Full Annular Rub in Mechanical Seals, Part II: Analytical Study

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This paper outlines the analytical study of the rotor lateral response under conditions of contact between the rotor and mechanical seal. The seal is modeled as an added stiffness and dry friction force, applied to the rotor during contact. The rotor is assumed sliding in the mechanical seal circular clearance due to unbalance force. The resulting equations allow for rotor synchronous circular response with stable and unstable branches of the amplitude-frequency characteristics. It is typical for jump phenomena. Nonlinear equations of motion are investigated for the stability of this particular solution. Root locus plots are utilized. Instability zones are determined to lead to the self-excited limit cycle reverse precession known as “dry whip”. Destabilizing factors are rotor-seal dry friction coefficient and seal stiffness. A stability study is also performed for the self-excited limit cycle reverse precession. The conditions for the rotor-mechanical seal parameters are derived to avoid “dry whip” under any amount of unbalance.

Keywords: Full annular rubs; Synchronous response; Stability; Self-excited vibration; Reverse precession

1. INTRODUCTION

Rotor-to-stator rub, an unwelcome contact between the rotating rotor and nonrotating component of a machine, such as a mechanical seal, is one of the most damaging malfunctions of rotating machinery. Part I of this paper presented the detailed experimental results of full annular rub. This part (Part II) of the paper interprets these results with a simplified mathematical model.

Full annular rub has been studied by many researchers and engineers. Black (1968) examined synchronous response of rotor-to-stator rub. Ehrich (1969) developed a formula that related ratio of rotor reverse whirl frequency to natural frequency and ratios of stator-to-rotor natural frequency and damping. Muszynska (1984) analyzed full annular rub with a 2-DOF model and showed measured results of dry-whip triggered by a hammer impact. Lingener (1990) and Crandall (1990) demonstrated dry-friction whirl and whip for a case when ratio of rotor radius to radial clearance reached 2.66. Childs (1993) discussed their work and indicated that dry-friction whirl and whip were only likely to occur at a large clearance and that triggering of contact normally required an outside disturbance.

Recent experiments, however, show that dry whip can be ensured to occur in a rotor kit with an attached mechanical seal, as indicated in experimental and analytical studies conducted by Yu et al. (1998, 2000), and Goldman and Bently (1998). No outside disturbance or impact is needed to trigger dry-whip. Moreover, it has been found that dry-whip can even occur for small clearance cases where ratio of rotor radius to radial clearance reaches 40.

The transition from synchronous response to dry whip due to mass unbalance was not noticed until the recent experiments. This paper reveals for synchronous response a dry-friction-related unstable regime in addition to a jump-related unstable regime, thus showing the effect of mass unbalance on triggering of dry whip. A criterion to avoid dry whip under any amount of mass unbalance is also developed based on stability analysis of dry whip.

2. MATHEMATICAL MODEL

A mathematical model of the rotor system shown in Figure 1 reflects an addition of rub-related normal force and dry friction at the rotor-to-stator contact. It assumes that the normal force generated between the rotor and

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FIGURE 1 Dynamic scheme of the rotor with full annular rub.

stator can be approximated by a spring-type relation:

$$\begin{aligned} M\ddot{r} + D\dot{r} + Kr &= Mh\Omega^2 e^{j\varphi}, \quad |r| \leq c \\ M\ddot{r} + D\dot{r} + Kr &= Mh\Omega^2 e^{j\varphi} - \\ &\quad - K_s r \left(1 - \frac{c}{|r|}\right) (1 + jf), \quad |r| > c \end{aligned} \quad [1]$$

where $j = \sqrt{-1}$, $r = x + jy$, M , D , K are modal mass, damping, and stiffness of the rotor system without rub, h is a center of mass eccentricity, $\varphi = \Omega t$, K_s is a stiffness added to the system due to the contact with the stator, c is a radial clearance, f is a friction coefficient between the rotor and the stator. The system of Eq. [1] can be presented as a single equation:

$$\begin{aligned} M\ddot{r} + D\dot{r} + Kr &= Mh\Omega^2 e^{j\varphi} - \frac{K_s r}{2} (1 + jf) \\ &\quad \left[\left(1 - \frac{c}{|r|}\right) + \left|1 - \frac{c}{|r|}\right| \right] \end{aligned} \quad [2]$$

Equation [2] can be rewritten in nondimensional form:

$$\begin{aligned} w'' + 2\zeta w' + w &= an^2 e^{j\varphi} - \\ &\quad - \frac{pw}{2} (1 + jf) \left[\left(1 - \frac{1}{|w|}\right) + \left|1 - \frac{1}{|w|}\right| \right] \end{aligned} \quad [3]$$

where

$$\begin{aligned} w &= r/c, \quad \nu = \sqrt{\frac{K}{M}}, \quad \tau = \nu t, \\ w' &= \frac{dw}{d\tau}, \quad \zeta = \frac{D}{2\sqrt{KM}}, \\ a &= h/c, \quad n = \Omega/\nu, \quad p = K_s/K \end{aligned}$$

Basically, the system response is determined by parameters a , ζ , p , and f .

3. SYNCHRONOUS RESPONSE

Equation [3] allows for the synchronous solution in the case without contact.

$$\begin{aligned} w &= \frac{an^2}{1 - n^2 + 2j\zeta n} e^{jnt}, \\ \text{if } |w| &= \frac{an^2}{\sqrt{(1 - n^2)^2 + 4\zeta^2 n^2}} \leq 1 \end{aligned} \quad [4]$$

Obviously, if $a < 2\zeta\sqrt{1 - \zeta^2}$, contact will never occur. If $a > 1$, contact will be reached at a rotative speed below the natural frequency and will never cease.

In the case of contact between the rotor and the stator, synchronous response in the form

$$w = Ae^{j(n\tau + \alpha)} \quad [5]$$

has to satisfy the following complex equation,

$$\begin{aligned} \left(1 + \frac{p}{\cos \delta} e^{j\delta} - n^2 + 2j\zeta n\right) Ae^{j\alpha} &= \\ &= an^2 + \frac{p}{\cos \delta} e^{j(\alpha + \delta)} \end{aligned} \quad [6]$$

where $f = \tan \delta$. It is equivalent to two equations in real numbers.

$$\begin{aligned} AS \cos \beta - p &= an^2 \cos \alpha \\ AS \sin \beta + p \tan \delta &= an^2 \sin \alpha \end{aligned} \quad [7]$$

where

$$\begin{aligned} \tan \beta &= -\frac{2\zeta n + p \tan \delta}{1 - n^2 + p} \\ S &= \sqrt{(1 - n^2)^2 + 4\zeta^2 n^2 + p^2 / \cos^2 \delta + 2p[(1 - n^2) + 2\zeta n \tan \delta]} \end{aligned} \quad [8]$$

The system of Eqs. [7] allows for the following two solutions.

$$\begin{aligned} A &= \frac{1}{S} \left[p \frac{\cos(\delta + \beta)}{\cos \delta} \pm \sqrt{a^2 n^4 - p^2 \frac{\sin^2(\delta + \beta)}{\cos^2 \delta}} \right] \\ \tan \alpha &= \frac{AS \sin \beta + p \tan \delta}{AS \cos \beta - p} \end{aligned} \quad [9]$$

The second solution (with the sign “-”) is unstable (as can be shown based on the characteristic equation in the next paragraph of this paper).

An example of the rotor response calculated by Eq. [4] for no contact situation, and by Eq. [9] for rubbing, is

FIGURE 2 Root locus of the rotor synchronous response stability under full annular rub conditions. $a = e/c = 0.1, = 0.025$, $K_s/K = 0.5$, $f = 0.15$. Note that with rotative speed increase the synchronous response experiences transition from stable to unstable then to stable again.

shown in Figure 2. This figure contains also the stability analysis results discussed in the following section.

4. STABILITY OF SYNCHRONOUS SOLUTION

The subject of this section is a stability analysis of the rotor synchronous response during the full annular rub. In order to investigate it, the solution $w = Ae^{j(n\tau + \alpha)}$ has to be perturbed, and the resulting equation is then linearized. Introducing small perturbation $w_1(\tau)$ as $w = Ae^{j(n\tau + \alpha)} + w_1$, and assuming that the rotor/stator contact is maintained all the time, the linearized equation for a small perturbation is derived,

$$w_1'' + 2\zeta w_1' + \left[1 + p_1 \left(1 - \frac{1}{2A}\right) e^{j\delta}\right] w_1 + \frac{p}{2A \cos \delta} \bar{w}_1 e^{j(2\varphi + 2\alpha + \delta)} = 0 \quad [10]$$

where $\bar{w}_1$ is complex conjugate of w_1 .

Equation [10] displays a dependence of its coefficients on the amplitude A , typical for nonlinear stability problems. It also contains a time-variable stiffness component, which indicates a higher stiffness in the direction of the rotor-to-stator contact at each half of rotational period. To avoid solving differential equation [10] with periodic coefficients, the perturbation w_1 can be considered in a rotating frame $w_1 = (\xi + j\eta) e^{j(\varphi + \alpha)} = ue^{j(\varphi + \alpha)}$. Then Eq. [10] can be presented as follows:

$$u'' + 2jnu' - n^2u + 2\zeta(u' + jnu) + \left[1 + \frac{p}{\cos \delta} \left(1 - \frac{1}{2A}\right) e^{j\delta}\right] u + \frac{p}{\cos \delta} \frac{1}{2A} \bar{u} e^{j\delta} = 0 \quad [11]$$

or in the form of real variables ξ, η :

$$\begin{aligned} \xi'' + 2\zeta\xi' - 2n\eta' + \xi(1 + p - n^2) + \\ + \eta[p(1 - 2\Phi)f - 2n\zeta] &= 0 \\ \eta'' + 2\zeta\eta' + 2n\xi' + \eta[1 - p(1 - 2\Phi) - n^2] + \\ + \xi(pf + 2n\zeta) &= 0 \end{aligned} \quad [12]$$

where $\Phi = 1 - (1/2A)$.

The solution of Eq. [12] can be found in the form:

$$\xi = C_1 e^{\chi t}, \quad \eta = C_2 e^{\chi t} \quad [13]$$

where χ is an eigenvalue, C_1, C_2 are components of an eigenvector. A substitution of the solution [13] into Eq. [12] allows deriving a characteristic equation:

$$\begin{aligned} (\chi + \zeta)^4 + A_2(\chi + \zeta)^2 + A_1(\chi + \zeta) + A_0 &= 0 \\ A_2 &= 2(1 + n^2 - \zeta^2 + p\Phi); \quad A_1 = 4npf\Phi; \\ A_0 &= (1 - n^2 - \zeta^2)^2 + 2p\Phi(1 - n^2 - \zeta^2) - \\ &\quad - p^2(1 + f^2)(1 - 2\Phi) \end{aligned} \quad [14]$$

where

$$\begin{aligned} A_2 &= 2(1 + n^2 - \zeta^2 + p\Phi); \quad A_1 = 4npf\Phi; \\ A_0 &= (1 - n^2 - \zeta^2)^2 + 2p\Phi(1 - n^2 - \zeta^2) - \\ &\quad - p^2(1 + f^2)(1 - 2\Phi) \end{aligned}$$

Note that coefficients of characteristic equation [14] depend on the amplitude of the synchronous regime, A , which is typical for nonlinear stability problems. In order to create root locus plots for Eq. [14] versus rotative speed ratio n , first an amplitude A has to be calculated using Eq. [9] for a particular value of n . Figure 2 presents root locus plot calculated for the case of damping factor $\zeta = 0.025$, unbalance ratio $a = e/c = 0.2$, stiffness parameter $p = K_s/K = 0.5$, and friction coefficient $f = 0.15$. It shows a situation when synchronous forward precession is stable at the beginning of rub, then becomes unstable and then stable again with increased rotative speed. This situation corresponds to the full annular rub response shown in Figure 3, where the stable branch of the full annular rub synchronous response is marked by a different type of line. In the case of low friction (for example, $f = 0.1$) the synchronous forward precession, after establishing full annular rub, remains stable until the rotor jumps out of the contact against the seal. The unrealized branch of amplitude-frequency characteristic (see Figure 2) is unstable

FIGURE 3 Rotor synchronous stable and unstable responses due to full annular rub. Note that the response starts as stable, then with increase of rotative speed becomes unstable and then stable again (see Figure 4).

FIGURE 4 Synchronous response for different values of unbalance parameter a with and without rub and its stability.

due to the real root turning positive. In the case of high friction (for example, $f=0.3$) the synchronous response is unstable from the start of rubbing. These three different scenarios correspond to the experimentally observed behavior (see Part I of this paper). Figure 4 shows how the stability of the synchronous response is changing with the change of the unbalance parameter a . The boundary of instability region does not change with a , but crossings of the synchronous response into instability zone are affected.

In a particular case of $f=0$, the characteristic equation [14] allows the analytical solution for four roots:

$$\chi_{1,2} = -\zeta \pm \pm j \sqrt{1 + n^2 - \zeta^2 + \sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2}} \quad [15]$$

$$\chi_{3,4} = -\zeta \pm \pm \sqrt{\zeta^2 - 1 - n^2 + \sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2}} \quad [16]$$

Obviously, $\chi_{1,2}$ is a pair of complex-conjugate roots, which are stable in the absence of dry friction. As for the other two roots $\chi_{3,4}$, they vary qualitatively in two frequency regions. In the frequency regions defined by the following inequalities:

$$1 - \zeta^2 + p(2\Phi - 1) > n^2, \text{ or } n^2 > 1 - \zeta^2 + p \quad [17]$$

They are also stable complex-conjugate roots. If

$$1 - \zeta^2 + p(2\Phi - 1) < n^2 < 1 - \zeta^2 + p \quad [18]$$

$\chi_{3,4}$ are real values. In this case, the root with the sign “+” in front of the radical in Eq. [16] can become unstable if

$$1 - \zeta^2 + 2p\Phi - \sqrt{4p\Phi(1 - \zeta^2) + p^2(1 - \Phi)^2} < n^2 < 1 - \zeta^2 + 2p\Phi + \sqrt{4p\Phi(1 - \zeta^2) + p^2(1 - \Phi)^2} \quad [19]$$

Comparison between inequalities [18] and [19] shows that they can be satisfied at the same time if $\zeta^2 < p(1 - \Psi)$. This instability is of a “saddle” type and is not caused by dry friction. As it follows from Figures 2, 6 and 7, this instability is caused by jump phenomenon.

Four analytically found roots are reflected on the root locus plot in Figure 2, with modifications due to friction. In general, two roots are complex conjugate at all rotating speeds. These roots can become unstable due to friction. The other two roots start as complex conjugate and then turn real. These two are not strongly affected by the change in friction coefficient. Analytically the influence of friction on the solutions of the characteristic equation [14] can be evaluated under the assumption of smallness of f .

$$\begin{aligned} \chi_{1,2} = & -\zeta + \frac{fnp\Phi}{p\Phi + \sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2}} \pm \\ & \pm j\sqrt{1 + n^2 - \zeta^2 + p\Phi + \sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2}} \\ \chi_{3,4} = & -\zeta - \frac{fnp\Phi}{\sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2} - p\Phi} \pm \\ & \pm \sqrt{\zeta^2 - 1 - n^2 - p\Phi + \sqrt{4n^2(1 - \zeta^2 + p\Phi) + p^2(1 - \Phi)^2}} \end{aligned} \quad [20]$$

The first two roots can obviously be driven unstable by the friction-related term, while the second pair of roots becomes more stable by friction (negative sign in front of additional term). The instability of the first two roots created by dry friction is of a “focus” type (complex roots), and is shown in Figure 2. It is logical to assume that this instability leads to the reverse precession limit cycle, observed in experiment (Yu et al., 1998).

In order to fully investigate the boundaries of stability of the rotor synchronous response, the eigenvalue χ has to

be assumed purely imaginary ($\chi = js$). In this case, the characteristic equation [14] is equivalent to the system of two equations:

$$\begin{aligned} (1 + n^2 - s^2 + p\Phi)^2 - 4\zeta^2 s^2 + \\ + 4n\zeta pf\Phi - 4n^2(1 - \zeta^2 + p\Phi) = \\ = p^2[\Phi^2 + (1 + f^2)(1 - 2\Phi)] \\ s\left(1 + n^2 - s^2 + p\Phi + \frac{f}{\zeta}np\Phi\right) = 0 \end{aligned} \quad [21]$$

The second of Eqs. [20] allows for zero solution ($s = 0$). The first of Eqs. [21] determines a stable-unstable range of rotative speed to natural frequency ratio n_{st} :

$$\begin{aligned} n_{st}^4 - 2n_{st}^2(1 + p\Phi - 2\zeta^2) + \\ + 4n_{st}fp\zeta\Phi + 1 + 2p\Phi - \\ - p^2(1 + f^2)(1 - 2\Phi) = 0 \end{aligned} \quad [22]$$

The polynomial equation [21] can be easily solved numerically for a given value of the synchronous response amplitude A ($\Phi = 1 - 1/(2A)$, $A > 1$). The resulting curves of $A(n_{st})$ are shown in Figures 3 and 4 as instability due to the real root boundaries.

These boundaries determine the boundaries between stable and unstable branches of the full annular rub-affected synchronous response (jump phenomenon). In order to determine the boundaries between the stable synchronous response and reverse self-excited precession, the case of $s > 0$ has to be considered. In this case, Eqs. [21] can be rewritten as follows:

$$\begin{aligned} (1 + n_{st}^2 + p\Phi - s^2) = \\ = 4(\zeta^2 + n_{st}^2)(1 + p\Phi) + \\ + p^2[\Phi^2 + (1 + f^2)(1 - 2\Phi)] \\ 1 + n_{st}^2 + p\Phi - s^2 = -n_{st}p\Phi\frac{f}{\zeta} \end{aligned} \quad [23]$$

where n_{st} is, as in the previous case, instability threshold to natural frequency ratio.

These two equations can be solved for s and n_{st} :

$$\begin{aligned} n_{st}^2 = \zeta^2 \frac{4\zeta^2(1 + p\Phi) + p^2[\Phi^2 + (1 + f^2)(1 - 2\Phi)]}{p^2f^2\Phi^2 - 4\zeta^2(1 + p\Phi)} \\ s^2 = 1 + p\Phi + n_{st}^2 + n_{st}p\Phi\frac{f}{\zeta} \end{aligned} \quad [24]$$

This solution has a physical meaning only if the right side term of the first of Eqs. [24] is positive. Since $\Phi = 1 - 1/(2A)$, $A \geq 1$, this condition can be satisfied only if

$$f \geq 2\frac{\zeta}{p}\sqrt{1 + p} \quad [25]$$

FIGURE 5 Full annular rub synchronous response instability boundaries for different values of dry friction coefficient f ($=0.025$, $p=0.5$). Instability leads to the limit cycle self-excited reverse response known as “dry whip”. Note destabilizing effect of dry friction coefficient f increases.

Inequality [25] is a necessary condition for the “focus” type instability to take place. The latter is shown in Figures 4 and 5 as unstable complex conjugate roots.

The instability threshold n_{st} described by the first of Eqs. [24] can be calculated for each value of the full annular rub synchronous response amplitude A . The result of such calculations is presented in Figures 3 and 4 as instability, due to the dry friction region boundaries.

There is another way to present such results by using Eqs. [9] to calculate the unbalance parameter a , which creates amplitude A at the rotative speed ratio n_{st} :

$$a = \frac{1}{n_{st}^2} \sqrt{A^2 S^2 + p^2 - 2ASp \frac{\cos(\delta + \beta)}{\cos \delta}} \quad [26]$$

The influence of the system parameter f on the boundary of instability of the “focus” type, calculated according to Eqs. [24] and [26], is shown in Figure 5. It is important to note that damping parameter ς plays a stabilizing role, while friction coefficient f and stiffness p have destabilizing effects. Qualitatively, the increase of parameters f and p , as well as mass eccentricity ratio a , leads to the increase of the friction force in the direction opposite the rotation direction. That should contribute to the destabilization of the synchronous response, which has a forward direction. An inequality [25] describes a necessary condition of the reverse self-excited precession known as a “dry whip”. According to it, a minimum required friction coefficient increases with rotor damping and decreases with stiffening of the seal. This result is in the agreement with stabilizing nature of damping and destabilizing effect of the seal stiffness.

5. SELF-EXCITED RESPONSE DUE TO FULL ANNULAR RUB

According to the experimental evidence (Yu et al., 1998), reverse precession takes place under the rotor/seal full annular rub either right after the rotor contact with the seal, when the rotating speed sufficiently increases, or not at all. Each of these three possibilities corresponds to one of the situations discussed above with the stability of synchronous response. This and the fact that reverse precession occurs with a frequency independent of rotative speed suggest that reverse precession during full annular rub is self-excited in nature. Besides creating the original contact between the rotor and seal, unbalance force is eventually immaterial for the self-excited vibration. The amplitude of the rotor synchronous response is significantly lower than that of self-excited reverse precession. The above considerations allow for a simplified version of Eq. [3]:

$$w'' + 2\varsigma w' + w = an^2 e^{j\varphi} - \frac{pw}{2} (1 + jf) \left[\left(1 - \frac{1}{|w|} \right) + \left| 1 - \frac{1}{|w|} \right| \right] \quad [27]$$

where the unbalance force is neglected, and the contact situation is assumed. Equation [27] allows for the particular solution

$$w = Be^{j(\omega\tau + \beta)} \quad [28]$$

where B , ω , can be determined from Eq. [27] by substituting Eq. [28] into it:

$$[1 - \omega^2 + p(1 + jf) + 2j\omega\varsigma]B = p(1 + jf) \quad [29]$$

and β is an arbitrary constant. Equation [29] has the following two solutions:

$$\begin{aligned}\omega_1 &= -\left(\frac{\varsigma}{f} + \sqrt{1 + (\varsigma^2/f^2)}\right); \\ B_1 &= \frac{p}{\left|p - 2(\varsigma^2/f^2)\left(1 + \sqrt{1 + (f^2/\varsigma^2)}\right)\right|}; \\ \omega_2 &= \sqrt{1 + \frac{\varsigma^2}{f^2}} - \frac{\varsigma}{f}; \\ B_2 &= \frac{p}{\left|p + 2(\varsigma^2/f^2)\left(\sqrt{1 + (f^2/\varsigma^2)} - 1\right)\right|}; \\ \omega_2 &= \sqrt{1 + \frac{\varsigma^2}{f^2}} - \frac{\varsigma}{f}; \\ B_2 &= \frac{p}{\left|p + 2(\varsigma^2/f^2)\left(\sqrt{1 + (f^2/\varsigma^2)} - 1\right)\right|}\end{aligned}\quad [30]$$

Note that the first solution, Eqs. [30], which describes a rotor self-excited reverse precession, has an amplitude more than 1, while the second one, Eqs. [31], which shows a forward precession, has an amplitude less than 1. Obviously, the second solution does not have any physical meaning. It is important to remember that both solutions are determined with an accuracy up to the arbitrary phase shift β . The meaningful solution [30] with reverse precession can be presented in original dimensional variables as follows:

$$r = \frac{K_s c \exp j\left(-\sqrt{K/M}\left(\varsigma/f + \sqrt{1 + (\varsigma^2/f^2)}\right)t + \beta\right)}{\left|K_s - 2K(\varsigma^2/f^2)\left(1 + \sqrt{1 + (f^2/\varsigma^2)}\right)\right|}\quad [32]$$

In order to investigate the stability of this self-excited solution, it has to be perturbed, and linearized equations for perturbations have to be considered. Following the procedure described in the previous section, the linearized perturbation equations in the system of coordinates, rotating with precession frequency ω_1 , can be derived by the substitution of n in Eqs. [11] by ω_1 . A similar statement is true regarding the characteristic equation [13] for the perturbed solution [30].

$$\begin{aligned}(\chi + \varsigma)^4 + A_2(\chi + \varsigma)^2 + A_1(\chi + \varsigma) + A_0 &= 0 \\ A_2 &= 2(1 + n^2 - \varsigma^2 + p\Phi); \quad A_1 = 4npf\Phi; \\ A_0 &= (1 - n^2 - \varsigma^2)^2 + 2p\Phi(1 - n^2 - \varsigma^2) - \\ &\quad - p^2(1 + f^2)(1 - 2\Phi)\end{aligned}\quad [33]$$

Note that in this case $\Phi = 1 - (1/2A)$. After the substitution of expressions [30] into Eq. [33], the characteristic equation

for self-excited reverse precession can be reduced to the following form:

$$\begin{aligned}\chi\{\chi^3 + 4\varsigma\chi^2 + (4\varsigma^2 + 3\omega_1^2 + p + 1)\chi + \\ + 2\varsigma(p + 1 + 3\omega_1^2) + \\ + 2f\omega_1(p - 1 + \omega_1^2)\} = 0\end{aligned}\quad [34]$$

Equation [34] has a zero root, which is the result of the system autonomy and does not affect stability.

The stability boundary is reached when the eigenvalue χ is purely imaginary ($\chi = js$). In this case, Eq. [34] is equivalent to the following system of equations:

$$\begin{aligned}s(s^2 - 4\varsigma^2 - 3\omega_1^2 - p - 1) &= 0 \\ \frac{\varsigma}{f}(2s^2 - 3\omega_1^2 - p - 1) &= \omega_1(p - 1 + \omega_1^2)\end{aligned}\quad [35]$$

Using an expression [30] for ω_1 , it could be shown that the second of Eqs. [35] can have a solution only if $s = 0$. Using the latter and an expression [30] for ω_1 , it can be rewritten as follows:

$$\frac{\varsigma}{f} = \frac{p}{2\sqrt{1+p}}\quad [36]$$

Equation [36] determines the stability boundary of the self-excited reverse precession regime. Obviously, it is stable if

$$\frac{\varsigma}{f} \leq \frac{p}{2\sqrt{1+p}}\quad [37]$$

It exactly coincides with the inequality [25], which determines the necessary conditions for the full annular rub synchronous response to become unstable. Using Eqs. [30], this inequality also allows evaluation of the upper boundary for a reverse precession frequency:

$$|\omega_1| \leq \sqrt{1+p}\quad [38]$$

or with the original dimensional parameters:

$$|\nu_1| \leq \sqrt{\frac{K + K_s}{M}}\quad [39]$$

It means that the frequency of the rotor self-excited reverse precession is always higher than the natural frequency of the rotor without rub, and lower than the natural frequency of a rotor with seal stiffness added to the original stiffness of the rotor.

6. CONCLUSIONS

- The analytical model developed in this report describes the rotor response under full annular rub conditions. The rub occurs between the rotor and mechanical seal due to the unbalance force. Physical rotor/seal contact is described as an added stiffness in radial direction and dry friction force in tangential direction.
- The model allows for synchronous response, which exhibits a typical jump phenomenon Bode plot with two branches of amplitude and phase characteristics. One branch is stable, while the other is unstable. It causes a jump – during startup, the rotor response jumps when the rotor disengages from the seal; during shutdown, a jump occurs when the contact between the rotor and the seal is originated. These two jumps occur at different rotative speeds, which explains the experimentally observed difference between the rotor startup and shutdown responses (Yu et al., 1998).
- Nonlinear equations of motion are investigated for the stability of the rotor synchronous response. The coefficients of the resulting characteristic equation depend on the system parameters (damping factor ζ , stiffness ratio $p = K_s/K$, dry friction coefficient f) and the synchronous response amplitude A . The latter involves the mass eccentricity to clearance ratio (unbalance parameter) a into the stability study.
- Instability zones are determined to lead to the self-excited limit cycle reverse precession known as “dry whip.” Destabilizing factors for the rotor synchronous response are rotor-seal dry friction coefficient and seal stiffness. Damping factor plays a stabilizing role.
- It is important to note that if

$$f < 2 \frac{\zeta}{p} \sqrt{1+p} \quad [40]$$

then a synchronous response is stable for any value of unbalance parameter a . Since a “dry whip” is highly destructive, Eq. [40] can be used as a criterion for choosing a mechanical seal for a particular rotor system,

which would not experience self-excited limit cycle reverse precession during rub.

- Self-excited limit cycle reverse precession (“dry whip”) stability conditions do not depend on the rotative speed. Therefore, once established, “dry whip” exists in a wide rotative speed range (till the lowest mode model of the rotor becomes invalid, or there is no slipping between the rotor and the seal). The stability conditions determine that a “dry whip” frequency is higher than the natural frequency of the rotor without rub and lower than the natural frequency of a rotor with seal stiffness added to the original stiffness of the rotor. Its value is a function of the damping factor to dry friction coefficient ratio, and is proportional to the rotor original natural frequency.

The analytical model describes all dynamic effects observed experimentally during the full annular rub conditions (Yu et al., 1998).

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