

Evaluation of Seal Effects on the Stability of Rotating Fluid Machinery

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The stability of typical rotating fluid machinery such as single and multi-stage pumps is evaluated by using the finite element method. The individual contribution of the impellers, bearings and seals to the stability and the dynamic interactions of these fluid elements are examined. Various types of bearings and seals, such as annular smooth, parallel grooved and damper seals, are compared for better rotor stability. The effect of the operating conditions on the stability is also investigated. The results show that rotor stability can be easily improved by replacing the unstable fluid elements.

Key Words: *Rotating fluid Machinery; Bearing; Impeller; Non-contacting seal; Rotor stability; Logarithmic decrement*

INTRODUCTION

In recent years the operating conditions of rotating fluid machinery tend towards higher speed and higher pressure along with the rapid progress made in the technology in industry and space development. In order to raise the efficiency and prevent leakage flow at high pressure, the clearance of fluid elements such as bearings, non-contacting seals and impellers has to be designed possibly smaller. However, with higher pressure, higher speed and smaller clearance, greater fluid forces occur and sometimes these forces cause unstable vibration.

To prevent such unstable vibration, firstly the dynamic characteristics of these fluid elements and their individual contribution to rotor stability must be made clear. Work is being done, and many results have been obtained by various researchers. Secondly, the combined effect of these fluid elements on rotor stability and their interaction must be investigated. Yang *et al.* [12] (1985) investigated the effect of annular smooth and taper seals on the stability of the single-stage pump rotor system.

Diewald *et al.* [3] (1987) showed a procedure to investigate the effect of annular smooth and grooved seals and impellers on the stability of the Jeffcott rotor and the multi-stage pump rotor system. These researches show that the rotor stability is strongly affected by the fluid elements, and the contribution of these elements to rotor stability varies according to the operating conditions.

In this paper, continuing the research of Yang *et al.* [12] (1985), the stability of typical rotating fluid machinery such as the single-stage and multi-stage pumps which consist of impellers, bearings and non-contacting seals is evaluated by using the finite element method. The individual contribution of the impellers, bearings as well as seals to the stability, and the dynamic interactions of these fluid elements are investigated. The contribution to the rotor stability is evaluated by the logarithmic decrement. For the linear and non-cross-coupled inertia rotor system, the total logarithmic decrement of the rotor system can be represented as the sum of the individual decrements. Therefore, the stability of the rotor system can be easily improved by changing the unstable fluid elements in the design stage. In the investigation, some

types of bearings and seals such as annular smooth, parallel grooved and damper seals are compared to seek better ones for the rotor stability. The effect of the operating conditions on stability is also studied.

EQUATIONS OF MOTION AND LOGARITHMIC DECREMENT

The analytical models of the single and multi-stage pumps are shown in Fig. 1 and Fig. 2. For general application, the non-symmetrical single-stage pump is taken. These rotor systems consist of bearings, impellers and seals. For the convenience of analysis, the impeller is looked upon as a disk which has the same mass and same moment of inertia as the practical impeller. If the rotor rotates with a steady angular velocity ω , the equation of motion of the disk elements in the coordinates illustrated in Fig. 2 is as follows:

$$([M_t^d] + [M_r^d])\{\dot{q}^d\} - \omega[G^d]\{\dot{q}^d\} = \{F^d\} \quad (1)$$

where $[M_t^d]$ and $[M_r^d]$ are respectively the mass matrices for translational and bending motions; $[G^d]$ is gyroscopic matrix; $\{F^d\}$ is the force vector acting on the disks.

The equation of motion of the journal elements is given by the expression

$$([M_t^e] + [M_r^e])\{\dot{q}^e\} - \omega[G^e]\{\dot{q}^e\} + [K^e]\{q^e\} = \{F^e\} \quad (2)$$

where $\{q^e\} = \{qt, qr\}^T = \{x, y, \phi_x, \phi_y\}^T$, $[M_t^e]$, $[M_r^e]$, $[G^e]$, $[K^e]$ are the mass matrices of translational and bending motions, gyroscopic matrix and stiffness matrix respectively; $\{F^e\}$ is the fluid force vector of bearings, impellers and seals, and they are expressed as follows:

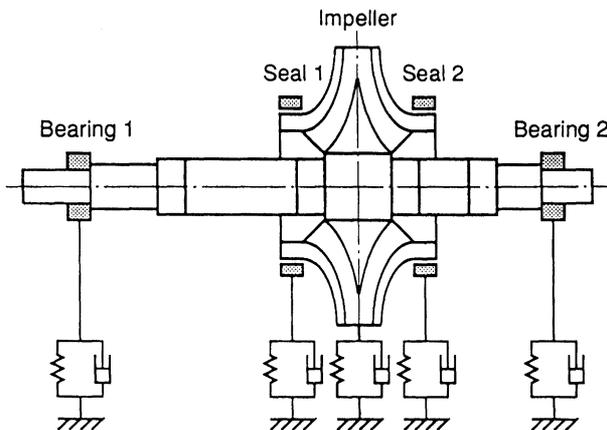


FIGURE 1 Model of single-stage pump.

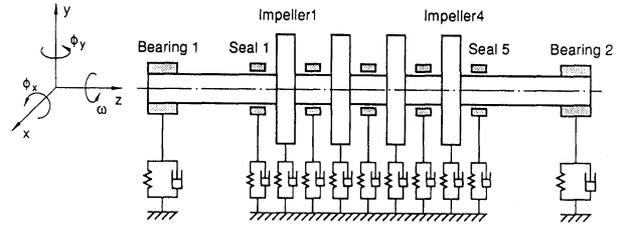


FIGURE 2 Model of multi-stage pump.

$$\begin{aligned} -\{F_B\} &= [C_B]\{\dot{q}_t\} + [K_B]\{q_t\} \\ -\{F_I\} &= [M_I]\{\ddot{q}_t\} + [C_I]\{\dot{q}_t\} + [K_I]\{q_t\} \\ -\{F_S\} &= [M_S]\{\ddot{q}_t\} + [C_S]\{\dot{q}_t\} + [K_S]\{q_t\} \end{aligned} \quad (3)$$

where $[M_I]$, $[M_S]$ are the inertia coefficient matrices; $[C_B]$, $[C_I]$, $[C_S]$ are the damping coefficient matrices; $[K_B]$, $[K_I]$, $[K_S]$ are the stiffness coefficient matrices. Substituting Eq.(3) into Eq. (2), and combining the equations of disk elements and journal elements, the equation motion of the rotor system can be obtained.

$$[M]\{\dot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F\} \quad (4)$$

where $\{F\}$ is the external force not including the fluid forces of bearings, impellers and seals. To determine the eigenvalues and eigenvectors, the characteristic equation of Eq. (4) is rewritten in the following form:

$$\begin{bmatrix} [I] & 0 \\ [C] & [M] \end{bmatrix} \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} + \begin{bmatrix} 0 & -[I] \\ [K] & 0 \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} = \{0\} \quad (5)$$

or

$$[D]\{\dot{z}\} + [E]\{z\} = \{0\} \quad (6)$$

Letting the generalized solution of Eq. (6) have the form

$$\{z\} = \{\bar{z}\}e^{\lambda t} \quad (7)$$

then Eq. (6) yields

$$\{[I]\lambda + [D]^{-1} + [E]\}\{\bar{q}\} = \{0\} \quad (8)$$

where λ and $\{\bar{q}\}$ are the eigenvalue and eigenvector, respectively. They can be obtained by solving Eq. (8).

In general, the eigenvalues are conjugate complex, and expressed in the following form:

$$\lambda_i = \alpha_i + j\omega_i, \quad \bar{\lambda}_i = \alpha_i - j\omega_i \quad (9)$$

where i means i -th natural mode. The logarithmic decrement of the rotor system is defined as

$$\delta_i = -2\pi \frac{\alpha_i}{\omega_i} \quad (10)$$

In order to investigate the effects of the fluid elements on the rotor stability, the logarithmic decrements of the fluid elements have to be determined. These logarithmic decrements can be obtained by the eigenvalue and eigenvector through the following transformation [9] (1982).

Substituting the i -th conjugate eigenvalues $\lambda_i, \bar{\lambda}_i$ and i -th conjugate eigenvectors $\{\psi_i\}, \{\bar{\psi}_i\}$ into the characteristic equation of Eq. (4), the following equations are obtained.

$$\begin{aligned} \lambda_i^2 [M]\{\psi_i\} + \lambda_i [C]\{\psi_i\} + [K]\{\psi_i\} &= \{0\} \\ \bar{\lambda}_i^2 [M]\{\bar{\psi}_i\} + \bar{\lambda}_i [C]\{\bar{\psi}_i\} + [K]\{\bar{\psi}_i\} &= \{0\} \end{aligned} \quad (11)$$

Furthermore, expressing the mass matrix, damping matrix and stiffness matrix as the sum of symmetric parts M^*, C^*, K^* and unsymmetric parts $\Delta M, \Delta C, \Delta K$, then the above equations become

$$\begin{aligned} \lambda_i^2 [M^* + \Delta M]\{\psi_i\} + \lambda_i [C^* + \Delta C]\{\psi_i\} + [K^* + \Delta K]\{\psi_i\} &= \{0\} \\ \bar{\lambda}_i^2 [M^* + \Delta M]\{\bar{\psi}_i\} + \bar{\lambda}_i [C^* + \Delta C]\{\bar{\psi}_i\} + [K^* + \Delta K]\{\bar{\psi}_i\} &= \{0\} \end{aligned} \quad (12)$$

where $[\Delta M] = -[\Delta M]^T, [\Delta C] = -[\Delta C]^T, [\Delta K] = -[\Delta K]^T$.

Premultiplying equations of (12) by $\{\bar{\psi}_i\}^T$ and $\{\psi_i\}^T$, and introducing the following expressions

$$\begin{aligned} \{\bar{\psi}_i\}^T [M^*]\{\psi_i\} &= \{\bar{\psi}_i\}^T [M^*]\{\bar{\psi}_i\} = m_i^* \\ \{\bar{\psi}_i\}^T [C^*]\{\psi_i\} &= \{\bar{\psi}_i\}^T [C^*]\{\bar{\psi}_i\} = c_i^* \\ \{\bar{\psi}_i\}^T [K^*]\{\psi_i\} &= \{\bar{\psi}_i\}^T [K^*]\{\bar{\psi}_i\} = k_i^* \\ \{\bar{\psi}_i\}^T [\Delta M]\{\psi_i\} &= -\{\psi_i\}^T [\Delta M]\{\bar{\psi}_i\} = j \Delta m_i \\ \{\bar{\psi}_i\}^T [\Delta C]\{\psi_i\} &= -\{\psi_i\}^T [\Delta C]\{\bar{\psi}_i\} = j \Delta c_i \\ \{\bar{\psi}_i\}^T [\Delta K]\{\psi_i\} &= -\{\psi_i\}^T [\Delta K]\{\bar{\psi}_i\} = j \Delta k_i \end{aligned} \quad (13)$$

the difference of the two equations of Eq. (12) can be written as follows:

$$(\lambda_i^2 - \bar{\lambda}_i^2) m_i^* + (\lambda_i^2 + \bar{\lambda}_i^2) \Delta m_i + (\lambda_i + \bar{\lambda}_i) c_i^* - (\lambda_i + \bar{\lambda}_i) \Delta c_i + 2 \Delta k_i = 0 \quad (14)$$

Because the unsymmetric mass Δm_i is caused by the impellers and seals, and it is usually much smaller than the symmetric mass m_i^* , it is ignored here. Substituting Eq. (9) into Eq. (14), the real part of the eigenvalues is obtained.

$$\alpha_i = -\frac{\omega_i c_i^* + \Delta k_i}{2\omega_i m_i^* + \Delta c_i} \quad (15)$$

Therefore, the logarithmic decrement is expressed as follows:

$$\delta_i = -2\pi \frac{\alpha_i}{\omega_i} = \frac{\pi \omega_i c_i^* + \pi \Delta k_i}{\omega_i^2 m_i^* + \omega_i \Delta c_i / 2} \quad (16)$$

For the present rotor systems, c_i^* and Δk_i can be expressed as the sum of the fluid elements.

$$\begin{aligned} c_i^* &= \{\bar{\psi}_i\}^T [C^*]\{\psi_i\} \\ &= \sum_{j=1}^{n_B} (\{\bar{\psi}_{Bj}\}^T [C_{Bj}^*]\{\psi_{Bj}\}) + \sum_{j=1}^{n_S} (\{\bar{\psi}_{Sj}\}^T [C_{Sj}^*]\{\psi_{Sj}\}) \\ &\quad + \sum_{j=1}^{n_I} (\{\bar{\psi}_{Ij}\}^T [C_{Ij}^*]\{\psi_{Ij}\}) \\ &= \sum_{j=1}^{n_B} (c_{Bj}^*) + \sum_{j=1}^{n_S} (c_{Sj}^*) + \sum_{j=1}^{n_I} (c_{Ij}^*) \\ \Delta k_i &= \{\bar{\psi}_i\}^T [\Delta K]\{\psi_i\} \\ &= \sum_{j=1}^{n_B} (\{\bar{\psi}_{Bj}\}^T [\Delta k_{Bj}]\{\psi_{Bj}\}) + \sum_{j=1}^{n_S} (\{\bar{\psi}_{Sj}\}^T [\Delta k_{Sj}]\{\psi_{Sj}\}) \\ &\quad + \sum_{j=1}^{n_I} (\{\bar{\psi}_{Ij}\}^T [\Delta k_{Ij}]\{\psi_{Ij}\}) \\ &= \sum_{j=1}^{n_B} (\Delta k_{Bj}) + \sum_{j=1}^{n_S} (\Delta k_{Sj}) + \sum_{j=1}^{n_I} (\Delta k_{Ij}) \end{aligned} \quad (17)$$

Substituting the above expressions into Eq. (16), the logarithmic decrement expressed as the sum of the individual elements is obtained.

$$\delta_i = \frac{\pi}{\omega_i^2 m_i^* + \omega_i \Delta c_i / 2} \left\{ \omega_i \left[\sum_{j=1}^{n_B} (c_{Bj}^*) + \sum_{j=1}^{n_S} (c_{Sj}^*) + \sum_{j=1}^{n_I} (c_{Ij}^*) \right] + \left[\sum_{j=1}^{n_B} (\Delta k_{Bj}) + \sum_{j=1}^{n_S} (\Delta k_{Sj}) + \sum_{j=1}^{n_I} (\Delta k_{Ij}) \right] \right\}$$

$$= \sum_{j=1}^{n_B} \delta_{Bij} + \sum_{j=1}^{n_S} \delta_{Sij} + \sum_{j=1}^{n_I} \delta_{Iij}$$

where n_B , and n_S , and n_I are the total numbers of the bearings, seals and impellers, respectively.

EXAMPLE OF ANALYSIS

Stability of Single-Stage Pumps

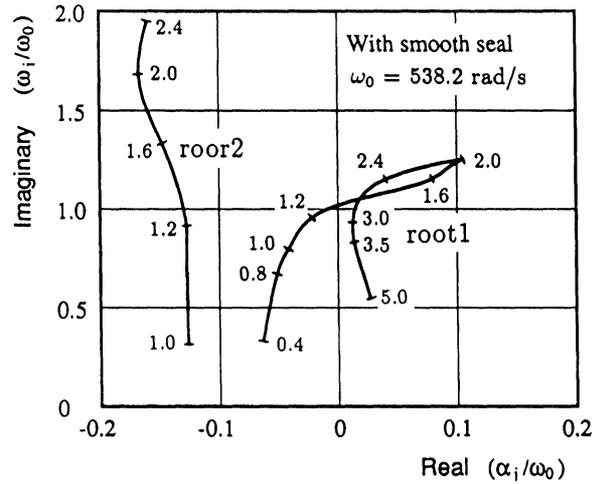
The calculation is based on the conditions in Table 1. In the calculation, the bearing dynamic coefficients and impeller dynamic coefficients of references [7] (1984) and [11] (1987) are used, while the dynamic coefficients of seals are obtained by the calculating method given in [5] (1989) and [6] (1990).

The loci of eigenvalues of Eq. (8) for single-stage pump rotor systems with smooth, damper and parallel grooved seals are illustrated in Fig. 3. Here, only the two eigenvalues in or near to the unstable region ($\alpha_i \geq 0$) are given. In these figures, ω_0 is the first eigenfrequency of the rotor without seals and bearings. The numbers marked on the loci are the ratios at rotating speed N to ω_0 . In the present rotating region, the stability of the rotor system is dependent on the eigenvalue of root 1. Therefore, the stability is discussed as to this eigenvalue by means of the logarithmic decrement.

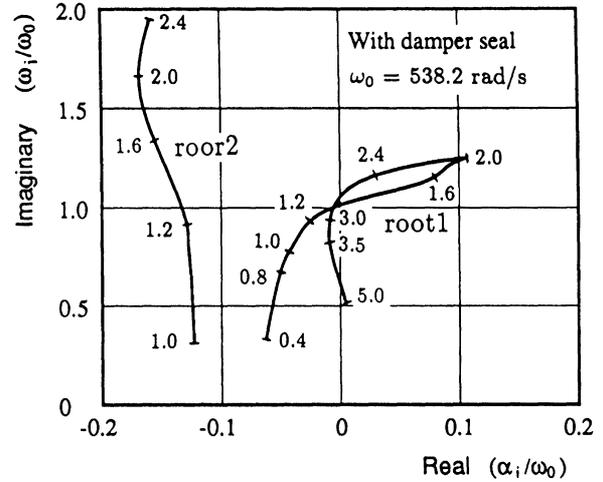
From the point of view of energy, the rotor system is releasing energy to the outside so that the system tends to

TABLE 1
Specification of single-stage pump

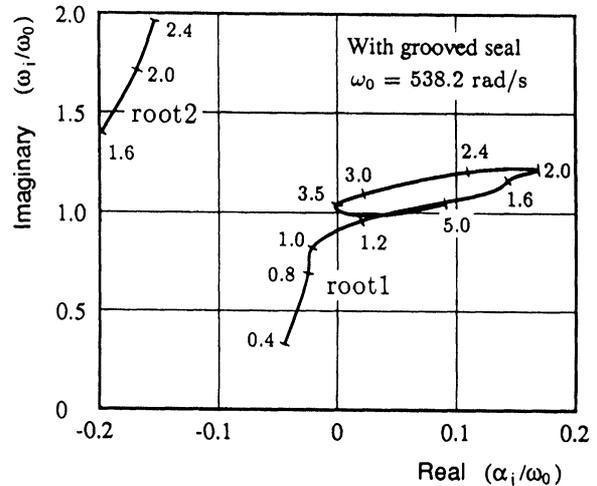
Bearing length	(mm)	60
Bearing diameter	(mm)	60
Bearing clearance	(mm)	0.06
Seal length	(mm)	20 ~ 40
Seal diameter	(mm)	120
Seal clearance	(mm)	0.2~0.6
Impeller width	(mm)	80
Impeller diameter	(mm)	300
Impeller clearance	(mm)	0.5
Pressure difference	(M Pa)	4.9
Oil viscosity	(mPa·s)	16.7
Oil temperature	(°C)	60
Water temperature	(°C)	50



(a) With smooth seals



(b) With damper seals



(c) With parallel grooved seals

FIGURE 3 Complex eigenvalues.

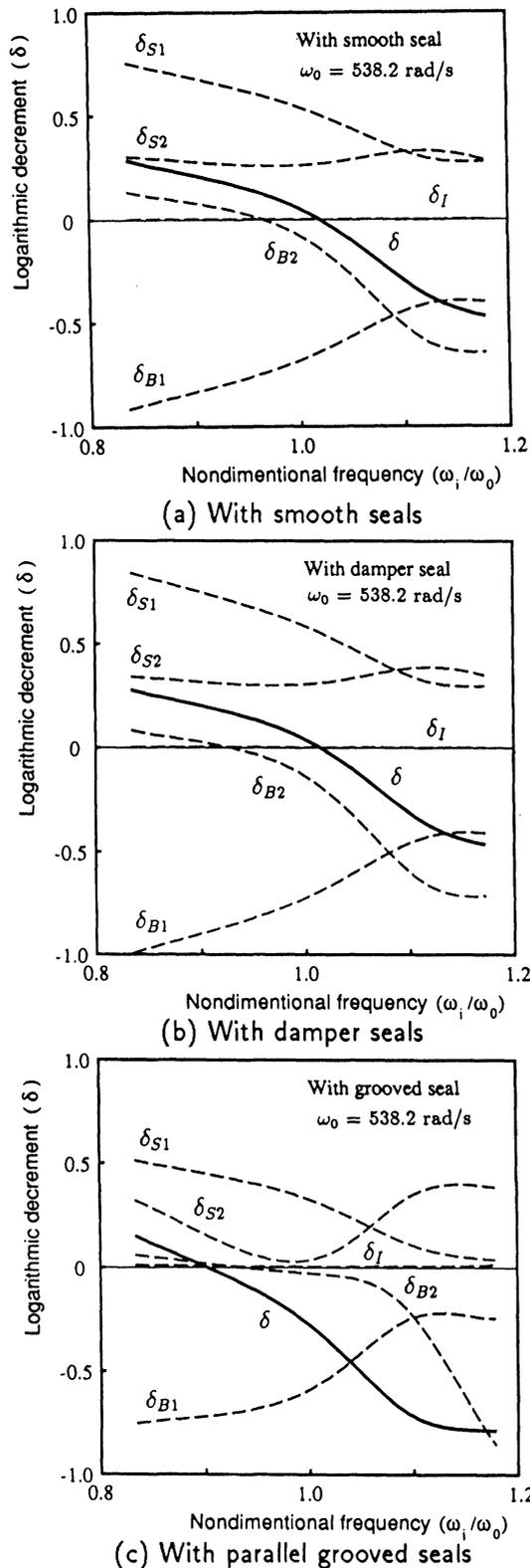


FIGURE 4 Logarithmic decrements.

stabilize with the lapse of time, if the logarithmic decrement is positive; while, if the logarithmic decrement is negative, the rotor system is absorbing energy from the outside so that the system enlarges the amplitude and tends to destabilize with the lapse of time [9] (1982). Figure 4 shows the logarithmic decrements of the bearings, seals, impellers and the total rotor systems. For this natural mode, the contribution of the impeller to the stability is very small. The logarithmic decrements of the seals are positive, which means the stable effect on the rotor stability, but the logarithmic decrements of the bearings are almost negative, which means an unstable effect on the rotor stability.

The logarithmic decrements of the rotor systems with smooth, damper and parallel grooved seals are shown in Fig. 5. The results show that the logarithmic decrements of rotor systems with smooth and damper seals are similar, and the stability of these systems are better than that of rotor systems with parallel grooved seals. In actual machinery, investigation of rotor stability with a specific rotating speeds or in a specific rotating region is usually necessary. The logarithmic decrements of rotor systems with different seals at specific rotating speed $N/\omega_0 = 1$ are shown in Fig. 6, where the mode shape of the rotor system is shown, too. These results are drawn in the form of bar graphs for the convenience of clarity. From these results the contribution of every fluid element to rotor stability can be immediately recognized. The rotor stability can be improved by replacing the unstable elements with stable elements. This will be discussed in next section.

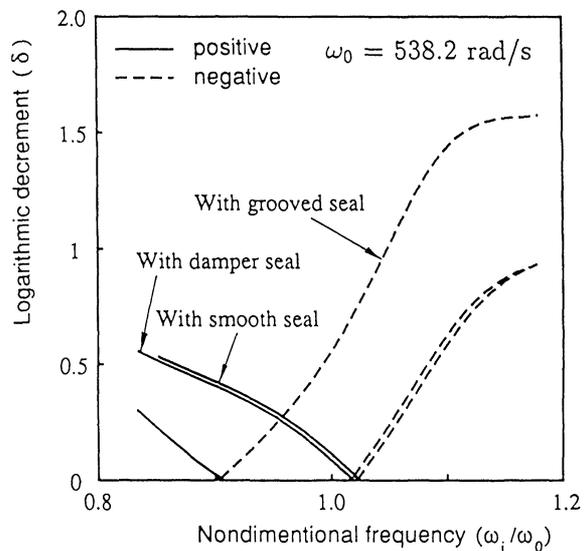


FIGURE 5 Comparison of logarithmic decrement.

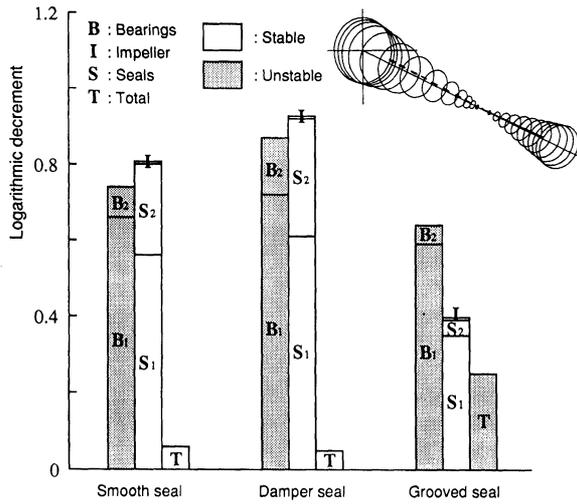


FIGURE 6 Logarithmic decrement at $(N/\omega_0=1)$.

The influence of preswirl velocity V_t in seals on rotor stability is investigated, and the results are shown in Fig. 7. This figure shows that positive preswirl velocity exerts an unstable influence on rotor stability; while negative preswirl velocity exerts a stable influence on rotor stability. This result is in agreement with the individual research of seals [4]~[6].

Stability of Multi-Stage Pumps

The specification of multi-stage pumps is illustrated in Table 2. The logarithmic decrements of rotor systems

TABLE 2
Specification of multi-stage pump

Bearing length	(mm)	80
Bearing diameter	(mm)	60
Bearing clearance	(mm)	0.06
Seal length	(mm)	20 ~40
Seal diameter	(mm)	100
Seal clearance	(mm)	0.2~0.6
Impeller width	(mm)	40
Impeller diameter	(mm)	300
Impeller clearance	(mm)	0.5
Pressure difference	(MPa)	4.9
Oil viscosity	(mPa·s)	16.7
Oil temperature	(°C)	60
Water temperature	(°C)	50

and fluid elements at $N = \omega_0 = 19671.6$ rpm are studied and shown in Fig. 8, where the mode shapes of rotor systems are shown, too. In the investigation, some of seals or bearings are changed in order to improve rotor stability and find the interactions of these fluid elements.

In the case of (a), five parallel grooved seals are used in the rotor system. The total logarithmic decrement shows a negative value because of the unstable effects of the seals and the bearings. If the parallel grooved seal 1 is replaced by a damper seal, the rotor system becomes stable (b). Furthermore, by replacing all the grooved seals with damper seals, the rotor system becomes more stable (c). It is found that after the replacement of the seals, not only the logarithmic decrements of the seals but also those of the bearings are changed. If the circle bearings are replaced with tilting pad bearings instead of the seals, as shown in (d), the rotor stability can also be improved. In this case, the logarithmic decrements of the seals are also changed. It is considered that the interactive variation of the fluid elements is caused by the variation of the natural mode, because the logarithmic decrement is dependent on the natural mode, which has been demonstrated in the previous section. According to the above discussion, the interaction of other elements must be considered when evaluating the effect of the fluid element on rotor stability.

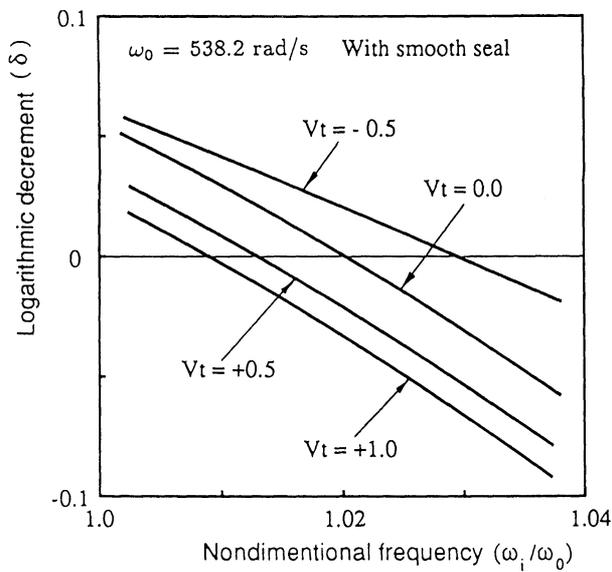


FIGURE 7 Effect of preswirl velocity on logarithmic decrement.

CONCLUSION

The present analysis supports the following conclusions:

1. In rotating fluid machinery, the investigation of the effect of the individual fluid element on rotor stability is an effective method for the purpose of the evaluation of rotor stability and dynamic design.

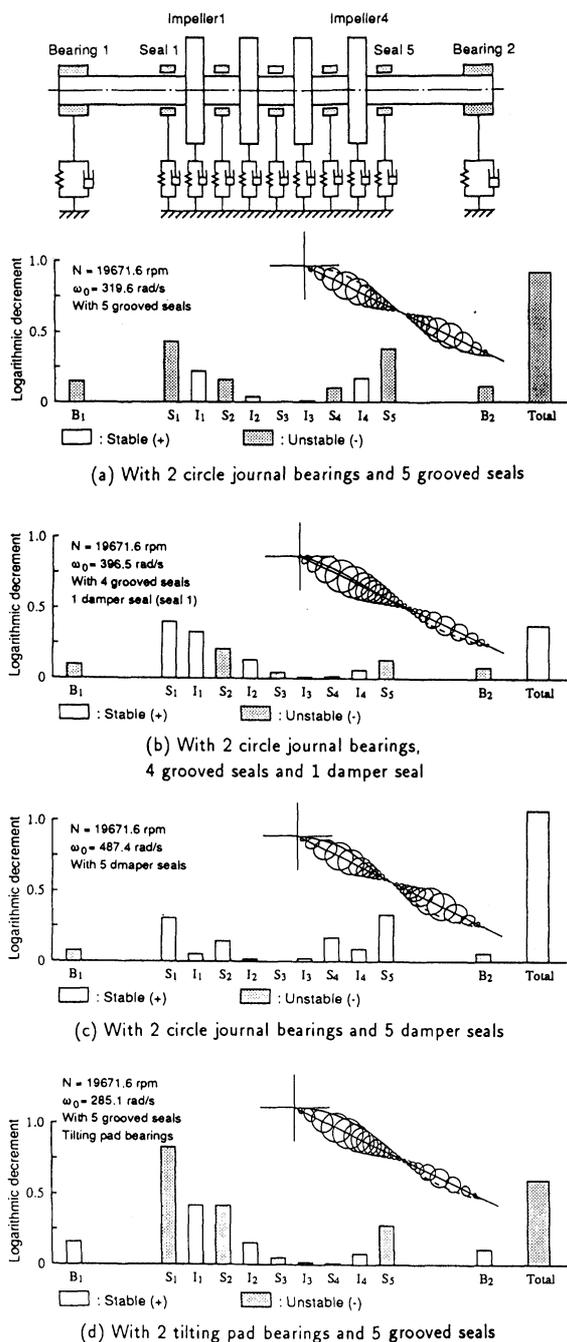


FIGURE 8 Logarithmic decrement of multi-stage pump.

2. Rotor stability can be improved by replacing the unstable or other elements. Such replacements usually cause a variation on the logarithmic decrements of the other elements.
3. The effect of preswirl velocity in seals on the stability is consistent with that of the individual research on seals.

Nomenclature

C ($N \cdot s/m$)	:	Damping coefficient
F (N)	:	Force
K (N/m)	:	Stiffness coefficient
M (kg)	:	Inertia coefficient
N (rpm)	:	Rotating speed of rotor
n	:	Number of fluid elements
V_i (m/s)	:	Preswirl velocity in seal
x, y, z	:	Coordinates
α	:	Real part of eigenvalue
δ	:	Logarithmic decrement
λ	:	Eigenvalue
\odot	:	Rotatory coordinates
ψ	:	Eigenvector
ω	:	Eigenfrequency or imaginary part of eigenvalue
ω_0	:	Eigenfrequency of rotor system without fluid

Subscript

B	:	Bearing
I	:	Impeller
S	:	Seal

Superscript

T	:	Transposed (matrix)
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