A Dynamic Analysis of a Flexible Rotor in Ball Bearings with Nonlinear Stiffness Characteristics

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This paper presents an effective analysis approach for a flexible rotor in ball bearings with nonlinear stiffness characteristics to obtain realistic dynamic behavior results. The ball bearing is modeled in five degrees of freedom and the nonlinear stiffness characteristics of the bearing are completely described as functions of combined loads and spin speed. For dynamic behavior analysis of the nonlinear rotor-bearing system, a transfer-matrix method is iteratively used until the bearing displacements and the shaft displacements at every bearing location converge to the same values. The results show that the nonlinear stiffness characteristics of ball bearings significantly influence system dynamic behaviors and the proposed analysis approach for the nonlinear rotor-bearing system is effective.

Keywords: Rotor-bearing system, ball bearing, nonlinear bearing stiffness, critical speed, unbalance response, whirl orbit

1 INTRODUCTION

High-Speed rotating machinery with ball bearings, such as aircraft engines and small capacity gas turbines, has been widely used because of easy maintenance, little rotordynamic instability, and low cost. Many authors have shown that a large number of parameters, which include system geometry, disk properties, stiffness distribution of a rotating shaft, and bearing stiffnesses can influence the dynamic behavior of a rotor-bearing system. Amongst them, the ball bearing stiffnesses in particular possess nonlinear

characteristics which vary with applied loads and spin speed (Harris [1991]). Recently, many researchers have studied on the analysis of a rotor-bearing system for design and diagnosis. However, their analysis results are difficult to apply because they usually considered the bearing stiffness in radial direction only and disregarded bearing stiffness variation with spin speed and applied loads.

The objective of this study is to propose an effective way of handling the nonlinear stiffness characteristics of ball bearings in the context of the system dynamic behavior analysis to obtain realistic results.

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Two major components necessary to achieve the objective are a ball bearing stiffnesses analyzer and a system dynamic behavior analyzer.

For the analysis of ball bearing stiffnesses, while [1979] calculated radial stiffness by using finite approximation, Gargiulo [1980] presented bearing stiffness formula in radial and axial directions, and Lim and Singh [1990] developed a numerical scheme to compute complete bearing stiffnesses. However, all of them neglected high speed effects such as centrifugal forces and gyroscopic moments of ball elements. Jones [1960] and De Mul *et al.* [1989] presented general analysis theories for the ball bearing under arbitrary load and speed conditions. In their analyses, however, the inner- and outer-raceway load-deflection factors, centrifugal force, and gyroscopic moment of every ball were assumed to have constant values for all the ball locations. These values, however, actually depend on contact angle values of each ball. This may often cause some errors in ball bearing analysis results under applied load and high speed conditions since contact angle values become quite different from ball location to ball location. To correct this defect, we devise an iterative ball bearing analysis scheme based on Jones' general theory, and then apply it in this study.

For the dynamic behavior analysis of a nonlinear rotor-bearing system, a transfer-matrix method (Murphy and Vance [1983]; Rao [1983]; Vance [1988]) and the proposed bearing analysis scheme are directly integrated and iteratively used until the bearing displacements and the shaft displacements converge to the same values within a specified tolerance at every bearing location.

The next section presents an analysis scheme for obtaining complete nonlinear ball bearing stiffnesses. The numerical procedure for the dynamic behavior analysis of a nonlinear rotor-bearing system is described in section 3. In section 4, numerical results are presented to illustrate the nonlinear stiffness characteristics of a high speed ball bearing, the effect of the nonlinear characteristics on the dynamic behavior of a rotor-bearing system, and the usefulness of the proposed analysis approach for a flexible rotor in ball bearings. Finally, concluding remarks are mentioned.

2 BALL BEARING ANALYSIS

The relationship between the bearing loads $F = \{F_x, F_y, F_z, M_y, M_z\}^T$ and the bearing displacements $\delta^b = \{\delta_x^b, \delta_y^b, \delta_z^b, \theta_y^b, \theta_z^b\}^T$, as shown in Figure 1, has to be determined for ball bearing analysis. In this study, an iterative bearing analysis algorithm based on Jones' theory [1960] is devised and used to calculate a complete stiffness matrix, and a brief flow chart of the algorithm is shown in Figure 2.

As shown in Figure 2, the input data such as bearing geometry, material, applied loads, and spin speed are specified, and the values of bearing displacements and inner- and outer-raceway contact angles at every ball location (α_{ij}^l and α_{oj}^l) are initially assumed at first. In this study, we use the free contact angle value for the initial contact angles. Next, we calculate the inner- and outer-raceway load-deflection factors at each ball location (K_{ij} and K_{oj}) as well as the centrifugal force and gyroscopic moment of each ball element (CF_j , GM_j), and their values are fixed as constant during the process of the inner loop. In the inner loop, the contact angles and deformations at each ball location are computed by solving a system of nonlinear algebraic equations (which consists of two compatibility equations and two ball equilibrium equations) with the values of bearing displacements assumed. Having obtained these values, the bearing displacements (δ^b) can be computed by solving a sys-

FIGURE 1 Loads and displacements of a ball bearing.

FIGURE 2 Flow chart for ball bearing analysis.

tem of five bearing equilibrium equations. This process of the inner loop is repeated until the displacements are converged. Now, the differences between the current contact angle values obtained at the end of the inner loop (α_{ij} and α_{oj}) and the previous ones (α_{ij}^l and α_{oj}^l) are tested against a specified tolerance (ϵ) for every ball location. If they are not within the tolerance, the contact angle values are set to the current ones, and the outer loop process is repeated until con-

FIGURE 3 Schematic diagram of a rotor-bearing system.

vergence. With the converged results, a complete stiffness matrix is finally calculated by differentiating bearing equilibrium equations.

3 NONLINEAR ROTOR-BEARING SYSTEM ANALYSIS

A rotor system supported by ball bearings can be modeled as an assemblage of disks, flexible shaft elements, and bearings as shown in Figure 3. The dynamic behavior of the system is nonlinear since bearing stiffnesses vary with spin speed, applied preload, and transmitted loads due to unbalance. For the analysis of the nonlinear rotor-bearing system, an overall procedure is shown in Figure 4.

As shown in Figure 4, bearing and rotor data are specified and a spin speed is given at first. Bearing loads ($\underline{F}_n^b$), which include an applied preload and loads transmitted from the rotor, and bearing displacements ($\underline{\delta}_n^b$) are also initially assumed at every bearing location. Having the assumed bearing loads

FIGURE 4 Overall procedure for the dynamic analysis of a nonlinear rotor-bearing system.

and displacements, bearing stiffnesses ($\mathbf{k}_n$) and displacements (δ_n^b) of every bearing are calculated in the bearing analysis module. The calculated bearing stiffnesses ($\mathbf{k}_n$) are fed into the unbalance response analysis module to compute the shaft displacements (δ_n^s) and internal shaft forces ($\underline{F}_n^s$) at every bearing location. Then, the differences between the bearing displacements (δ_n^b) and shaft displacements (δ_n^s) are compared against a given tolerance for every bearing. If they are not within the tolerance, the bearing loads and displacements ($\underline{F}_n^b, \delta_n^b$) are updated to have the values of the shaft internal forces and displacements ($\underline{F}_n^s, \delta_n^s$), and the above process is repeated until convergence. Finally, using the bearing stiffnesses ($\mathbf{k}_n, n = 1, \dots, m$) converged, the eigenvalues of the system at the given spin speed are computed in the eigenvalue analysis module. In this study, the scheme described in the previous section is used for ball bearing analysis and a transfer matrix method (Murphy and Vance [1983]; Rao [1983]; Vance [1988]) for the eigenvalue and unbalance response analyses.

For the dynamic behavior analysis of a nonlinear rotor-bearing system in a range of spin speeds, the above procedure can be repeatedly used to sweep the range.

4 NUMERICAL RESULTS

To evaluate the usefulness of the suggested dynamic analysis approach for a nonlinear rotor-bearing sys-

TABLE I Ball bearing specifications

Number of ball elements	13
Pitch diameter (mm)	46.0
Ball diameter (mm)	9.526
Young's modulus (MPa)	206000
Initial contact angle (deg.)	40.0
Outer groove curvature radius (mm)	4.950
Inner groove curvature radius (mm)	4.950

tem, a computer program is developed to implement the numerical procedures described in sections 2 and 3. In the following, the numerical results are presented to illustrate nonlinear stiffness characteristics of a ball bearing and the effect of the nonlinear characteristics on the dynamic behavior of a rotor-bearing system.

4.1 Ball Bearing Stiffness Analysis

The specifications of the angular contact ball bearing used in this study are listed in Table I. To investigate the effects of spin speed and loads on bearing stiffnesses, we simulated the variation of bearing stiffnesses within the speed range of 0–40000 rpm under the loads of $F_x = 1500$ N, $F_z = 1000$ N, $M_y = 10$ N·m.

Simulation results are shown in Figure 5. The variations of radial, coupling, and rotational stiffnesses are plotted as a function of spin speed. Further, the stiffness results analyzed by the proposed scheme are compared with those analyzed by Jones' scheme. As

FIGURE 5 Nonlinear stiffness characteristics of the angular contact ball bearing (—: proposed scheme,: Jones' scheme).

FIGURE 6 A multi-stepped rotor-bearing system.

can be seen in Figure 5, the effect of spin speed on bearing stiffnesses is significant and the stiffness characteristic is anisotropic due to the force in the z direction and the moment in the y direction. The discrepancy between the results analyzed by the two schemes increases as spin speed increases, because the proposed iterative scheme well considers the effects of centrifugal force and gyroscopic moment of each ball element on bearing stiffness characteristics.

4.2 Dynamic Behavior Analysis of a Nonlinear Rotor-Bearing System

In this study, the ball bearing analysis module, the system unbalance response analysis module, and the system eigenvalue analysis module are directly integrated and applied to the dynamic behavior analysis of a multi-stepped rotor which is supported by two angular contact ball bearings as shown in Figure 6. The shaft with varying cross-sectional area is divided

into 18 elements and the bearings of Table I under the preload of 1500 N in axial direction are located at stations 11 and 15. One disk with a fixed mass is located at station 5. The details of rotor configuration data are listed in Table II.

To investigate the effect of nonlinear bearing stiffness characteristics on system critical speeds, the unbalance response with constant bearing stiffnesses and that with nonlinear stiffnesses are obtained and the results are shown in Figure 7 and Figure 8, respectively. The constant bearing stiffness values used here are the ones calculated at the spin speed of 22000 rpm, and the unbalance at the disk is assumed as $1.0 \times 10^{-10} \text{ kg}\cdot\text{m}$ in both cases. Comparison of Figure 7 and Figure 8 shows that the discrepancies of the first and the second critical speeds of two results are 14% and 10%, respectively. Figure 9 shows that the whirl orbit results with nonlinear bearing stiffnesses at 19000 rpm, 22000 rpm, and 24400 rpm are circular. This implies that the isotropic characteristics of the bearings are hardly affected since the forces and moments transmitted by the unbalance are not noticeable compared with the applied preload.

To examine the effect of unbalance magnitude on the dynamic behavior of the nonlinear rotor-bearing system, we analyze system eigenvalues, unbalance response, and whirl orbits with the unbalance of $1.0 \times 10^{-6} \text{ kg}\cdot\text{m}$, and the simulation results are shown in Figure 10, Figure 11, and Figure 12, respectively. The

TABLE II Multi-stepped rotor configuration data

Element node no.	Node location (m)	Bearing & disk	Outer radius (m)	Inner radius (m)	Element node no.	Node location (m)	Bearing & disk	Outer radius (m)	Inner radius (m)
1	-0.1790		0.0051		11	-0.0139	Bearing	0.0127	
2	-0.1663		0.0102		12	0.0115		0.0152	0.0052
3	-0.1282		0.0076		13	0.0496		0.0152	0.0052
4	-0.1028		0.0203		14	0.0877		0.0127	
5	-0.0901	Disk	0.0203	0.0152	15	0.1080	Bearing	0.0127	
6	-0.0774		0.0330		16	0.1258		0.0381	
7	-0.0723		0.0330	0.0152	17	0.1360		0.0203	0.0152
8	-0.0647		0.0254	0.0178	18	0.1664		0.0203	0.0152
9	-0.0520		0.0254		19	0.1791			
10	-0.0444		0.0127						
Disk:		Location (m)	Mass (kg)	Polar inertia (kg·m)	Diametral inertia (kg·m)				
		-0.0901	1.401	0.0020	0.00136

FIGURE 7 Unbalance response results with constant bearing stiffnesses (Unbalance = 1.0×10^{-10} kg-m: — Response at the left bearing, Response at the right bearing).

FIGURE 8 Unbalance response results with nonlinear bearing stiffnesses (Unbalance = 1.0×10^{-10} kg-m: — Response at the left bearing, Response at the right bearing).

comparison of Figure 8 and Figure 11 reveals that the effect of unbalance magnitude on the first and the second forward critical speeds is not noticeable. With relatively large unbalance, however, backward critical speeds appear as shown in Figure 10 and Figure 11. The comparison of Figure 9 and Figure 12 shows that the whirl orbits near the critical speeds become quite elliptical as unbalance magnitude increases.

FIGURE 10 Whirl speed map of a multi-stepped rotor-bearing system.

FIGURE 11 Unbalance response results with nonlinear bearing stiffnesses (Unbalance = 1.0×10^{-6} kg-m: — Response at the left bearing, Response at the right bearing).

This reflects that the bearing stiffness characteristics become anisotropic due to the loads transmitted from the rotating shaft.

FIGURE 9 Whirl orbits of nonlinear rotor-bearing system (Unbalance = 1.0×10^{-10} kg-m: — Response at the left bearing, Response at the right bearing).

FIGURE 12 Whirl orbits of the nonlinear rotor-bearing system (Unbalance = 1.0×10^{-6} kg-m: — Response at the left bearing, Response at the right bearing).

5 CONCLUDING REMARKS

This paper proposes a new approach of directly integrating a ball bearing analyzer and a dynamic behavior analyzer for the dynamic analysis of a nonlinear rotor-bearing system. At first, a numerical scheme for calculating the speed- and load-dependent stiffnesses of a ball bearing is developed in which the dependency of load-deflection factors, centrifugal force, and gyroscopic moment on contact angle values is well regarded. Then, an iterative procedure for analyzing the dynamic behavior of a nonlinear rotor-bearing system in a truly integrated fashion is developed and applied to the analysis of a multi-stepped rotor supported by two angular contact ball bearings.

Numerical results demonstrate the importance of including the nonlinear bearing stiffness characteristics for dynamic behavior analysis by showing the discrepancy between the analysis results with nonlinear bearing stiffnesses and those with constant bearing stiffnesses. Comparison of the results also illustrates that the effect of unbalance magnitude on system dynamic behavior can be remarkable.

Even though the suggested analysis approach is applied only to the rotor system supported by ball bearings in this study, we believe that the same approach can be applied to the rotor system supported by sliding bearings.

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NOMENCLATURE

CF	: centrifugal force [N]
$\underline{F}$	: load vector
GM	: gyroscopic moment [N-m]
K	: load-deflection factor [$\text{N/m}^{3/2}$]
$\mathbf{k}$	: stiffness matrix
k_{pq}	: stiffness in p direction due to the load in q direction
m	: number of ball bearings
z	: number of ball elements
Ω	: spin speed [rpm]
$\underline{\delta}$	: displacement vector
α	: contact angle [degree]
ϵ	: specified tolerance for convergence

Superscript

b	: bearing
I	: initial guess
s	: shaft

Subscript

i	: inner-raceway
j	: ball azimuth location
n	: ball bearing location
o	: outer-raceway

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