Rotordynamic Coefficients of Long Staggered Labyrinth Gas Seals

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Experimental and theoretical investigations concerning a long staggered labyrinth gas seal are presented. Accurate static measurements help to assess the influence of the entry swirl, the rotational speed and the pressure difference on the conservative and the nonconservative force. For a better understanding of the phenomena the forces in each cavity are investigated. A coupling between the circumferencial flow and the axial flow in the cavity is revealed. The theoretical results are obtained by a one-volume bulk-flow theory which is essentially based on the momentum equation in circumferencial direction and therefore neglects some effects caused by the axial flow. The experimental results are compared to calculated results.

INTRODUCTION

High power density turbomachines often work at parameter values very close to the stability limit. In the unstable region the rotor performs self-excited bending vibrations with a frequency which is normally close to the first critical speed. Self-excited vibrations are essentially caused by fluid mechanisms in journal bearings and labyrinth seals transferring energy into the rotor vibration. The unacceptable high amplitudes of the self-excited vibrations can severely affect the safe and reliable operation of the turbomachine. An exact prediction of the stability limit requires a good modelling of the exciting mechanisms and, as a consequence, a reliable evidence about the forces acting on the rotor. Based on the assumption of small deflections out of the centered position, a linearized formulation of the labyrinth seal forces is usually used.

\[
\mathbf{F} = -\begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} - \begin{bmatrix} E & e \\ -e & E \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \quad (1)
\]

The force acting on the rotor can be splitted in a conservative restoring and a nonconservative cross force. In most cases, the main source of instability is the cross-coupled stiffness. The experimental identification of the direct and the cross-coupled stiffness is realized by so called static measurements (Hauck [1982], Thieleke and Stetter [1990], Benckert and Wachter [1980] Leong and Braun [1984]). Despite the fact that the restoring force is conservative it has however an influence on the critical speeds and the deflection of the vibrational modes, and as a consequence on the stability. This can be relevant especially when the rotor is slim and/or labyrinth seals are placed in a central position between the bearings.

Dynamic measurements are effected to identify the dynamic damping and inertia coefficients of the labyrinth seal (Childs et al. [1986], Nordmann and Massmann [1984]). These coefficients are highly important especially when the fluid is dense and/or when the whirling frequency is high.

The various theoretical approaches can be validated by means of these experimental results. In addition, the involved models can be improved, if necessary, and the used simplifications reviewed. The computational effort is low when a lumped parameter model—either a one-, two-, or three-volume model is used (Baumgartner [1987], Childs and Scharrer [1986], Nordmann and Weiser[1990]). When computational fluid dynamic methods are applied, the geometry of the investigated cavity is discretized by a grid and the local variables (velocity,
pressure) are calculated (Nordmann and Weiser [1988]). This case requires a higher computational time and, therefore renders, parametric studies more difficult.

Extensive experimental investigations on multistage gas labyrinth seals with various cavity geometry were carried out by Benckert and Wachter [1980]. Among others, a staggered 10-cavity labyrinth geometry with grooves and tooth on stator and fins on rotor was investigated. For long seals the rotation turned out to have a significant influence. Benckert and Wachter [1980] stated that the cross-coupled stiffness arose primarily because of the circumferential velocity which depends not only on the rotating speed but also on the ratio of rotating and nonrotating cavity walls. No information was given about the cavity to cavity distribution of the forces. The maximum adjusted pressure ratio was 0.35. Leong and Brown [1984] tested a number of labyrinth geometries with strips attached to the stator. The maximum pressure ratio investigated was 0.28. They found considerable decentering forces and negative stiffness coefficients due to rotation without entry swirl. Thieleke and Stetter [1990] presented additional test results for staggered labyrinth seals with maximum eight cavities in a teeth on stator configuration using the apparatus of Benckert and Wachter [1980].

In this paper experimental investigations performed on a 14-cavity staggered teeth on stator labyrinth gas seal are presented. The stiffness coefficients are measured with the static identification method. First results showing the overall tendencies are presented by Kwanka et al. [1993]. The flow-induced reaction forces yielded decentering restoring forces exceeding the magnitude of the cross forces for all parameters. The influence of the entry swirl on the cross force was considerably higher than that of the rotational speed. It was the change of the circumferential velocity which was decisive for the magnitude of the cross force, not the absolute value. The measured force-deflection relationships were reasonably linear out to an eccentricity ratio of 0.8. Therefore, the linear model used in Eq. (1) is justified.

Despite the numerous efforts in the past some discrepancies between experimentally measured and theoretically predicted labyrinth forces still cannot be explained. It is necessary to examine the cavity to cavity distribution of the forces in order to obtain a better insight into the prevailing physical phenomena. The static identification method used guarantees a high accuracy of the measured stiffness coefficients, which in most cases are of outstanding importance for the dynamic behavior of the rotor. The comparison with calculated results obtained by an one-volume bulk-flow model would then show the suitability of the theoretical approach.

**EXPERIMENTAL INVESTIGATIONS**

The test rig was designed and built to measure the radial forces which are induced by the flow through labyrinth seals on the eccentric rotor. An overall layout of the test rig is shown in fig. 1a. Two identically grooved labyrinth seals with fourteen cavities each are located symmetrically to the inflow region. Cooled, dried and metered air from a compressor enters the test rig through an inflow device (fig. 1b). By combination of radial and circumferential flow components the inlet swirl can be varied.

One of the seals is used to measure the pressure distribution in circumferential direction and, as a consequence, the acting force in each cavity as well as on the rotor as a whole. Ten pressure sensing holes are located on the outer circumference of each cavity. The circumferential pressure distributions were integrated to obtain forces which were split into components parallel and perpendicular to the displacement. The 160 pressure lines are connected to a multifunction pressure measurement rig. The inlet swirl and the flow field in the cavities
can be measured through outlets in the casing of the opposite seal by using a probe.

In fig. 2 the geometrical data of the staggered labyrinth are indicated. Some additional test rig dimensions and experimental conditions are listed in table 1. The static eccentricity can be adjusted in horizontal direction by moving the casing relative to the rotor forwards as well as backwards. When the test rig is supplied only via the tangential feeders, the maximum preswirl velocity direct upstream of the seal is about 22 m/s. This velocity is nearly constant for all pressure ratios investigated in this paper due to the simultaneous increase of the mass flow and the density. Choked flow prevails in the gap between the exit seal tip and the rotor when the highest pressure difference of $\Delta p = 460$ kPa is adjusted. Further details about the test rig and the experimental procedure are given by Steckel [1993]. In a staggered labyrinth the geometrical shape of the cavities changes alternately. The change from the grooved to the toothed rotor section has a greater impact on the axial flow than the change from the toothed to the grooved section. This was confirmed by numerical investigations which show that the axial flow and the strength of the cavity whirl induced by this flow depends on the geometry (Baumgartner [1987], Nordmann and Weiser [1988]. As a consequence, the pressure is declining in big and small steps alternately (fig. 3). Together with the geometry-dependent flow area this causes alternately high and low flow coefficients (Kwanka et al. [1993]).

In the center position of the rotor the circumferential pressure distribution is nearly constant and therefore no forces act on the rotor as a whole. Nevertheless, the small forces for each cavity in normal and cross direction in center position can be used to correct measurements performed with the eccentric rotor (fig. 4). These forces are mostly insensitive to the rotational speed and the entry swirl whereas the pressure ratio has a great impact. Even if the forces in centered position are small, they still represent the main cause for divergencies and are due to random variations in the actual fin clearance. The use of the forces in centered position for correctional purposes is not necessary when the data points at positive and negative eccentricity are averaged. This procedure is applied to all following measurements with eccentric rotor.

Although forces induced in gas seals are at least one order of magnitude lower than in liquid seals, the

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**TABLE 1**

<table>
<thead>
<tr>
<th>Test rig dimensions and experimental conditions</th>
<th>D [mm]</th>
<th>L [mm]</th>
<th>d [mm]</th>
<th>e [mm]</th>
<th>$\Delta p$ [kPa]</th>
<th>$n$ [rpm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (stator)</td>
<td>240</td>
<td>150</td>
<td>0.5</td>
<td>max. 0.4</td>
<td>460</td>
<td>*8000</td>
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<tr>
<td>Length (seal)</td>
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<td>Clearance ($e=0$)</td>
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<tr>
<td>Eccentricity</td>
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<td>Pressure difference</td>
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<tr>
<td>Rotating speed</td>
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**FIGURE 3** Pressure drop in the labyrinth seal.

**FIGURE 4** Cross forces in centered position of the rotor ($e=0.0$ mm).
measurements display a good reproducibility. In addition to the averaged values four particular measurements are shown in fig. 4. The mean uncertainty of the averaged force per cavity is about 0.33 N.

The restoring force which was measured in eccentric position shows a behavior, similar to the correction forces (fig. 5). Again the preswirl and the rotation have only little influence on the course and the magnitude of the forces. The pressure difference on the other hand has a considerable influence and finally indicates that the restoring force is generated by the axial flow (fig. 6). In the cavities where the grooved section is followed by a tooth the restoring forces are considerably higher than in the other cavities.

The restoring force arises due to pressure differences which occur on the perimeter and which basically should influence also the circumferential flow. The same alternating behavior of the restoring force was found by Thieleke and Stetter [1990] in that case; however, the total force was about zero.

In a next step the cross force will be examined. When small pressure differences are adjusted, the density of the air at the entrance section of the seal is small, too. Therefore, for a given entry swirl, which is a nearly constant value for all pressure differences \( \omega_s \approx 22 \text{ m/s} \), the momentum in circumferential direction is low for small densities. In addition to the swirl, the axial flow seems to have an influence on the generation of the cross force (fig. 7a). The peaks of the cross force correspond to the peaks of the restoring force in the front part of the seal. This phenomenon shows a good reproducibility for all rotating speeds.

Nevertheless, the cross force declines smoothly in the first part of the seal when the circumferential flow in the cavity is dominated by swirl due to a high pressure level (fig. 7b). In the rear part of the seal the influence of swirl decreases and again the cross force seems to be controlled by the axial flow.

In case of no shaft rotation, mainly the part of the cross force resulting from the entry swirl appears (fig. 8). The above-mentioned change in the behavior of the cross force comes about gradually and depends clearly on the pressure level.
THEORETICAL TREATMENT AND COMPARISON TO EXPERIMENT

With the help of a bulk-flow theory the circumferential pressure distribution due to the eccentric position of the rotor relative to the casing is obtained. Details of the calculation method can be seen in Baumgartner [1987].

The principal equations are the same as used by other authors (Childs and Scharrer [1986], Thieleke and Stetter [1990]):

\[ \frac{\partial}{\partial t}(p_i \nu_i) + \frac{\partial}{\partial x_i}(\rho c_{ui} \nu_i) + m_{i+1} - m_i = 0 \]
\[ \frac{\partial c_{ui}}{\partial t} + c_{ui} \frac{\partial c_{ui}}{\partial x_i} + m_i - c_{ui-1} \]
\[ + k_s \nu_i^2 - k_R(R_i \omega - c_{ui})^2 = -\frac{\partial p_i}{\rho R_i \omega \varphi} \]
\[ K_{Si} = \lambda_{Si} \frac{U_S}{2l_i} \text{sgn}(c_{ui}) \]
\[ K_{Ri} = \lambda_{Ri} \frac{U_S}{2l_i} \text{sgn}(R_i \omega - c_{ui}) \]

This is the well-known set of partial differential equations. In order to find an analytical solution, the differential equation system is linearized by dividing the main variables into an average value (for the centric position) and an additional value (for the eccentric position). This perturbation method is applied for eight terms.

For the calculation of the mass-flow coefficient \( \mu \) the method of Neumann is used in the program. The calculated mass-flow coefficient is adapted to the geometry investigated here, but not changed from cavity to cavity as found out by the experiments. Eqs. (6) are inserted in eqs. (2)-(5), and an exponential formulation for the perturbation term leads to the cross force acting on the rotor.

\[ F = -R_{p0} \rho \int_0^{2\pi} \xi \sin \varphi d\varphi \]
Δp=460 kPa; e=0.4 mm; 6000 rpm; max. swirl

<table>
<thead>
<tr>
<th></th>
<th>Steckel (exp.)</th>
<th>Ortinger (calc.)</th>
<th>Serkov (calc.)</th>
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<tbody>
<tr>
<td>Cross force [N]</td>
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<tr>
<td>0</td>
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<td></td>
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FIGURE 9  Comparison of calculation and experiment in case of rotating shaft.

only the circumferential velocity is involved in calculating wall shear stress which is assumed to be constant at the stationary and rotating boundaries of the cavity respectively. As shown in this paper, the pressure difference has an impact on the radial forces acting on the rotor. The comparison of calculated and measured total cross forces shows that the tendency depending on the pressure difference is predicted well (fig. 10). (The total cross force divided by eccentricity actually corresponds to the cross coupled stiffness of the seal.) Further improvements in the absolute value will probably require a more accurate modelling of the flow among others.

CONCLUSIONS
A fourteen cavity staggered labyrinth gas seal is investigated in order to obtain a better insight into the influence of preswirl, rotation of shaft and axial flow on the radial forces which act on the rotor. The cavity by cavity distribution of the forces is obtained from the circumferential pressure distribution when the rotor is moved out of his central position.

The pressure declines in big and small steps due to alternating flow coefficients. The normal force which acts in decenterizing direction changes depending on the geometrical shape of the cavity. The normal force is mainly influenced by the pressure difference adjusted on the labyrinth seal and is essentially generated by the axial flow. The normal forces indicate that a circumferential pressure differences exist which under certain circumstances can even influence the cross forces. In this case, the course of the cross force shows the same alternating behavior as the normal force.

The prediction of the cross force distribution by calculation is good when the rotor does not rotate. The discrepancies in the course of the force between calculations with different friction factors are considerable when the shaft rotates. The one-volume bulk flow theory does not predict the part of normal force induced by the axial flow and therefore cannot consider that influence on the cross force.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>C, E, K</td>
<td>Damping, inertia, stiffness coefficients</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>H</td>
<td>Labyrinth height</td>
</tr>
<tr>
<td>U</td>
<td>Part of cavity perimeter</td>
</tr>
<tr>
<td>(c_{\omega}), (c_{ax})</td>
<td>Circumferential velocity, axial velocity</td>
</tr>
<tr>
<td>(e)</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>(f, l)</td>
<td>Cross-sectional area, length of cavity</td>
</tr>
</tbody>
</table>
ROTODYNAMIC COEFFICIENTS

\( \dot{m} \) Mass flow related to circumference
\( n \) Rotating speed
\( p \) Pressure
\( R \) Radius of rotor
\( \delta \) Radial clearance
\( \varphi \) Peripheral angle
\( \kappa \) Isentropic coefficient
\( \lambda \) Friction factor
\( \mu \) Flow coefficient, dynamic viscosity
\( \psi, \xi, \eta, \varepsilon \) Perturbation terms
\( \omega \) Angular velocity
\( \rho \) Density

Indices
* Central position

References


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