

Research Article

Correlated Energy Exchange in Drifting Sea Ice

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The ice floe speed variations were monitored at the research camp North Pole 35 established on the Arctic ice pack in 2008. A three-month time series of measured speed values was used for determining changes in the kinetic energy of the drifting ice floe. The constructed energy distributions were analyzed by methods of nonextensive statistical mechanics based on the Tsallis statistics for open nonequilibrium systems, such as tectonic formations and drifting sea ice. The nonextensivity means the nonadditivity of externally induced energy changes in multicomponent systems due to dynamic interrelation of components having no structural links. The Tsallis formalism gives one an opportunity to assess the correlation between ice floe motions through a specific parameter, the so-called parameter of nonextensivity. This formalistic assessment of the actual state of drifting pack allows one to forecast some important trends in sea ice behavior, because the level of correlated dynamics determines conditions for extended mechanical perturbations in ice pack. In this work, we revealed temporal fluctuations of the parameter of nonextensivity and observed its maximum value before a large-scale sea ice fragmentation (faulting) of consolidated sea ice. The correlation was not detected in fragmented sea ice where long-range interactions are weakened.

1. Introduction

The Arctic sea ice cover (ASIC) is the open thermodynamic system that exhibits well-pronounced scaling properties [1–5]. The ASIC dynamics is determined by the sea ice drift, which is caused, predominantly, by irregular wind forcing [6–8]. The phenomenology of the sea ice drift and its impact on the climate change was reviewed by Morison et al. [9].

Overland et al. [10, 11] regarded the drifting ice as a hierarchic system, the dynamic properties of which are self-similar at all scale levels. Rothrock and Thorndike [12] were the first who reported the power-law floe size distribution and formulated the concept of scale invariance of fragmented sea ice. More recent studies showed that both the energy [13] and temporal [4, 5] parameters of the drift are characterized by power-law distributions that are well known in geodynamics (Gutenberg-Richter law [14]), as well as in other nonequilibrium processes in nature including human activity (see [15]).

However, the power-law relations for frequent-vs-size distributions of energy-change-related events in open systems are essentially empirical relations, which cannot be

derived in the framework of classical thermodynamics that is valid for closed equilibrium systems. The Boltzmann-Gibbs distribution implies the independence of individual “events” (local perturbations), since the effect of each of them on neighboring structures decays exponentially with the distance. Fast decreasing leads to the additivity of the contribution of individual events to the total process (i.e., dynamic extensivity). Long-range interactions between components in externally driven systems break their dynamic isolation, and cause deviations from a simple summation of transferred energy. In other words, in the case of power-law decaying, the correlation radius of each local perturbation exceeds the size of a single perturbed site, thus inducing a cooperative response of affected sites to the external forcing.

The discrepancy between the results expected from the Boltzmann-Gibbs statistics and actual phenomenology was resolved by Tsallis [16, 17] who developed the nonextensive statistical mechanics (NESM) as a generalization of classical statistics for the case of the nonexponential energy distribution in a nonequilibrium system characterized by long-range interrelation between energy release/exchange events.

In the very recent years, the nonextensive statistics was successively applied for analyzing the seismic activity in relation with the problem of predictability of earthquakes [18–25]. This work is aimed at expanding the applicability of NESM over another dynamic geophysical object, the ASIC, which is comparable with geostuctures both in dimensions and character of mechanical processes.

We shall demonstrate that the sea ice drift dynamics can be described adequately both by an empirical relation identical to Gutenberg-Richter law [14] and by analytical expressions deduced from the basic principles of the Tsallis statistics. We used in our considerations the concepts and models developed in geophysics for describing dynamic processes in geostuctures. We believe that a common approach to studying large-scale phenomena in solid Earth and ASIC is justified by the morphological closeness of two-dimensional tectonic plates and ice floes, as well as by their very similar mechanical behavior (cracking, faulting, shearing, and stick-slip motions). A similarity between dynamic processes in solid Earth and Arctic ice pack was discussed in review [26].

The monitoring of motions of an “individual” ice floe was performed at the ice research camp North Pole 35 established on Arctic ice pack in 2008. The ice floe displacements were detected using the GPS technique in regular 2 min intervals. The obtained records were used for determining the variations of the kinetic energy of the ice floe during its spontaneous drift at approximately 86° north of the Nova Zemla in the period of time January to March, 2008.

2. Energy Distribution

2.1. NESM. “Scaling” in the given context means the validity of the scaling equation that establishes the following dependence of events frequency on event energy:

$$N(\lambda E) = \lambda^{-b} N(E), \quad (1)$$

where N is the number of events characterized with the energy E ; λ is the constant (scaling factor); b is the constant. The term “event” refers to any local structural or dynamic perturbation that resulted in a change of energy. For example, any signal from established seismograph is regarded as an evidence of fracture event accompanied with a certain release of elastic energy. In our case, we shall regard as an “event” every detected change in the kinetic energy of drifting ice floe. The correlation radius of so-defined event is determined by the physical distance, at which the energy change of the given floe affects the behavior of neighboring pack structures. Large correlation radius implies long-range interactions between components, such as collisions or disengagements, through which a local excitation is transferred to the environment.

The only function that verifies (1) is a power law:

$$N(E > E') \propto E'^{-b}, \quad (2)$$

where $N(E > E')$ is the number of events characterized with the energy E exceeding a threshold value E' ; b is the constant. Equation (2) is an alternative designation of the celebrated

Gutenberg-Richter law, $\log N(m > m') \propto -bm$, where N is the number of earthquakes having magnitudes m greater than m' .

The power-law function means much slower decay of an event effect on adjacent sites than that which takes place in the case of exponential decay. This disturbs the entropic additivity of the process, which in the simplest case of two independent *equilibrium* subsystems A and B can be expressed as

$$S(A + B) = S(A) + S(B), \quad (3)$$

where S is the entropy.

The main idea forwarded by Tsallis was to introduce into additive expression (3) a member that would take into account some interactions between subsystems through a “parameter of nonextensivity” q :

$$S(A + B) = S(A) + S(B) + \frac{S(A)S(B)(1 - q)}{k}. \quad (4)$$

Here, k is the Boltzmann constant; the right-hand cross-term characterizes the interaction between the subsystems.

Accordingly, the classical logarithmic definition for probability of states in multisite system

$$S = -k \sum_{i=1}^w p_i \ln p_i \quad (5)$$

(here p_i are the probabilities of w virtual configurations in the system) transforms into the power-law expression:

$$S_q = \frac{k(1 - \sum_{i=1}^w p_i^q)}{(q - 1)}, \quad (6)$$

where S_q is the “nonextensive” entropy.

Passing over details of the Tsallis’ conjecture (see his book [17] for details), we note that the Boltzmann-Gibbs statistics responds the limit $q \rightarrow 1$ ((6) transforms into (5)); for $q < 1$ the formalism imposes a high-energy cutoff, that is a limited statistics in equilibrium system (see an example below); finally, the value $q > 1$ signalizes the presence of long-range interactions in non-equilibrium dynamic system—this is the case of power-law dynamics, which is incompatible with the entropic additivity. Being the parameter q a measure of nonextensivity, its value characterizes indirectly a correlation radius of interactions in a non-equilibrium system.

The nonextensive paradigm was applied in seismological [21–23], hydrological [27], climatological [28], and atmospheric [29] studies. Issuing from the Tsallis formalism, Sotolongo-Costa and Posadas [18] obtained a formula for magnitude distribution of earthquakes, which reproduced actual distributions in a wider range of magnitudes than the empirical Gutenberg-Richter relation [14]. The formula was modified by Silva et al. [19] who replaced the linear energy density suggested in [18] by a more realistic volumetric relation, and obtained the following expression for the number

of earthquakes with magnitudes, m , larger than a value m' normalized to the total number of events N :

$$\begin{aligned} \log N(m > m') \\ = \log N + \left(\frac{2-q}{1-q} \right) \log \left[1 - \left(\frac{1-q}{2-q} \right) (10^{2m} a^{-2/3}) \right], \end{aligned} \quad (7)$$

where a is the volumetric energy density. In terms of the energy released ($m \approx 1/3 \log E$), the expression for the energy distribution in the nonextensive dynamic process takes the following form:

$$N(E > E') = N \left[1 - \left(\frac{1-q}{2-q} \right) \left(\frac{E}{a} \right)^{2/3} \right]^{(2-q)/(1-q)}. \quad (8)$$

Equations (7) and (8) were obtained on assumption of the proportionality between the probability of the given release of elastic energy in a fracture event and the size of formed fragment. In a similar way, the kinetic energy of drifting ice floe depends linearly on its size. In fact, (7) is a generalized form of (2) [30]. Very recently, Balasis et al. [31] applied (7) for analyzing the solar flare and magnetic storm intensities and obtained the excellent coincidence between calculated and measured values without making any assumptions on the mechanism that governs the probability of the energy release. The authors argued for the universality of the nonextensive energy distribution in the form (7) for a wide range of nonlinear phenomena characterized by long-range interactions.

The relation (8) allows one to determine the parameters q and a by fitting them to the plot $N(E > E')$ versus E' constructed on the base of experimental data. Variations of the q -value reflect changes in the thermodynamic state of the system, and, correspondingly, in the degree of correlation of events. In this work, we utilized (8) for characterizing the sea ice drift dynamics and for a comparison of the results of nonextensive analysis with those obtained from the assessment of the scaling parameter b .

2.2. Kinetic Energy. Nonuniform sea ice drift in the Arctic Ocean induces localized and extended ice pack fragmentations, which, in turn, affect the local drift rate through collisions and shearing of mobile ice floes. Therefore, the kinetic energy of individual fragments of ice pack is substantially interrelated with the fracture process, which affects their motions. The average speed of drift of consolidated sea ice is about 0.1-0.2 m/s, while the speed of an individual ice floe varies in the range 0.02 to 0.4 m/s [32].

In this work, the local speed changes were derived from the data on the ice floe displacements measured using a couple of GPS transmitters that were placed on the ice floe at the vicinity of the camp North Pole 35 at the distance of about 180 m. The data were collected using a field PC in sampling intervals of two minutes. A detailed analysis of the accuracy of drift speed determination from the GPS data [4] showed that the standard error in values V measured in 2 min sampling intervals was about 0.009 m/s, while the

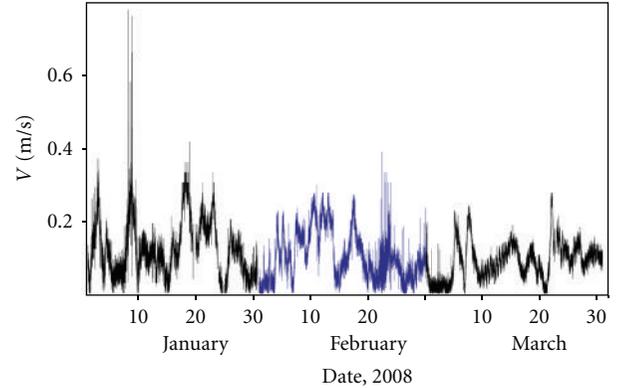


FIGURE 1: Ice floe speed measured at the camp North Pole 35 from January 1 to March 31, 2008.

changes in the drift speed, ΔV , were much larger. Only the values ΔV that exceeded the standard error of speed determination two times or more were taken into account in the subsequent data processing.

Figure 1 shows a series of the values of drift speed V calculated from readings of one of the GPS transmitters during the period of time from January to March 2008. This period of observations covered two significant anomalies in “stationary” ice floe drift occurred on 1 to 6 February and on 5 to 8 March. A few 180° turns in local drift direction were detected in these time intervals (Figure 2). The event of 5–8 March coincided in time with a large-scale ice pack fragmentation occurred at about 150 km from the research camp (Figure 3).

The kinetic energy change, ΔE , is directly proportional to the speed change squared, $\Delta E \propto \Delta V^2$ (to simplify formulae, we shall denote ΔE as E hereafter). The distributions of changes in the kinetic energy of the drifting ice floe are depicted in Figure 4 in 15-day intervals covering the period of the GPS monitoring.

In left-hand panels of Figure 4(a), the experimental data were approximated by log-linear portions that represent the power-law dependences given by (2). One can see that the power-law behaviour takes place in all time windows with the exception of the period of time 16–21 March when the log-linear portion (if ever) does not cover even an order of magnitude (Figure 4(f)). In other words, the ensemble of interacting ice floes exhibits scaling properties in the sense of (1) most of the time.

The power exponent, b , calculated from the slope of straight line drops significantly (from 3.20 to 2.30) in the period of time 16 to 29 February preceding the large-scale sea ice fragmentation.

In the right-hand panels of Figure 4, the same experimental dependences $N(E > E')$ versus E' were approximated using (8). The best-fitting procedure was applied to determine the parameters q and a .

One can see that the found q -values exceed unity in all time intervals with a decrease in the value of this parameter down to $q = 0.93$ after the faulting of 5–8 March. The q -value less than unity means the prevalence of noncorrelated events

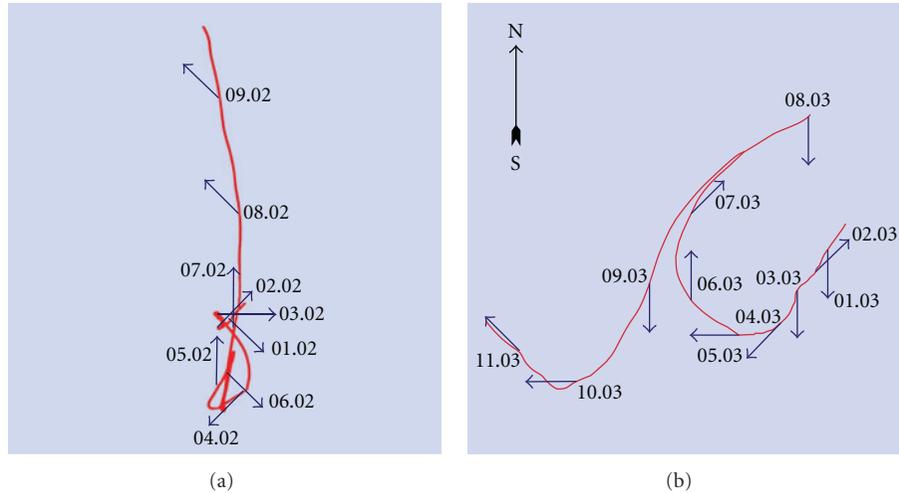


FIGURE 2: Fragments of drift trajectory showing two series of the ice research camp North Pole 35 in periods of dramatic changes in the direction of the drift. Arrows indicate the prevailing wind directions.

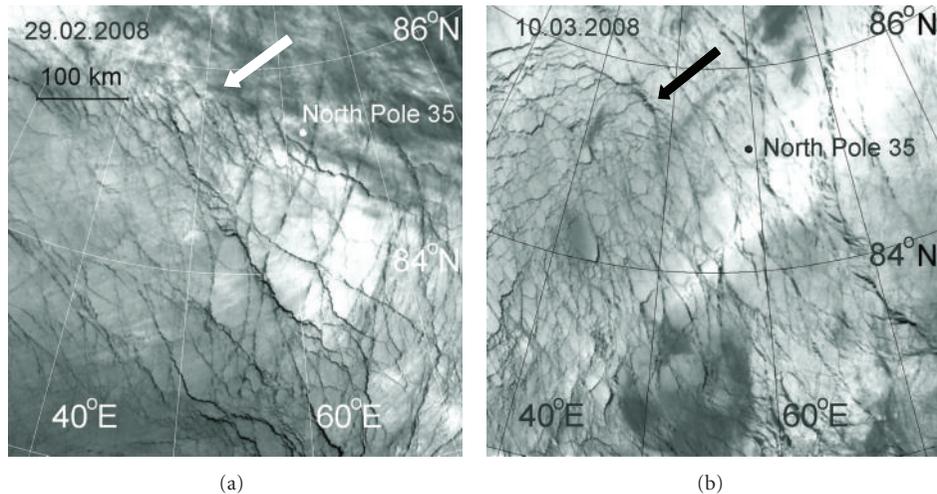


FIGURE 3: NOAA satellite image of a fault formed on 5–8 March 2008 in the vicinity of the ice-research camp North Pole 35. Arrows show the region of “future fault” (a) and the actual fault (b).

which one could expect in the fragmented pack. As distinct from the extensive dynamics at $q = 1$, which is typical for a highly connected system that admits the occurrence of correlated behavior in *limited* time intervals (when the system deviates spontaneously from the thermodynamically equilibrium state [33]), the state characterized by $q < 1$ excludes the long-range energy exchange because of the disintegration of the system in individual, low-connected components with rare interactions (subextensivity [17]). Obviously, this is the case for sea ice dynamics after faulting.

3. Discussion

In this work, the data on the kinetic energy variation in drifting sea ice were analyzed using both the empirical relation (2) and the analytical function $N(E > E')$ (8) based on the nonadditive definition of entropy in (4).

Both used fittings, that is those through the b -value and the q -value, approximate the experimental energy distributions with a certain inaccuracy. In the range of mid- and high-energy kinetics, the approximation through the b -value represents well log-linear portions of the N versus E' dependences. Low-energy portions of dependences follow neither (2) nor (8) but the q -value approximation reflects a trend to nonscale behavior of small events. This is explained by significant decay of low-energy excitations in dissipative media [34, 35].

The q -value as a measure of nonextensivity [16] has a clear physical sense: it characterizes a deviation of the given dynamic system from the equilibrium state. To assess the relative degree of nonextensivity of the drift dynamics, one can compare the found q -values with those known for tectonic process. Summarizing the data reported in the literature [18, 19, 22–25, 30], we conclude that

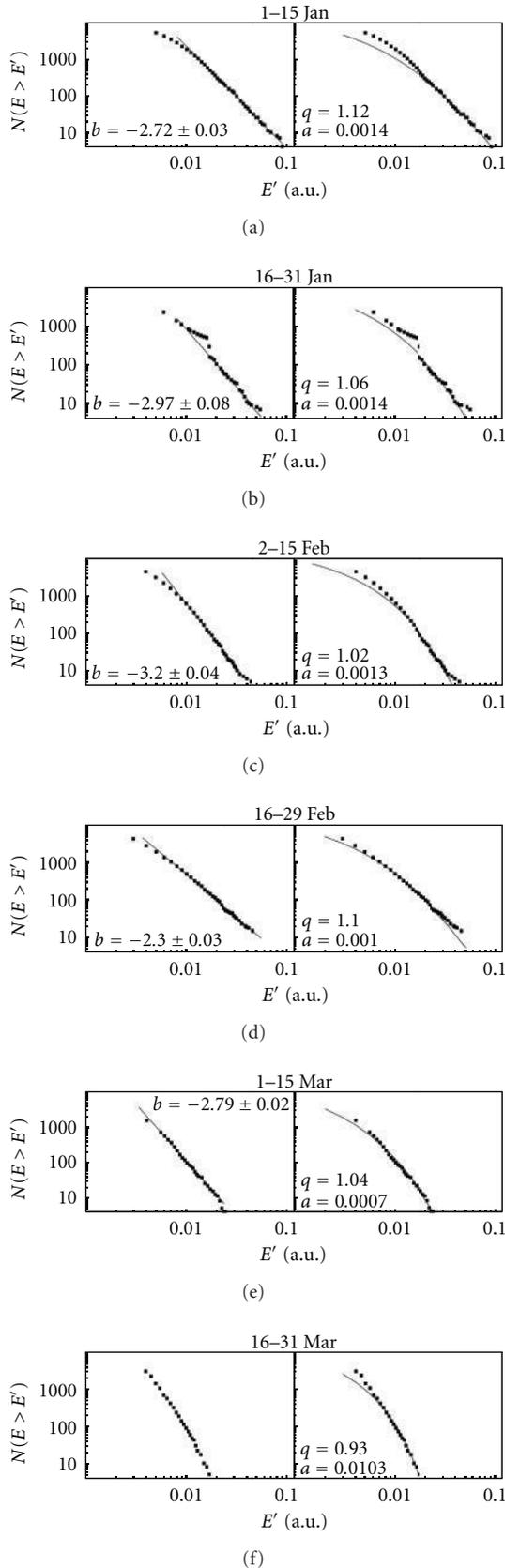


FIGURE 4: Experimental $N(E > E')$ versus E' dependences (squares) approximated by curves using (2) (left hand panel) and (8) (right hand panel).

the seismicity-related q -values fall in the range 1.5 to 1.8 for different seismic zones. In our case, the q -value reached only 1.12 in consolidated pack, and dropped down to 0.93 after a cycle of large-scale sea ice fragmentation. This comparison points out a low level of long-range correlations in drifting ice. Motions of each individual floe affect insignificantly the dynamics of neighboring floes as compared to motions occurring in the system of tectonic plates. The fault formation resulted in the disturbance of the pack connectedness and, correspondingly, in noncorrelated drift dynamics ($q < 1$) in March. We suppose that the decrease of the q -value down to 1.02 on 2–15 February was also caused by a cycle of moderate (or local) pack fragmentation, which was not, however, detected in available satellite images.

The found values of both the power exponent b and the parameter q are in good agreement between each other from the viewpoint of their physical meaning. According to the computational spring-block model developed by Olami, Feder and Christensen (OFC) [34], the b -value is correlated with the energy conservation in the system: the higher b -value, the lower conservation. In our case, we deal with high- b -value process. For reference, the literature data collected by Vallianatos [36] contain the following power-law exponents for various geomechanical processes: earthquakes 0.5 to 0.8; landslides 1.0 to 1.6; rockfalls 0.4 to 0.7. The values of this parameter shown in Figure 4 ($b = 2.3$ – 3.2) evidences a low energy conservation in drifting sea ice. In low-conservative systems, the energy exchange is restricted by neighboring sites, and the nonextensive process should be characterized by values of the parameter q close to unity.

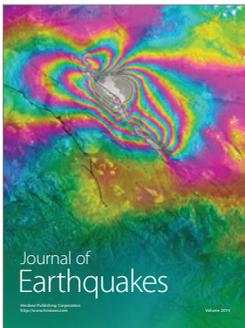
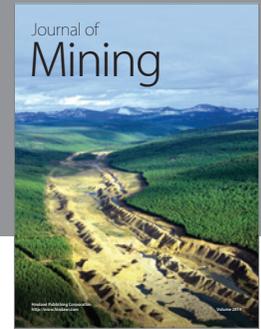
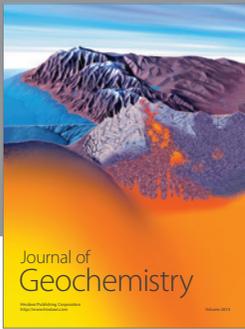
It is worthy to note that the variation of the q -value reflects the changes in the b -value with the opposite sign: the higher b , the lower q . In words, the lower energy conservation, the weaker long-range interactions. Thus, we obtained a reasonable agreement between the empirical and analytical approaches in descriptions of the nonextensive properties of the sea ice drift dynamics.

4. Conclusion

In this work, some models and concepts developed in recent years in tectonophysics were applied for assessing the actual thermodynamic state of drifting sea ice. A common approach to mechanical processes in solid Earth and Arctic ice pack is justified by a close similarity in the nature of dynamics of two-dimensional tectonic plates and ice floes, which is characterized by cracking, shearing, and stick-slip motions. We have shown that the nonextensive analysis based on the Tsallis statistics for open systems allows one to estimate deviations of ice pack from the equilibrium state, and these deviations signalize the enhancing of correlated dynamics with increased probability of avalanche-like energy release, such as strong icequakes. On the other hand, we detected a transient period of equilibrium state of sea ice after the formation of a fault consisting of highly fragmented sea ice. This thermodynamic volatility reflects changes in the dynamic connectedness of drifting sea ice.

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