ± 1) particles, with spin angular momentum (SAM) of $\hbar$ for right circular polarized light and $-\hbar$ for left circular polarized light. In the circular basis decomposition

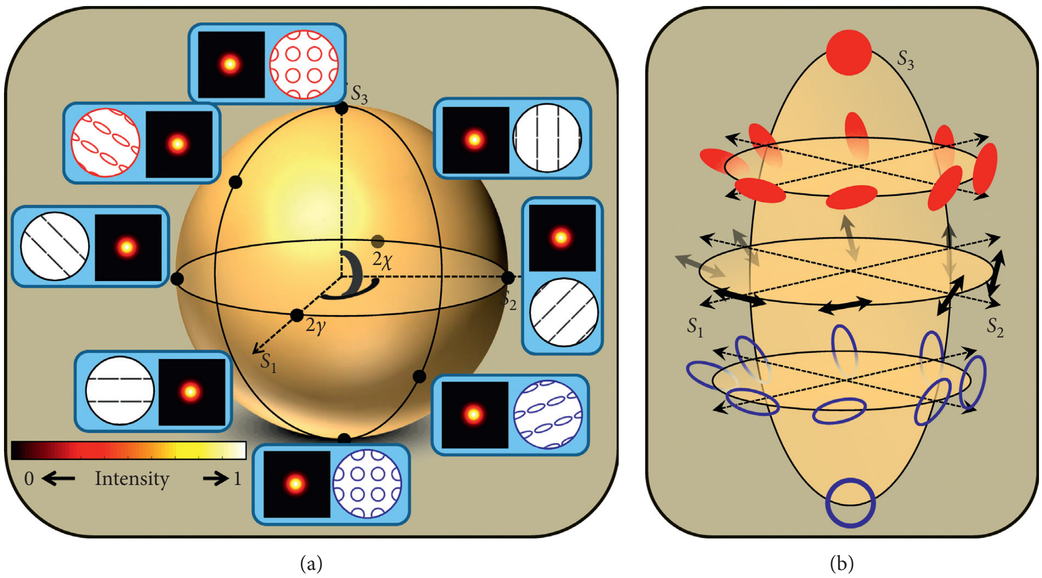

Figure 8: (a) A Poincaré sphere: geometric representation of homogeneous polarization distribution; (b) polarization variation along azimuth and ellipticity contours.

Table 1: Stokes parameters: cyclic rotation of Stokes parameters, S_i, $(i = 1, 2, 3)$ in three different decompositions.

Decomposition	Linear	Linear diagonal	Circular
$\lvert E_p\rvert^2 + \lvert E_q\rvert^2$	S_0	S_0	S_0
$\lvert E_p\rvert^2 - \lvert E_q\rvert^2$	S_1	S_2	S_3
$2\mathrm{Re}\,(E_p{}^* E_q)$	S_2	S_3	S_1
$2\mathrm{Im}\,(E_p{}^* E_q)$	S_3	S_1	S_2

The subscripts "p" and "q" in linear basis represent "x" and "y"; in diagonal basis, they represent "+45°" and "−45°"; in circular basis, they represent "R" and "L," respectively.

(equation (6)), we have seen that any polarization state of light is shown as a superposition of right and left circular polarization components (superposition of positive and negative spin states). The component amplitudes decide the helicity (or handedness) of the superposition state. If the right circular polarization component is larger than the left circular polarization component $(a_R > a_L)$, then the resulting elliptical polarization state is said to be right-handed ellipse state and so on. Traditionally, right-handed (left-handed) polarizations are considered as positive (negative) helicity (handedness). For example, a left elliptically polarized light has negative helicity (handedness) but has both positive and negative spin components. Therefore, spin and helicity are different parameters. For linear polarization, both the right and left circular polarization components are equal $(a_R = a_L)$, and therefore, there is no handedness associated with the linear states.

6. Stokes Phase

Using the Stokes parameters, Stokes fields can be constructed. These fields are mathematical constructions. For example, using Stokes parameters S_1 and S_2, a complex field, namely, $S_{12} = S_1 + iS_2 = A_{12}\exp\{i\phi_{12}\}$ field, can be constructed [8, 9, 109, 110]. The Stokes phase $\phi_{12} = \tan^{-1}(S_2/S_1)$ and is equal to 2γ, as given in equation (8). Hence, the phase vortices of complex S_{12} Stokes field are the polarization singularities. Therefore, constructing a Stokes field from the measured Stokes parameters is helpful in identifying the polarization singularities. In Figure 9(a), in the polarization distribution, presence of two polarization singularities with opposite I_C index can be identified as two phase singularities in the Stokes phase distributions. A V-point singularity and its Stokes phase distribution is shown in Figure 9(b).

However, there are certain limitations in using the Stokes phase. The Stokes phase distribution does not distinguish the right- and left-handed C-point singularities. It does not distinguish between dark and bright C-point singularities. Also, it does not distinguish between integer charged C-points and V-point singularities. As an example, four different polarization distributions with the same Stokes phase distribution are illustrated in Figure 9(c). The Stokes index $\sigma_{12} = (1/2\pi)\oint \nabla\phi_{12}\cdot dl$ is connected to I_C index and Poincaré Hopf index η by the relation $\sigma_{12} = 2I_c = 2\eta$. Similar to Stokes phase degeneracy, intensity degeneracies have been observed in interference and diffraction of polarization singularities [111].

6.1. Stokes Phase and Scalar Fields. Phase distribution of S_{12} Stokes field is found to be related to phase difference between RCP and LCP components in circular basis. From Table 1, the corresponding expressions for S_1 and S_2 in circular basis are

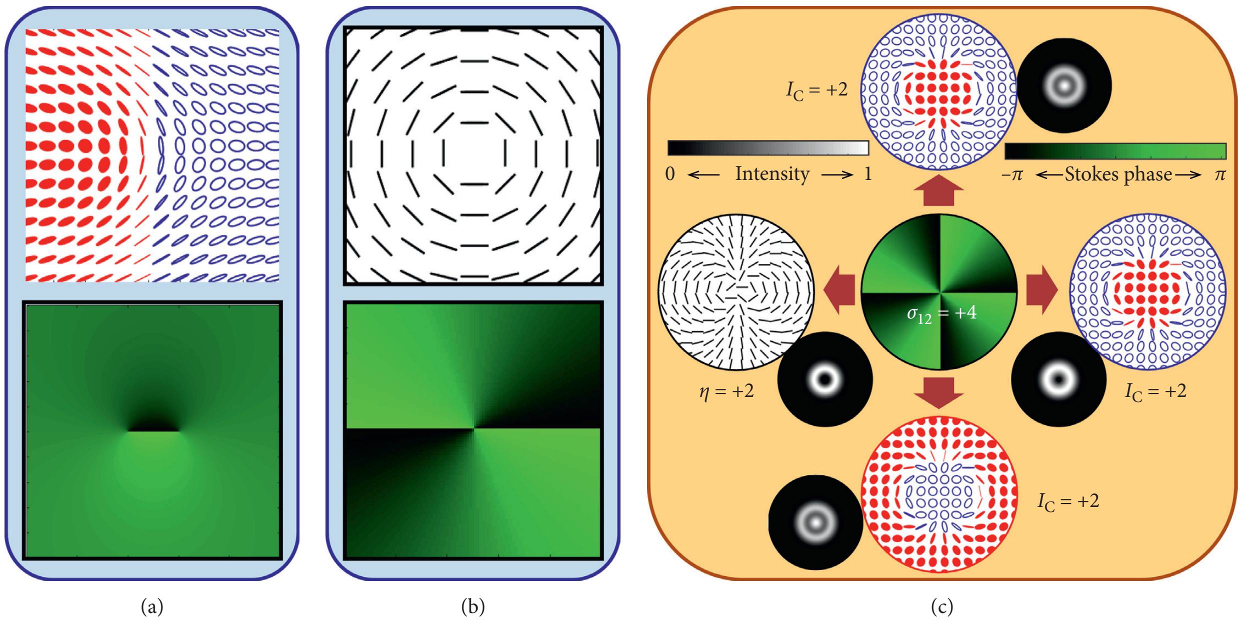

Figure 9: Polarization distribution and Stokes phase of (a) dipole field and (b) a V-point field. (c) Limitations in Stokes phase: the Stokes phase has same spatial distribution for a V-point of index $\eta = 2$, right-handed and left-handed bright C-point of index $I_C = 2$, and a dark C-point of index $I_C = 2$, and thus, the Stokes phase is degenerate.

$$S_1 = 2\mathrm{Re}(E_R^* E_L) = 2a_R a_L \cos(\phi_L - \phi_R),$$
$$S_2 = 2\mathrm{Im}(E_R^* E_L) = 2a_R a_L \sin(\phi_L - \phi_R). \tag{9}$$

Using equation (9), we construct S_{12} Stokes field as

$$\begin{aligned} S_{12} &= S_1 + iS_2 = A_{12}\exp\{i\phi_{12}\} \\ &= 2a_R a_L\{\cos(\phi_L - \phi_R) + i\sin(\phi_L - \phi_R)\} \\ &= 2a_R a_L \exp\{i(\phi_L - \phi_R)\}. \end{aligned} \tag{10}$$

Likewise, using expression of S_2 and S_3 from Table 1, the phase distribution of S_{23} Stokes field is found to be related to phase difference between $\hat{x}$ and $\hat{y}$ components in linear basis as

$$S_{23} = S_2 + iS_3 = A_{23}\exp\{i\phi_{23}\} = 2a_x a_y \exp\{i(\phi_y - \phi_x)\}. \tag{11}$$

Similarly using Table 1, phase distribution of S_{31} Stokes field is found to be related to phase difference between 45° and 135° components in linear diagonal basis as

$$S_{31} = S_3 + iS_1 = A_{31}\exp\{i\phi_{31}\} = 2a_{45^\circ}a_{135^\circ}\exp\{i(\phi_{135^\circ} - \phi_{45^\circ})\}. \tag{12}$$

It is surprising to see by using component intensities in linear states, the phase difference between circular basis states can be obtained. That is by using S_1 and S_2 the phase difference between left and right circular basis states $\phi_L - \phi_R = \phi_{12}$ can be obtained. Similarly, by using S_2 and S_3 the phase difference between vertical and horizontal component states ϕ_{23} can be obtained. Note neither S_2 nor S_3 need intensity measurements in horizontal and vertical states. Similar arguements hold good for ϕ_{31}.

7. Phase Distribution and Phase Difference Distribution

The phase distribution of a scalar field is that of the complex field, whereas the Stokes phase distribution is that of phase difference distribution. Phase distribution of a scalar field corresponds to wavefront structure, which is seen in homogeneous polarization distribution. In the interference of two scalar beams both in the same SOP, the resulting intensity variation depends on the phase difference between the interfering beams. These intensity modulations are referred as intensity fringe pattern in text book. But if the two interfering scalar fields are such that one is in one SOP and other in the orthogonal SOP to the first one, then the phase difference between these beams does not modulate the intensity but changes the SOP in the resultant field. In other words, one can realize polarization fringes instead of intensity fringes by changing the phase difference between orthogonal polarization components of any field. Interestingly, this phase difference distribution is found to be linked to phase distribution of a complex Stokes field. Referring to equations (10)–(12), the Stokes phases are related to phase differences and are reproduced as $\phi_{12} = \phi_L - \phi_R$, $\phi_{23} = \phi_y - \phi_x$, and $\phi_{31} = \phi_{135^\circ} - \phi_{45^\circ}$. Surprisingly, the Stokes phases which are phase differences between orthogonal polarization states can be obtained from the Stokes parameters which are pure intensity

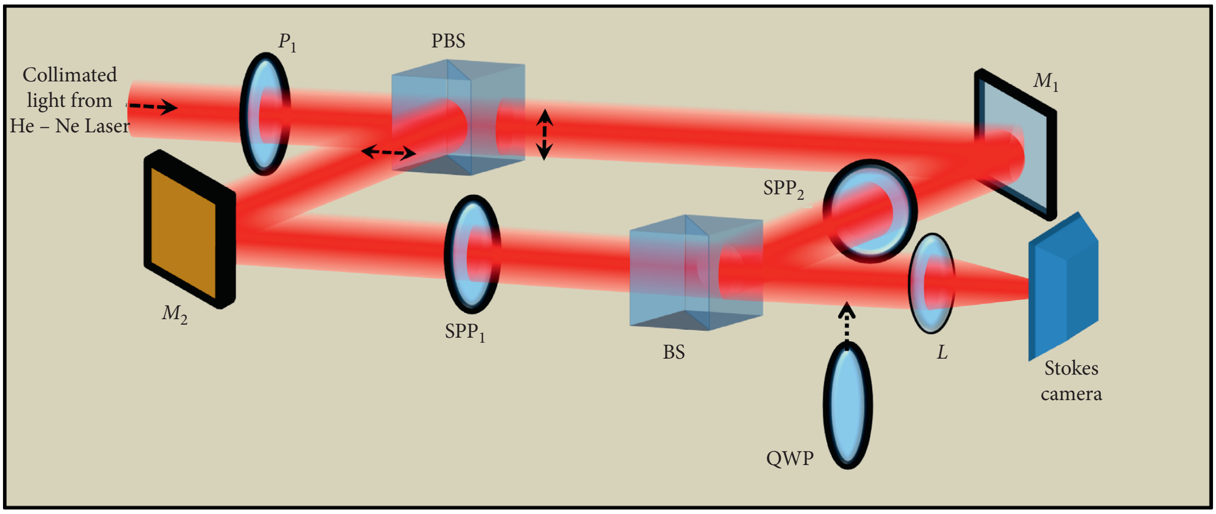

FIGURE 10: Experimental generation of Stokes phases and polarization singularities using scalar vortices. P, polarizer; (P)BS, (polarizing) beam splitter; M_1 and M_2, mirrors; SPP, spiral phase plate; QWP, quarter wave plate; L, lens; stokes camera.

measurements. Of the three Stokes phases, the ϕ_{12} Stokes phase is related to the azimuth of the SOP, and hence, it plays an important role in the polarization singularities as they have circulating $\nabla\gamma$. This can be seen from equations (3) and (4). Polarization singularities are ϕ_{12} Stokes phase vortices. Presence of a vortex in ϕ_{12} Stokes phase indicates that there may be vortices in ϕ_{23} and ϕ_{31} Stokes phases also [110, 112].

8. Experimental Realization of Polarization Vortices from Phase Vortices

In this section, we demonstrate experimental generation of polarization singularities such as bright C-points (lemons and stars), dark C-points, polarization flowers, and spider webs. The interferometer presented in Figure 10 is universal in the sense that it can be used for generating any type of phase as well as polarization singularities.

8.1. Experimental Setup. Collimated He-Ne laser light illuminates a polarizer at 45°. The light coming from the polarizer is equally split into two arms by a polarizing beam splitter (PBS). By inserting spiral phase plates (SPPs) of different charges m and n in the two arms of the Mach–Zehnder-type configuration, it is possible to realize bright C-points, dark C-points, and V-points at the output. In the two arms of the interferometer, SPPs transform the orthogonal linearly polarized plane waves ($\hat{x}$ and $\hat{y}$) into $\hat{x}$ and $\hat{y}$ polarized vortex beams of charges m and n, respectively. Note that the beams coming from the two arms are homogeneously polarized, and each contain a phase vortex. Therefore, by blocking one of the arms of the interferometer, this setup can be used for the generation of phase singularities. Since these beams are homogeneously polarized, we can call them as scalar vortices. The beam splitter (BS) combines these two scalar vortices to form polarization singularities. For superposition to be in circular basis, a quarter wave plate (QWP) at 45° is used after the BS. The Stokes camera (SALSA: Full Stokes Polarization Imaging camera, Bossa Nova, USA) is used to record the experimental Stokes parameters, and these parameters are used to plot the corresponding polarization distributions. The different combinations of m and n lead to generation of bright C-points, dark C-points, and V-points. These are illustrated in Figure 11 and elucidated in following sections.

8.2. Case 1: Bright C-Points ($m = 0$ and $n \neq 0$) or ($m \neq 0$ and $n = 0$). In interference, bright C-points are generated when one of the superposing beams is a nonvortex beam, i.e., the beam has a plane wavefront. Figures 11(a)–11(d) depict bright C-points with (m, n) combinations as $(0, 1)$, $(1, 0)$, $(3, 0)$, and $(4, 0)$, where m and n are topological charges of scalar vortices in right circular polarization (RCP) and left circular polarization (LCP) states, respectively. The index of the C-point is mentioned in each figure and is given as $I_C = (n - m)/2$. Note that, in each case, the handedness of the C-point is decided by the handedness of the nonvortex beam. In the superposition, the amplitude corresponding to the vortex core is zero, whereas the amplitude corresponding to the nonvortex beam is nonzero at the same point. Hence, the resultant state is circular, and its handedness is that of nonvortex beam. At every other neighbourhood points, since the amplitudes of the two beams are unequal in the circular basis superposition, elliptical states result. The phase difference between the two beams (Stokes phase) is that of helical phase, and this leads to rotation of azimuth around the C-point. The C-point in Figure 11(a) is right-handed (RH), whereas they are left-handed (LH) in Figures 11(b)–11(d). C-points can occur at any value of intensity. For example, in Figures 11(a), 11(b), and 11(d), C-points occur at intensity maxima, whereas C-point in Figure 11(c) occurs at intermediate value of intensity. Note for the C-point generation, both the interfering beams must have dissimilar amplitude distribution, and the phase difference between them must be helical.

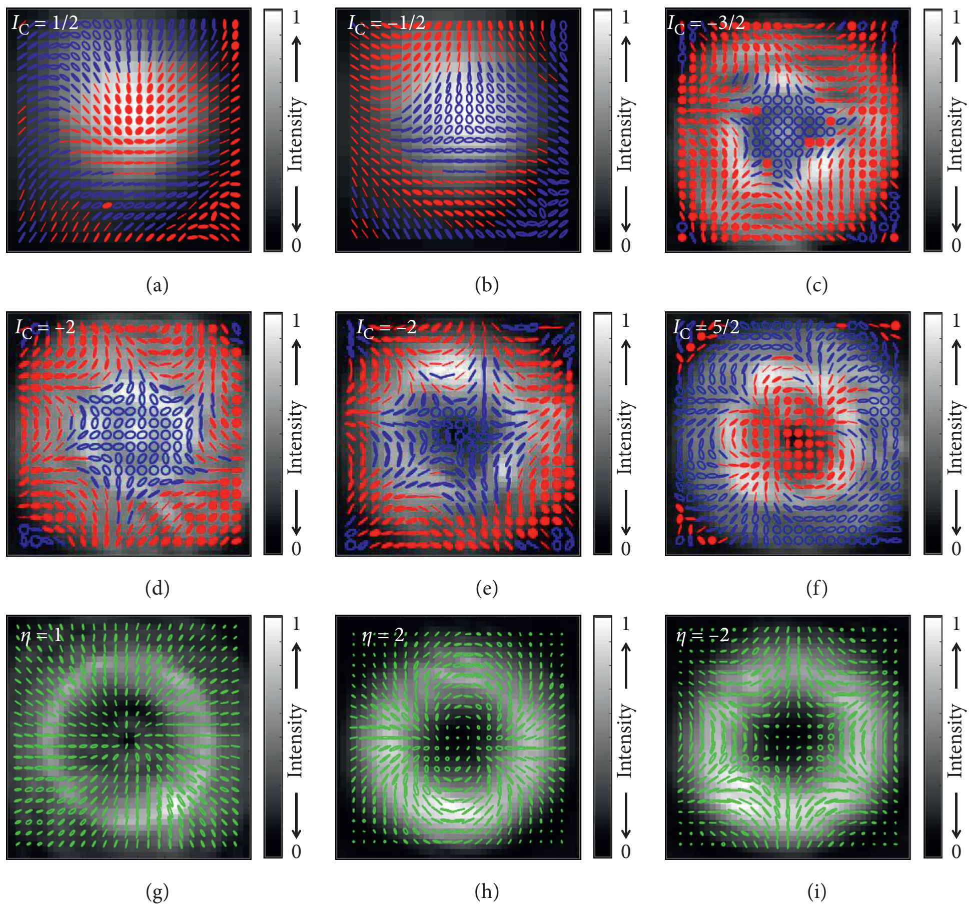

FIGURE 11: Experimentally recorded polarization distributions containing (a–f) C-points and (g–i) V-points. Indices of C-points and V-points are mentioned above each figure. (a–d) Bright C-points; (e-f) dark C-points. V-points always occur at intensity null. (g) A generic V-point; (h) a polarization flower; (i) a spider web.

8.3. Case 2: Dark C-Points ($m \neq n \neq 0$). When both the beams in the interferometer contain vortices such that $m \neq n \neq 0$, dark C-points are produced. Note this combination satisfies the amplitude and phase difference condition for C-point generation. The only difference here is that since both the beams have dark vortex cores, the resulting C-point is a dark C-point. Two examples of dark C-points with indices $I_C = -2$, for the m and n combination $(3, -1)$, and $I_C = 5/2$, for the m and n combination $(-1, 4)$, are shown in Figures 11(e) and 11(f), respectively. Note that, in case of a dark C-point, the handedness is decided by the circular polarization component that has a lower magnitude of topological charge. As the magnitude of vortex in LCP (RCP) component has a lower value, the dark C-point in Figure 11(e) (Figure 11(f)) is LH (RH).

8.4. Case 3: V-Points ($m = -n$). To generate a V-point, same intensity variation should exist in two orthogonal circular polarization components. This can be achieved by introducing SPPs in two arms of the interferometer such that $m = -n$. This combination of vortices generates V-points with index $\eta = (n - m)/2$. Some experimentally generated V-points with index $\eta = 1$, $\eta = 2$, and $\eta = -2$ are shown in Figures 11(g)–11(i), respectively. The respective (m, n) combination for each case is $(-1, 1)$, $(-2, 2)$, and $(2, -2)$. As both the circular components contain a vortex, V-points always occur at intensity minima, as depicted in Figures 11(g)–11(i). Figure 11(h) is an example of a polarization flower as it has two petals, whereas Figure 11(i) is an example of a spider web.

9. Superpositions

We have seen that the ellipse field and vector field singularities can be expressed as the superposition of beams in orthogonal spin and orbital angular momentum states:

$$\vec{E}(r, \theta) = Ar^{|m|} \exp(im\theta)\hat{R} + Br^{|n|} \exp(i(n\theta + \theta_0))\hat{L}, \tag{13}$$

where $\hat{R}$ and $\hat{L}$ are right and left circular unit basis vectors, respectively. A and B are constants, m and n are vortex charges in each beams, and θ_0 is the constant phase shift. In the above equation, the superposition is between two phase vortex beams of unequal charges, and they are in orthogonal polarization states. As far as spatial modes are concerned, two vortex beams with unequal charges themselves are

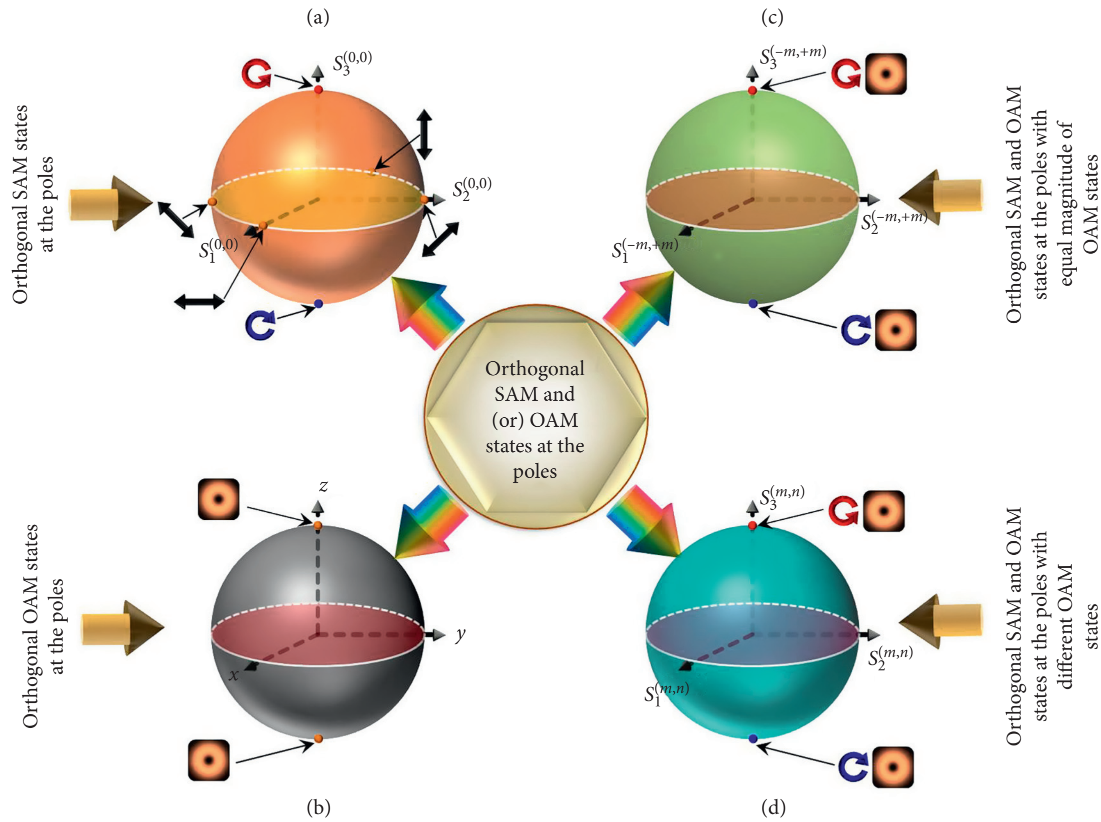

FIGURE 12: Construction of various spheres. (a) Fundamental Poincaré sphere: orthogonal spin angular momentum (SAM) states at the poles. (b) Modal sphere: orthogonal orbital angular momentum (OAM) states at the poles. (c) Higher order Poincaré sphere: orthogonal SAM and OAM states at the poles with equal magnitude of OAM states. (d) Hybrid order Poincaré sphere: orthogonal SAM and OAM states at the poles with different OAM states. Note that in each sphere, the eigenstates are different.

orthogonal to each other. By considering different combinations of vortex charges in right and left circular polarization states, different polarization singularity distributions can be constructed and that state can be represented as a point on either one of the spheres, namely, higher order Poincaré sphere (HOPS) [113–117] or hybrid order Poincaré sphere (HyOPS) [118–121]. Figure 12 depicts construction of various spheres by considering two orthogonal OAM states in same SOP, as basis states to form modal sphere, and then two orthogonal OAM states in two orthogonal SAM states, as basis states to form other types of spheres.

For every pair of vortex state in right and left circular polarization states, a HOPS or HyOPS can be constructed. With infinite number of orthogonal vortex states available, it is therefore possible to realize infinite number of these types of spheres and hence infinite number of polarization singularity distributions. In each sphere, every point represents a polarization singularity distribution, and all the points on the sphere can be realized by changing the values of A and B in that superposition. All the possible polarization singularity distributions in a given sphere have the same polarization singularity index. Because of the way these spheres are constructed, the beams represented by points on each of these spheres have different topological features. Beams represented by points on HOPS are constant ellipticity fields, and beams represented by points on HyOPS are Poincaré beams [112, 122–129]. So far introduced, V-points lie on the surface of HOPS (equatorial points), whereas C-points lie on the surface of HyOPS (equatorial points). It is interesting to note that superposition of vortex beams in orthogonal linear polarization states will also produce Poincaré beams [112]. Elliptically polarized base states were also tried for vector field generation [130]. In the reverse conversion, it is also possible to realize homogeneous polarization distributions as superposition of polarization singularity distributions [131]. We have seen that HOPS beams and HyOPS beams are superpositions. These beams are also called spin-orbit beams or beams in nonseparable states of spin and orbital angular momenta. A separable state is a state that can be written as a single product of spin state and orbital state similar to the way separable functions are defined, for example, $f(x, y) = f(x) f(y)$. Beams having homogeneous polarization across the beam cross section such that the spin part and orbital part can be separately determined are said to be in separable state, like in scalar vortex beams. But polarization singularities are in nonseparable states. Each point on the equator of a HyOPS represents a C-point, whereas each point on the equator of a HOPS is a V-point polarization singularity. Beams represented by these spheres are called HyOPS beams and HOPS beams. Nonpolar and nonequatorial points on HOPS represent a vector vortex beam (VVB) with constant, nonzero ellipticity, and varying azimuth. For the generation of HyOPS beams, use of hologram [132], sectorial phase plate [133], and electrically driven devices [134] have been reported. These HyOPS beams can be considered as Poincaré beams, and there is lot of interest in them [122–129, 135, 136]. By anisotropic polarization modulation [137], Poincaré beams can be generated. Other beams such as Mathieu–Poincaré beams [138], entangled vector vortex beams [139], and Poincaré–Bessel beams [140] are also subject of interest. These elliptic field singularities are also generated in photoelastic stressed medium [141, 142] and studied. Tight focusing of full Poincaré beams [143] and the forces exerted by these beams on submicron particles were also studied [144].

10. Literature Survey of Phase and Polarization Singularities

10.1. Phase Singularities. In early seventies, the idea of phase singularity in electromagnetic waves was first introduced by Nye and Berry [11] while studying radio echoes from the bottom of Antarctic ice sheets. Like the crystal defects, optical wavefronts also exhibit phase defects. These defects are found in large numbers in random fields [15, 68, 145–149]. Presence of point phase defects in laser modes [150] is reported in 1983. There are also some early studies in the 80s [151, 152]. Vortices appear as solutions of wave equations in cylindrical coordinates. Many authors have studied their propagation characteristics [39, 153–156], with aberrations such as astigmatism [157, 158] and coma [159]. Role of Gouy phase during propagation was studied in [160]. Propagation through obstructions also came under study [161–164]. The concept of optical vortices has also been applied in phase retrieval algorithms [55, 165–167].

In scalar optical fields, the properties of phase singular beams came under study [31, 36]. The topological properties of vortices are studied in detail [37–39, 168–173]. The studies on critical points such as maxima, minima, saddles, and singular points in vortex rich optical fields came under thorough study [174–176]. There are also sign rules that describe the distribution of these critical points [12, 17, 174–177]. Vortex trajectories with the help of topological manifolds, leading to formation of knots, links, and loops, were studied [170, 171, 178–184]. Other types of phase defects such as edge and mixed-type phase defects [32, 34], anisotropic vortices [185], and perfect [76, 186–194] and fractional vortices came under study [195–204]. Some of the earliest optical elements capable of producing phase singularities were reported [58, 59, 61]. But the widely cited papers on the use of spiral zone plate for vortex generation were reported [60, 205] in 1992.

At the same time, reports on orbital angular momentum [206] carried by the helical waves appeared. Research articles on orbital angular momentum of light described [206–215] the central role played by the phase singularities. Other than LG beams, beams with helical wavefronts such as Bessel beams [216], Mathieu beams [217], and Ince–Gaussian beams [218] were also found to carry orbital angular momentum. Helico-conical beams also are known to contain OAM states [219, 220] since they are the product of helical and conical waves [221]. These beams have self-healing property [222, 223]. Vortex preserving statistical optical beams [224] were also reported. The analogy between paraxial optics and quantum mechanics was used [225, 226] to explain the OAM in light beams. Divergence of vortex beams was studied [227–231] to understand the propagation characteristics [39, 40, 160, 185, 232]. In [233], the transformation of vortex beams from fractional fork holograms due to Gouy phase was demonstrated.

The energy flows in a vortex beam have circulating components about the vortex core. The transverse flow has two contributions coming from spin and orbital angular momentum. The virtual spin part [234] is due to polarization of light. The energy flow in an optical field can be visualized by Bekshaev–Bliokh–Soskin method [235–239]. Helmholtz–Hodge decomposition method has also been applied to demonstrate internal energy flows in scalar optical fields [35, 240]. An analogy between azimuthons (found in nonlinear media) and rotating transverse energy flow structures in paraxial beams is demonstrated in [241]. Patterns of energy flows in dipole vortex beams were studied using a knife-edge test [242]. Effect of astigmatism [243] and coma [244] on the transverse energy distributions was also investigated. Mechanical action of the spin part of the energy flow was reported in [245, 246]. The skew angle of Poynting vector in a helically phased beam was measured in [247]. The handedness and azimuthal energy flow in optical vortex beams is also reported [248]. The radial [38, 169, 249] components of the propagation vector in a vortex beam came under scrutiny of few groups. Several other theoretical [25, 26, 250–254] and experimental [255–260] investigations exist for Poynting singularities in transverse energy flow.

10.1.1. Generation. Conversion between Hermite–Gaussian modes to Laguerre–Gaussian modes using cylindrical lenses is one of the early methods [261, 262] of vortex generation. Optical vortices can be generated in diffraction orders [263, 264] of specially designed diffraction grating. Use of chiral fiber grating is also reported for vortex generation [265]. Intracavity generation by introducing a spot defect in one of the resonator mirrors is also demonstrated [266–268]. Mirrors bent in the shape of a ramp [213, 269, 270] and digital micromirror device [192] can also be used for phase singularity generation. There are also reports [271–279] on the generation of vortices in optical fibers. Vortex generation in high power laser is possible with the use of fused silica fibers [280]. Use of micropatterned optical fiber tip has also been reported for the generation of optical vortices [281].

For the vortex generation use of diffractive lens [62] and newly designed diffractive optical elements (DOE) [282–284], mode converters [261, 262], spiral phase plate [57, 285], and a new element called q-plate [286–288] have been reported. Other methods of vortex generation include use of nonspiral phase plates [63], plexi glass [64], wedge plates [156, 289, 290], stack of wedge plates [291], spatial light modulators [187, 292], anisotropic media [293], spatial filtering [65], laser etched mirrors [66], micro-electromechanical systems [294], adaptive mirrors [67, 295, 296], and laser with large Fresnel number [23]. Use of photo polymerization and micromachining has also been introduced [297, 298] for SPP generation. It is also possible to generate vortices using nonspiral phase plate [63], where half of the beam cross section passes through a glass plate which is curved and adjustable phase plates [64] in which the amount of twist given to the plate can yield higher charges. Generation of vortices is also possible by controlling polarization of light [299] and by engineering astigmatism [300]. Spin to orbital momentum conversion methods [301, 302] were also reported for vortex generation. Spin to orbital angular momentum conversion is also possible by focusing, imaging, and scattering [303]. Tunable vortex generation method was also reported [304].

Spiral zone plates can be used [13, 60, 305] for vortex generation. Other than wedge plates, local tilts introduced in parts of the wavefront [19, 20] can be used for the generation of array of vortices of same charge. Sagnac interferometer can be used for vortex generation [306]. In the array form also, these vortices were generated [307]. There are many interference-based methods reported [71, 308–313] for vortex array generation in which multiple beams with non-coplanar propagation vectors are made to interfere. Vortex array generation by spiral Dammann zone plates [314] and spiral square zone plate [315] was also reported.

10.1.2. Detection. Formation of fork fringes by interference was suggested for vortex detection [78, 316], and this was the first reported and widely used method. On the contrary, fork gratings can be used for vortex generation also [317]. For phase singularity detection, however, initially, there were very few methods reported [318]. Later, many diffraction-based methods were reported [80, 81, 263, 319–323]. Some interference-based methods for vortex detection include phase shifting [190, 324], modified Mach–Zehnder [325, 326], and Fizeau interferometer [327]. Another interference method, based on lateral shear interferometry, was reported [33, 79] in 2008. Being a self-referencing method, this is one of the simplest methods available for vortex detection. A correlation-based detection technique was also employed for vortex detection [328]. The effect of aberrations, on vortices [158, 329–334], was also studied for vortex detection. Other methods include the use of Shack–Hartmann sensor [335]. Some other vortex sorters are presented in [336] and [337]. It has been shown that OAM content can be determined by using a cylindrical lens pair [338]. Diffraction patterns produced by phase singular beams are different, and they can be used for the detection of vortices in a beam. Normally, a higher topological charge vortex is unstable [339] and disintegrates into unit charged vortices under perturbation. Diffraction by single [80, 340], double [38, 319, 341], and multiple slits [320] was studied and used for vortex detection. Multiple slits correspond to grating, and special gratings were also designed [263, 321, 342] for the diffraction study. Apertures of different shapes ranging from triangular [81, 343], circular [344, 345], diamond-shaped [161], hexagonal [346], regular polygon [347], and annular [151, 155, 348, 349] were also used in diffraction experiments. Additional methods for vortex charge determination based on using twisted phase element [350], hyperbolic grating [351], axicon [352], spiral spectra [353], single point detector [354], and Talbot effect [355] were also reported in literature.

10.1.3. Applications. Singular beams found many applications. They are useful in optical meteorology for wavefront tilt measurement [356, 357], wavefront reconstruction [41, 358], phase unwrapping [165–167], vortex sign [359, 360], and vortex charge determination [361, 362] by optical vortex interferometer. Optical vortices are also used for collimation testing [42] and in spiral interferometry [44, 363, 364] in which the peak and valley can be detected unambiguously. Problems arising during misalignment of vortices is discussed in [365]. By using elements having vortex transmittance function, radial Hilbert transform mask isotropic edge enhancement [46, 61, 366] is possible. Modification to this radial Hilbert mask leads to selective edge-enhancing capabilities [202, 366–370] in the spatial filtering systems. The spiral phase filter introduced in microscopy [364, 371, 372] leads to phase contrast imaging. Optical vortices can also be used to perform high precision astronomy and tip/tilt correction [373] and used in coronagraphs [48, 374]. Annular intensity pattern of STED beam can be achieved using a vortex phase plate [375–380]. Introduction of vortex in diffracting field can offer better phase retrieval [45] algorithms with better capabilities. Studies on vortices lead to the construction of speckle-free reconstruction of phase randomized holograms [54, 56, 381] and diffractive optical elements. Application of singular optics arising due to the OAM carried by vortex beams [382–391], include trapping [392–399] and rotation [210, 400–407] of microscopic particles. These OAM states being orthogonal can be used as transmitting channels [22, 52, 53, 408–410] in free space as well as in fibers for communication. Vortex states were also tried for underwater communication [411] and free space communication [412]. There are some weak measurements [413, 414] suggested with singularities. Vortices also have healing properties [415, 416] so that restoration is possible.

Shaping the focal structure in optical systems has been an active area of research for a long time. Focal shaping is possible by pupil function engineering by modifying amplitude, phase, and polarization distribution of the wave that is focused. In high numerical aperture (NA) systems, the polarization distribution of the beam also plays a vital role. Focusing of singular beam leads to generation of a doughnut structure in the focal plane of a lens in contrast to the well-known Airy pattern. Doughnut intensity structure is useful in several applications in fields such as microscopy, optical trapping, lithography, and astronomy. It has been observed that even with optics considered well-corrected, the intensity distribution of a singular beam gets distorted in the presence of small amount of azimuthally dependent aberrations, in comparison to that of the nonsingular beam [417]. Structural modifications in the focused structure of the singular beam have been carried out in [418]. Singular beams focused by aberrated system disturb the doughnut pattern significantly. The focal plane intensity distribution under the influence of spherical aberration [419–422], astigmatism [329, 330, 423, 424] and coma [425–428] are also studied. Astigmatism was used to invert the sign of the topological charge of a vortex [429].

10.2. Polarization Singularities. Polarization singularities occur naturally in daylight sky [18, 430] and has been subject to various atmospheric studies [431–435]. Polarization patterns consisting of polarization singularities in cosmic sky have also been investigated [436, 437]. Topological singularities have also been observed in disclinations in liquid crystals [438, 439], fingerprints [440, 441], and umbilic points in the curvature of surfaces [442, 443].

In optical fields, polarization singularities may occur where state of polarization varies with position. There is a large interest in paraxial fields with slowly varying polarization distributions in recent years [82–84, 444–446]. Singularities in the polarization state of partially coherent wavefields are also gaining interest [447]. Ellipse fields have spatial distribution of elliptical SOPs, whereas vector fields have spatially varying linear SOPs. In polarization singularities, the direction of polarization azimuth is a crucial parameter [109]. Similar to phase in optical phase vortex, all possible values of polarization azimuth occur at the polarization vortex point. In an inhomogeneous polarization distribution of ellipse fields, C-points are points of circular polarization state, whereas L-lines are linear polarization states at which polarization azimuth and handedness are, respectively, undefined. The neighborhood of C- point has SOP distribution consisting of polarization ellipses with their azimuths oriented in clockwise or anticlockwise directions. In a spatially varying distribution of linear states, polarization singularities are V-points. The sense and number of rotation(s) of the azimuth in one complete closed path around the singularity decides the polarity and index of the polarization vortex, respectively. The polarization singularities form optical Mobius strips in three-dimensional fields [448, 449]. In three dimensions, circular polarization occurs along C lines and linear along L surfaces [448, 450–453]. Poincaré vortices [454] are another type of Stokes singularity at which the SOP is linear. There are reports on refraction [455] and reflection [456] of C-lines. During propagation, the SOP distribution in the polarization singular beams are found to rotate due to Gouy phase [457, 458]. The pattern can also undergo rotation by acquiring the Pancharatnam phase while travelling through appropriate optical elements [97]. Hamiltonian based on the Maxwell–Schrödinger equation has been used for the analysis of Pancharatnam–Berry phase of VVBs [459].

Similar to phase vortices, polarization singularities (which essentially are phase vortices in Stokes phase) in a distribution are also governed by sign rule. According to the sign principle, adjacent vortices along a stokes phase contour must alternate the index sign [10, 460–462]. Fields laced with only C-points [110, 463] and of purely V-points but with opposite indices [102] have been reported. Interference field distributions interlaced with mixtures of C-points and V-points are also there [93, 109, 464]. Sign conservation is followed during diffraction also. It has been observed that a V-point with index $\pm\eta$ disintegrates into 2η number of C-points, each with index 1/2, and same polarity as that of a V-point. Of these generated C-points during diffraction, half the number of C-points are right-handed and half are left-handed. This means that handedness conservation is also observed during diffraction.

10.2.1. Generation. One of the methods to generate polarization singularities is by using a universal interferometer presented in Section 8. Usage of a spiral phase is unavoidable in all the interferometric setups. These methods are highly sensitive and demand precise alignment of the cores of the overlapping vortices [465, 466]. Other efficient methods to generate radially polarized beams are by an image rotating resonator [467] or with a double interferometer [468, 469]. Polarization singularity generation by using a Twyman–Green interferometer [470] and a Wollaston prism [471] have also been reported. Vector beams embedded with polarization singularities can also be realized by a non-interferometric technique [472].

Cylindrical vector beams can be generated by introducing polarization-selective mirror inside a laser resonator cavity [473–475]. In this technique, the resonator is made polarization selective by a binary dielectric diffraction grating which is etched at the backsurface of the mirror substrate. Use of calcite crystal [476], windows [477], axicon [478], polarization selective grating mirror [479], image rotating mirror arrangement [467], polarization selective GIRO (giant reflection to zero order) mirror [473], conical Brewster prism [480], polarization-based beam displacer [481], an undoped c-cut YVO_4 crystal [482], conical prism [483] inside a resonator have been reported for the generation of cylindrical vector beams. Generally, these methods produce positive index V-point singularities. To produce their negative counterparts, a HWP can be inserted outside the cavity. Intracavity methods can be employed for generation of only V-points and not for C-points. Generation of radially polarized beams can also be achieved by conical diffraction [484], spatially varying subwavelength grating structures [485–487] and diffractive optical elements [488].

Commercially available spatial light modulators (SLMs) can also be used to generate polarization singularities. As SLMs respond to only one linear polarization, this fact can be used to tailor the phase of the beam [107, 489–494]. Simultaneous generation of multiple vector beams on a single SLM is reported in [495]. Similar to SLMs, recently developed digital micromirror devices (DMDs) are being used for producing structured light fields [496, 497].

Another method of generation of vector vortex beams utilizes segmented spatially varying waveplates [498, 499] or subwavelength grating structures [485, 486, 500, 501] or metasurfaces [502–505] that work on diffraction phenomenon. Generation of VVB with a single plasmonic metasurface is depicted in [506]. S-waveplates are also used for generation of vector beams. S-waveplates are segmented half waveplates, with each segment having different orientations of crystal's optic axis. Depending on the input plane of polarization of linearly polarized light, radial or azimuthal or superposition states of radial and azimuthal can be realized. Similar to S-waveplates, liquid crystal-based q-plates are also used to generate and manipulate vector beams [69, 507–511]. The q plates can be combined with HWPs to obtain higher order polarization singularities [512]. Use of q-plate as a coupler has been demonstrated recently [513]. There is also increased research activity in q-plate fabrication methods [286, 507, 510, 514–520].

Generation of vector vortex beams using polarization gratings has also been reported recently [521]. Vector vortex beams were also generated in optical fibers [522–524]. Generation methods also include metamaterial-based Pancharatnam–Berry phase elements [525–528]. There are also other methods reported for VVB generation [130, 529–532]. The topology of VVB was also discussed [258, 533]. Generation of vector beams using a double-wedge depolarizer [534], ring resonator [535], caustic-based approach [536], and parametric oscillator [537] was also demonstrated. Generation of broadband vector beams with tunable phase and polarization has been demonstrated in [538]. Achromatic VVBs can be produced from a glass cone [539]. Fractional polarization vortices by Sagnac interferometer [540] and radially polarized fractional vortex beam [541] were also generated and studied. A single integrated on chip device has been proposed that allows switchable radially and azimuthally polarized VVB [542].

Polarization singularities can also be found in random fields. In structured distributions, they appear in lattice form. There are several reports on lattices of only C-points [110, 463], only V-points [102], and C-points interlaced with V-points [93, 109, 464]. A spatially varying lattice of C-points and V-points has also been generated [543]. All these engineered lattice fields obey sign rule and are generated by interference of multiple plane waves.

10.2.2. Detection. In inhomogeneous polarization distributions, Stokes parameters are spatially varying and can be measured by adopting standard Stokes polarimetry technique. Polarization singularities in an optical field can be identified by measurement of these Stokes parameters, as they are phase vortices of S_{12} Stokes field [110, 112, 497]. The quality of the vector beams can be measured by vector quality factor (VQF) tool [544]. Another deterministic detection mechanism for vector vortex states utilizes classical and weak coherent states [545].

Detection of C-points in a field distribution is based on the identification of closed L-lines (s-contours) in multiple recorded interferograms [546]. The information about the presence of fork fringes in the interferograms can be used to locate and track path taken by L-lines. L-lines separate regions of right and left handedness in a polarization distributions and enclose a C-point singularity. Detection of V-points is relatively easier than C-points. For a V-point singularity with index η, a polarizer produces $2|\eta|$ lobes of intensity pattern. Another method of detection of V-points is based on diffraction. One of the first experiments on diffraction of V-points through triangular apertures of two types was reported recently [547]. In this method, both diffraction and polarization transformations were used to uniquely determine all states of V-points [548]. Diffraction of VVB through diamond-shaped aperture [549] and circular aperture [550, 551] has been reported recently. In diffracted near fields, exceptional polarizations structures have been found [552].

10.2.3. Applications. Polarization distribution of the beam becomes important in shaping the point spread function (PSF) in high NA focusing [553–557], and manipulation of the PSF by polarization distribution is referred to as polarization engineering [558]. Radially and azimuthally polarized light can be used to realize smallest focal point beyond scalar optics limit [554, 555, 559, 560]. These beams can also be used for particle acceleration [561, 562], trapping of metallic Rayleigh particles [563], optical manipulation [564], and optical signal processing [370, 565, 566]. Both point and edge phase dislocations are present in radial and azimuthal polarized beams [86] depending on which polarization component (either linear or circular) is extracted. Depolarization effects of laser beam propagation in turbulent atmosphere was studied [567, 568]. Propagation of radial/azimuthally polarized beams through turbulence also caught the attention of researchers [569–571]. Synthesized vortex beams are also subject of interest in the turbulent atmosphere [572]. Irradiance [573] and scintillation [574, 575] of radial/azimuthally polarized beams and Poincaré beams [576] propagating through atmosphere were examined. Use of C-point beams in turbulence-resistant robust beam generation has been proposed [577–580]. Reports on the use of polarization singularities for enhancing chiral light matter interaction are also there [88, 581–583]. Data-carrying fiber vector eigenmode multiplexing has also been reported [584] in communication [585, 586]. The nonseparability of VVB can also be used to encode information for optical communication [587]. Switching between phase and polarization singularity using metasurfaces [588] is also reported. Vector beams under the effect of perturbations form pair of fundamental and stable singularities that may be useful in weak field measurements [589, 590]. Splitting of C-points can be used as a tool for weak measurement of elliptical dipole moments [414]. Entanglement studies are also reported [591]. Characterization and manipulation of these vector vortex beams is a subject of study for laser matter interaction experiments [592]. Deep learning algorithms are also applied for turbulence aberration correction for VVBs [593].

The research area of polarization vortices (vector vortex beams or Poincaré beams) is relatively a new field compared to its scalar counterpart, i.e., optical phase vortices. Polarization vortices are still an emerging and active area of research with few review articles [18, 29, 594–596] published. There is also a review article [597], highlighting the applications of vector beams. Few review articles are also there in the area of structured light [598–600]. They are at a much higher and more eclectic level and hence may not be suitable for beginners. This article is believed to bridge the gap between the researchers at two extremes of the spectrum.

11. Conclusion

In conclusion, we have presented the method of generating polarization singularities from phase singularities from first principles. The azimuthal phase variation of the scalar vortices is converted into Stokes phase variation in a Mach–Zehnder-type interferometer. The interferometer presented here is universal in the sense that it can be used for generating both phase as well as polarization singularities. This interferometer can produce azimuth vortices with different attributes such as dark, bright, left-handed, right-handed, ellipse, and vector field singularities. We have presented new and intuitive explanations for Stokes phases and the connection between phase and polarization singularities. Experimental and simulations are included. In the first part, tutorial on the subject of polarization singularities as a natural extension of phase singularities is presented. Towards the end, a survey of activities on these two areas is presented.

Data Availability

The experimental data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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