Research Article

A Facile Approach to Evaluate Thermal Insulation Performance of Paper Cups

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Paper cups are ubiquitous in daily life for serving water, soup, coffee, tea, and milk due to their convenience, biodegradability, recyclability, and sustainability. The thermal insulation performance of paper cups is of significance because they are used to supply hot food or drinks. Using an effective thermal conductivity to accurately evaluate the thermal insulation performance of paper cups is complex due to the inclusion of complicated components and a multilayer structure. Moreover, an effective thermal conductivity is unsuitable for evaluating thermal insulation performance of paper cups in the case of fluctuating temperature. In this work, we propose a facile approach to precisely analyze the thermal insulation performance of paper cups in a particular range of temperature by using an evaluation model based on the MISO (Multiple-Input Single-Output) technical theory, which includes a characterization parameter (temperature factor) and a measurement apparatus. A series of experiments was conducted according to this evaluation model, and the results show that this evaluation model enables accurate characterization of the thermal insulation performance of paper cups and provides an efficient theoretical basis for selecting paper materials for paper cups.

1. Introduction

The first modern paper cup was initially developed by Lawrence Luellen in 1907. After a hundred years of development, paper cups have become one of the life's necessities for serving hot and cold drinks; they are widely used in fast-food restaurants, coffee shops, and offices, among others, because they are inexpensive, biodegradable, and renewable and because there are environmentally friendly properties of paper [1, 2]. The consumption of paper cups in China has increased sharply in recent years. More than 25 billion paper cups were consumed in 2011, and the number will continue to increase, with an estimated future annual growth rate of 12.84%. Paper cups are sometimes used to serve hot drinks or food, and the thermal insulation performance of paper should be considered because of the possibility of serious scalding accident. However, there is a lack of uniform standards, such as national standard or industrial standard for the convenient and accurate evaluation of the thermal insulation performance of paper cups.

Heat is transferred via conduction, convection, and radiation [3], and the impact of each mode of heat transfer depends

on the application. The thermal insulation performance of paper cups is determined by the paper materials. Paper is considered to be a type of porous fibrous material with a porosity of 40%~70%. When the volume fraction of cellulose fibers is greater than 1%, heat transfer by convection is negligible because the air cells in the fiber system are too small to support convection or turbulence [4–6]. Heat transfer by radiation is a linear function of temperature cubed (T^3) according to the Rosseland approximation [7]. Hence, the radiative heat transfer is also inappreciable at the relatively low operating temperature of paper cups [8, 9]. Therefore, an effective thermal conductivity is generally used to characterize the conductive heat transfer and thermal insulation performance of paper materials.

Schuhmeister first proposed a model for the thermal conductivity of fibrous materials [10]. Many researchers have since worked on modifications of this model [11–13]. The most basic expression for defining an effective thermal conductivity in porous media was given by Bhattacharyya [14] and Bankvall [12]

$$k_{\text{eff}} = \alpha k_f + (1 - \alpha) k_g. \quad (1)$$

The above equation assumes that the conductive heat travels through both the fibers and the interstitial fluid (often air). Therefore the effective thermal conductivity was based on a weighted average of the thermal conductivity values of the fibers and the interstitial fluid. The first term in the right-hand side of (1) overpredicts the conductivity of the solid phase. The second term is expected to predict the conductivity of the fluid phase. However, there are some questions regarding this equation. The heat transfer does not occur in a parallel mode through the paper because of the random fiber system, and the thermal barrier resistance should be considered when heat flows through a number of fiber-to-fiber contact areas [15]. The impact of each parameter on the thermal insulation performance of paper was studied by computer simulation. Through this simulation, we can predict the effective thermal conductivity of the whole media [15, 16].

However, a variety of parameters such as temperature and pressure [17], moisture [18, 19], the nature of the raw materials, and the macrostructure and microstructure of the paper [20] will affect the veracity of the effective thermal conductivity [21]. Another problem with an effective thermal conductivity is that the numerical value changes with temperature. Paper cups are always used at a relatively low temperature (below 100°C) that fluctuates. Therefore, an effective thermal conductivity is unsuitable for practical applications. In this study, we proposed a facile approach to precisely analyze the thermal insulation performance of paper cups by establishing an evaluation model. For the first time, we propose a new parameter named “temperature factor,” which is a simple and convenient parameter for characterizing the thermal insulation performance of a paper cup. Moreover, an apparatus for the experiment is also designed, as shown in Section 2.

2. Model Establishment

On the basis of MISO (Multiple-Input Single-Output) technical theory [22], the equation for the effective thermal conductivity can be expressed as shown below

$$k_{\text{eff}} = f(k_f, k_g, \beta, \gamma, \dots). \quad (2)$$

This equation denotes the effective thermal conductivity as a function of the factors that influence the thermal insulation performance of paper cups. When this approach is used, only the value of k_{eff} is required to appraise the thermal insulation performance of paper cups. However, Fourier's law states that thermal conductivity is independent of the temperature gradient but necessarily of temperature itself. The thermal conductivity is a function of temperature. That means the effective thermal conductivity was suited for evaluating the thermal conductivity of materials at a single temperature [23]. For paper cups, evaluation of the thermal insulation performance in a particular temperature range is more suitable for practical applications. Therefore, for assessing the thermal insulation performance of paper cups, this study includes the proposal of the temperature factor and the design of a corresponding apparatus for measurement

FIGURE 1: Schematic of setup for thermal insulation measurement.

of this new parameter. The temperature factor represents the thermal insulation performance of paper cups within a certain temperature interval, and the apparatus can help to quickly determine the value of the parameter.

2.1. Experimental Apparatus. To evaluate the thermal insulation performance of paper cups, an apparatus to test the temperature difference between the inside and outside of the cup wall was set up, and its structure is shown schematically in Figure 1. This apparatus was assembled based on the method used by Iioka [24], which was comprised of a heating section, a support frame, and a measuring part.

The sample is held between two heat-conducting fins to reduce the small amount of radiative heat as much as possible and to maintain stable heat transfer between the interface of the paper and the other phases. The internal surface temperature is the same as the heating liquid and is tuned by changing the heating power. A digital thermometer is utilized to monitor the in situ temperature of the liquid. A patch type thermocouple is posted on the heat-conducting metal sheet at the outside of the sample to monitor the cold side temperature. The temperature of the heating liquid is initially the ambient temperature and rises at a rate of 2°C/min. The temperatures on both sides of the sample are synchronously recorded every 5°C.

Pretreatment of paper materials is necessary to maintain a steady state heat transfer in paper media. First a 50 × 50 mm sample is cut from the cup and placed in a constant temperature humidity chamber at a temperature of 20°C and a relative humidity of 10% for 8 hours. A thin polyethylene film is then laminated onto each side of the paper to prevent moisture from penetrating the paper media. This allows the convective heat transfer on both surfaces of the paper to be neglected in the next measurement. What is more, the thermal insulation effect of the PE layer is neglected as the thickness of the laminated film is much thinner than the fibrous layer.

2.2. Establishment of the Numerical Model. Figure 2 illustrates the relationship between the hot surface temperature and the cold surface temperature of paper materials.

FIGURE 2: Cold surface temperature as a function of hot surface temperature for the paper cups. The area of the shaded part (the shape with the oblique lines) is ω .

The temperatures on both sides of the material will be the same when the material has no insulating effect, as in the diagonal line in Figure 2 referred to as “standard line.” The cold surface temperature-hot surface temperature curves will migrate downward from the standard line because the temperature of the cold side must be lower than that of the hot side for thermal insulation materials. These curves are labeled deviation curves and are the dot and dash curves in Figure 2 (the circle and the square symbols represent different types of paper materials). Moreover, the further the deviation from the standard line, the better the thermal insulation performance. Hence, the deviation curves are used to qualitatively analyze the thermal insulation performance of paper materials.

Furthermore, each fitting curve is obtained by curve fitting based on the original experimental data and is given by

$$t_c = f(t_h). \quad (3)$$

The area between the standard line and a fitting curve in a particular temperature interval $[t_1, t_2]$ is then used to quantitatively analyze the thermal insulation performance of the paper material in a particular temperature range, such as the shaded part (depicted with oblique lines) in Figure 2. We define the temperature factor (ω) as the area of the shaded part given by the expression

$$\omega = \int_{t_1}^{t_2} \Delta t dt = \int_{t_1}^{t_2} [t - f(t)] dt. \quad (4)$$

The higher the numerical value of ω for a paper material in the closed interval $[t_1, t_2]$, the better the thermal insulation performance of the paper cup in a particular temperature range.

TABLE 1: Data of the paper cup samples.

Number	Grammage g/m ²	Thickness mm	Structure type
1	252	0.335	I
2	280	0.333	I
3	248	0.321	I
4	537	0.672	II
5	504	0.646	II
6	479	0.820	III with air layer
7	606	0.780	III with air layer
8	282	0.512	III with foam surface
9	606	0.950	III with air layer
10	356	0.706	III with foam surface
11	299	0.361	I
12	543	0.695	II

3. Experimental

3.1. Materials. Paper cups with different structures were collected from the market and divided into three types based on the study of Iioka et al. [25–27], as shown in Figure 3. The main body of the paper cup was a single fibrous layer for structure I and a double-layer structure for structure II. Structure III was a composite structure composed of material of different natures. The structure type and basic properties of each paper sample are given in Table 1.

3.2. Heat Insulation Performance Test. All of the samples in Table 1 are pretreated and tested based on the methods mentioned in Section 2.1. The experiment was performed

FIGURE 3: Structure type classification: (a) structure I, (b) structure II, and (c) structure III.

at 23°C and 50% RH ambient conditions. The operating environment for evaluation of the paper cups was set to be the closed interval from 70 to 85°C. The temperatures of the cold and hot surfaces were recorded synchronously for every 5°C temperature change as the liquid temperature was varied from 45 to 85°C.

4. Results and Discussion

4.1. Qualitative Analysis for Thermal Insulation Performance of Paper Cups. Figure 4(a) shows the relationship between the hot surface temperature and the cold surface temperature of each sample in Table 1. With increasing temperature of the hot surface, increases in both the cold surface temperature and the amount of deviation from the standard line were observed. Furthermore, most of these curves are approaching each other and are hard to identify. Moreover, some curves crossed or overlapped in the closed interval, as shown in the high magnification image of Figure 4(a), for instance, number 4, number 5, number 8, number 10, and number 12.

Curve fits for each deviation curve in Figure 4(a) were obtained with Origin 8 software (Figure 4(b)). The broken lines with symbols were replaced by a group of continuous lines of different colors. The phenomenon of crossing and overlapping of the curves dramatically decreased with the use of the fitting curves, although these problems still exist. The samples were ranked by approximate thermal insulation performance as below (> means better than, ≈ means similar):

$$\text{No. } 7 > 6 > 9 > 12 \approx 10 \approx 4 \approx 5 \approx 8 > 3 > 11 > 1 > 2. \quad (5)$$

The thermal insulation performance is proportional to thickness [28], and the thermal insulation performance of the air is better than that of a solid, so structure III with an air layer can effectively increase the thermal insulation performance of paper cups. In addition, the thermal insulation performances of number 7, number 6, and number

TABLE 2: Fitted curve equations, linear correlation coefficients, temperature factors, and temperature factors per unit thickness.

Number	Equation	R	ω °C ²	ω/b °C ² /mm
1	$y = 3.15722 + 0.881x$	0.99926	90.98	271.58
2	$y = 4.43 + 0.88467x$	0.99974	67.62	203.06
3	$y = 4.52167 + 0.84633x$	0.99979	110.82	345.23
4	$y = 5.92444 + 0.808x$	0.99911	134.33	199.90
5	$y = 7.39833 + 0.79233x$	0.99819	130.44	201.92
6	$y = 7.23556 + 0.72133x$	0.98962	215.42	262.71
7	$y = 16.14278 + 0.55233x$	0.99237	278.27	356.76
8	$y = 6.32333 + 0.80733x$	0.99874	129.13	252.21
9	$y = 9.11389 + 0.72833x$	0.99568	179.11	188.54
10	$y = 6.54056 + 0.79767x$	0.99938	137.10	194.19
11	$y = 6.045 + 0.83367x$	0.99932	102.68	284.43
12	$y = 7.22556 + 0.78867x$	0.99425	137.29	197.54

9, which all have structure III with an air layer between the fibrous layers, were better than the others. However, it was difficult to discriminate the differences in the thermal insulation performances of number 4, number 5, number 8, number 10, and number 12.

4.2. Quantitative Analysis for Thermal Insulation Performance of Paper Cups by Temperature Factor ω . The temperature factor ω was used to quantify the thermal insulation performance of paper cups. First, the fitting curves and equations were established using linear fitting. The equations and the linear correlation coefficients are shown in Table 2. The process for calculation of the temperature factor ω was as follows.

Assume that the linear regression equation for the fit to the deviation curve is

$$y = A + Cx. \quad (6)$$

FIGURE 4: (a) Cold surface temperature as a function of hot surface temperature for the paper cups. (b) The fitting curve of each deviation curve in (a).

The equation of the standard line is

$$y = x. \quad (7)$$

Subtracting (7) from (6) gives

$$y = -A + (1 - C)x. \quad (8)$$

Let $1 - C = B$. Equation (8) can then be expressed as

$$y = -A + Bx. \quad (9)$$

The temperature factor ω is calculated by

$$\begin{aligned} \omega &= \int_{70}^{85} (-A + Bx) dx = 1162.5B - 15A \\ &= 1162.5(1 - C) - 15A. \end{aligned} \quad (10)$$

As shown in Table 2, each sample's temperature factor in the particular range $[70, 85]^{\circ}\text{C}$ was calculated from the fitted curve equations and (10).

FIGURE 5: Temperature difference as a function of hot surface temperature for the paper cups.

The utilization of the temperature factor clarifies evaluation of each sample's thermal insulation performance in a particular temperature range. We can exactly and accurately rank these paper cups according to their values of ω in Table 2 as

$$\text{No. } 7 > 6 > 9 > 12 > 10 > 4 > 5 > 8 > 3 > 11 > 1 > 2. \quad (11)$$

Rank (11) is almost the same as rank (5). For paper sample numbers 12, 10, 4, 5, and 8, whose thermal insulation performances were approximately equal in rank (5), the specific heat insulation performance order can now be obtained by comparing their values of ω , which are 137.29, 137.10, 134.33, 130.44, and 129.13, respectively. Moreover, the utilization of ω could avoid difficulties associated with the possible overlapping of fitting curves as in Figure 4(b).

4.3. The Application of ω to Paper Cup Design and Selection of Paper Materials. To compare the thermal insulation performance of paper cups with different structures, we calculated the temperature difference at each measurement temperature and plotted the temperature difference curves (Figure 5). The color of each temperature difference curve corresponds to the structure of the sample: black for structure I, red for structure II, and blue for structure III, as shown in Figure 5.

Figure 5 clearly shows the change of temperature difference with the hot surface temperature. Generally, the difference at high temperature is larger than at low temperature. Furthermore, the thermal insulation performance of a sample with a higher curve is better than that of a sample with

a lower one. The order of descending thermal insulation performance of the structures is structure III, structure II, and structure I, and the corresponding ranges of temperature factors are 129.13 to 278.27, 130.44 to 137.29, and 67.62 to 110.82, respectively. The degree of tightness of the curves for samples of the same structure in Figure 5 is consistent with the width of the temperature factor interval for that structure. Hence, we can choose the best structure for paper cups according to the required thermal insulation performance (the range of ω).

In addition to the structure, another factor in the design of paper cups is the selection of paper materials. The temperature difference between the cold and hot surfaces of paper is proportional to the thickness according to Fourier's law and the equation for heat conduction through a flat single-layered wall [28]:

$$Q = -kA \frac{dt}{dx}, \quad (12)$$

$$\Delta t = t_2 - t_1 = \frac{bQ}{kA}. \quad (13)$$

Simultaneous equations (4) and (13) give

$$\omega = \int_{t_1}^{t_2} \Delta t dt = b \int_{t_1}^{t_2} \left[\frac{Q}{kA} \right] dt. \quad (14)$$

Equation (14) demonstrates that the value of the temperature factor ω is also proportional to the thickness of the paper material. Thus, the temperature factor ω per unit length in the thickness direction could be expressed as

$$\frac{\omega}{b} = \frac{1}{b} \int_{t_1}^{t_2} [t - f(t)] dt. \quad (15)$$

Equation (15) also manifests the thermal insulation performance of paper cups per unit length in the thickness direction. From the simultaneous equations (10) and (15), the thermal insulation performance of each sample per unit length in the thickness direction was obtained and given in Table 2. According to the numerical values of ω/b in Table 2, the paper materials in order of descending thermal insulation performance per unit length in the thickness direction are

$$\text{No. } 7 > 3 > 11 > 1 > 6 > 8 > 2 > 5 > 4 > 12 > 10 > 9. \quad (16)$$

Obviously, rank (16) is quite different from rank (11). A sample with better thermal insulation performance of the whole media may not be outstanding when compared on a per thickness basis. Therefore, many benefits will flow from this finding. For instance, it can be speculated that if the fiber layer of sample 4 was instead by the fiber as sample 3, it would have better thermal insulation performance than present sample 4.

In conclusion, the structure of paper cups significantly influenced the thermal insulation performance. The difference in thermal insulation performance can be attributed to a multitude of factors, such as fiber type, paper bulk, porosity,

moisture, and production process. Numerous parameters cause accurate measurement of the effective thermal conductivity to be difficult and time consuming. However, it will be more convenient to use the temperature factor ω when characterizing the thermal insulation performance of paper cups in practical applications and, of course, during the design of thermally insulating paper cups.

5. Conclusion

- (1) This work proposed a facile approach to analyze the thermal insulation performance of paper cups by establishing a novel evaluation model that includes a characterization parameter named “temperature factor” ω and a measurement apparatus.
- (2) The temperature factor is defined as $\omega = \int_{t_1}^{t_2} [t - f(t)]dt$, which is the integrated area between the standard line and the fitting curve in a particular temperature range $[t_1, t_2]$. Moreover, the value of the temperature factor is proportional to the thickness of the paper samples.
- (3) The thermal insulation performance of paper cups with different structures is quantitatively analyzed based on the value of the temperature factor in the temperature range of 70 to 85°C. The calculation results of the temperature factor of 12 paper cup samples show that the structure type of the cups is the most important parameter which strongly influences the thermal insulation performance of the paper cups.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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