International Journal of Rotating Machinery, 8(2): 99–108, 2002

1023-621X/02 $12.00 + .00

Balancing of an Experimental Rotor without Trial Runs

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Several methods attempting rotor balancing without trial runs have been published in the past. There are, however, no reports of systematic application of these procedures to field balancing of large rotating machinery. This suggests that some practical difficulties have still to be solved. An analysis on such difficulties shows that balancing a rotor without trial runs is not possible if the mode shapes are not known. Trial runs are also necessary when the residual vibration at normal operating speed, produced by the influence of higher unbalanced modes, is too high to allow continuous operation of the machine. There are, also, additional difficulties related with the angular position of the vibration transducers, which allow the determination of the magnitude and phase of the correction masses only if their position coincide with the direction of the system principal axes of stiffness. This paper describes a balancing procedure incorporating all these elements and its application to the balancing of an experimental rotor rig.

Keywords: Rotor balancing; Balancing of flexible rotors; Mixed modal balancing; Balancing of rotors without trial runs

1. INTRODUCTION

Balancing a rotor consists in the compensation of an eccentric mass distribution that generates large centrifugal forces and high levels of vibration. This compensation is done attaching correction masses in predefined balancing planes. The objective of the balancing procedure is to determine the magnitude and angular position of the correction masses and their axial distribution along the shaft.

Two different strategies have been adopted giving origin to the two families of balancing methods more widely used until present days: the influence coefficient methods and the modal balancing methods.

The influence coefficient methods are based on a cause–effect philosophy, their only assumption being a direct proportion between the response of the rotor and the unbalance. Several numerical techniques can be applied to minimise a large number of vibration readings.

The basic consideration of the modal balancing method is that the unbalance response of a rotor can be expressed as a series of modal components, each one corresponding to a degree of freedom with a characteristic natural frequency and a mode shape. Similarly, the unbalance forces may be expressed as a series of modal unbalances. Thus, the deflection of the shaft is made up of contributions from the mode shapes of the system. the scale of each mode shape being affected by a suitable scale factor, which is a function of the corresponding modal unbalance. In this method the unbalance is eliminated for each mode in turn, taking care not to upset the vibration modes previously balanced during the process.

One common characteristic of the two methods is the large number of trial runs required to determine the correction masses. In the influence coefficient method the number of trial runs is defined by the number of balancing planes and cannot be reduced. Additional runs may be even necessary each time the rotor is not able to travel through a critical speed.

The modal balancing method requires at least one trial run for each mode to be balanced, although additional runs may be necessary to determine the mode shapes and to reduce the influence of higher modes. However, the number of trial runs can be reduced correcting several modes at the same time, although in the practice this is more easily said than done.

In recent years, attempts have been made to combine the influence coefficient and the modal balancing methods, giving place to a number of unified balancing techniques. These techniques, however, have not changed the mode-by-mode approach used in the modal balancing method.

Received in final form 30 June 2000.
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Several researchers have considered the reduction or complete elimination of trial runs. Some of the proposed procedures are similar to the influence coefficient method, but the trial runs are simulated in a computer using a mathematical model of the rotor. The main difficulty in this case is the determination of the support characteristics, which have a predominant effect on the results of the simulation. Experience has shown that these parameters are difficult to estimate without having to use elaborate experimental procedures, and even then, the numerical simulations are usually unable to replicate the response of the rotor with sufficient accuracy to allow the determination of the required correction masses.

Other procedures used to eliminate the trial runs are closer to the modal method. In this case the calculation of the correction masses requires the previous determination of the following parameters: (a) the modal vibration vectors for each resonance, (b) the modal damping ratios, (c) the mode shapes and (d) the equivalent mass of the rotor for each mode. Different authors have proposed specific procedures for the determination of these parameters, but there are no published reports describing the practical application of such procedures to the balancing of large rotating machines in the field using no trial runs. This suggests that some of the practical difficulties still need to be overcome, which keeps the door open for further research on this area.

An analysis of such difficulties shows that a rotor cannot be balanced if the mode shapes are not known. Trial runs are also necessary when the residual vibration produced by higher modes at normal operating speed is too high to allow continuous operation of the machine. Furthermore, there are additional difficulties related with the vibration transducers, which need to be placed in the same direction as the principal axes of stiffness to allow the determination of the correction masses. The following sections describe a balancing procedure incorporating all these elements together with its application to the balancing of an experimental rotor.

2. BALANCING WITHOUT TRIAL RUNS

The response of a flexible rotor may be expressed as a series of characteristic functions, so that

$$v(z,t) = \sum_{r=1}^{n} q_r(t) \cdot \varphi_r(z) \qquad [1]$$

where n is the number of vibration modes with a significant effect on the rotor response, $q_r(t)$ represents the rth principal coordinate of the system and $\varphi_r(z)$ is the rth characteristic function or mode shape for the free undamped vibrations.

Similarly, the eccentricity distribution $e(z)$ may be expressed as a series of characteristic functions as shown in the following equation.

$$e(z) = \sum_{r=1}^{n} \varepsilon_r \varphi_r(z) \qquad [2]$$

where the elements $\varepsilon_r\varphi_r(z)$ represent the modal components of eccentricity. Each one of these modal components excites a single mode of vibration. Multiplying Eq. [2] by $\rho A(z)\varphi_r(z)$ and integrating along the shaft gives

$$\varepsilon_r = \frac{1}{m_r} \int \rho A(z) e(z) \varphi_r(z) dz \qquad [3]$$

The modal mass, m_r, represents the equivalent mass of a single degree of freedom that would generate a response similar to that observed in rth mode of vibration.

The characteristic function in the previous equations represents the shape adopted by the shaft in its rth mode of vibration. This shape is defined as the ratio between the vibration values along the shaft and a reference value. In practice, this reference value is usually the corresponding vibration measured by one of the transducers.

Thus, the characteristic function at each point along the shaft is a fixed parameter known as *mode shape factor*. If the reference vibration transducer is located at $z = z_i$, the rth mode shape factor corresponding to the axial position $z = z_j$ is given by

$$\varphi_r(z_j) = (\varphi_r)_{ji} = \frac{v_r(z_j, t)}{v_r(z_i, t)} \qquad [4]$$

The relative nature of the characteristic functions shows that the modal component of eccentricity ε_r depends on the deflection used as reference value. The same conclusion applies for any modal parameter expressed as a function of the mode shapes, such as the modal mass, for instance.

The vibrations in Eq. [4] represent modal components. Use of global values would result in errors when calculating the mode shape factors.

The vibration v_{ri} produced by the rth modal component of unbalance ε_{ri} at the position of the reference transducer is

$$v_{ri} = \frac{\Omega_r^2 \varepsilon_{ri}}{\sqrt{(1-\Omega_r^2)^2 + (2\zeta_r\Omega_r)^2}} = A_r(\omega) \cdot \varepsilon_{ri} \qquad [5]$$

where Ω_r is the ratio between the rotation frequency and the rth natural frequency and ζ_r is the rth damping ratio. The amplification factor $A_r(\omega)$ is the same at any point along the shaft.

The rotor may be balanced in its rth vibration mode adding a single mass to cancel the centrifugal force

generated by the rth component of eccentricity. Considering a balancing plane located at $z = z_j$, the correction mass U_{rj} is given by

$$U_{rj} = -\frac{m_{ri} \cdot \varepsilon_{ri}}{R_j \cdot (\varphi_r)_{ji}} = -\left(\frac{m_{ri}}{R_j \cdot A_r(\omega) \cdot (\varphi_r)_{ji}}\right) \cdot \upsilon_{ri} \qquad [6]$$

where m_{ri} is the modal mass observed at the position of the reference transducer and R_j is the radius of the balancing plane.

Thus, the correction mass required to eliminate the rth component of unbalance can only be found without performing any trial runs if the elements between parentheses in the right hand side of Eq. [6] can be determined by some other means. The negative sign in this equation indicates that the correction mass has to be attached to the rotor at 180° from the position of the unbalance force, which also needs to be found.

Once the single correction mass required to compensate the original unbalance has been defined, it is necessary to transform it into an equivalent set of masses, which produces the same effect on the corresponding unbalance, but produces no effect on other modes.

The following considerations show that, in principle, it is possible to determine the correction mass without the necessity of doing any trial runs. A trial run implies that the rotor is taken from rest up to normal operating speed two times: one without trial mass and one with the trial mass (or masses). The difference between the two vibration signals is the result of the added mass (or masses) and a numeric relationship may be established between cause and effect.

Balancing a rotor without trial runs, on the other hand, means that only the original measurement is done. It is required, however, that the rotor reaches normal operating speed in order to register the vibration levels that require correction in the complete operating range. If the rotor fails to reach normal operating speed due to high levels of vibration, the concept of balancing without trial runs can only be applied for the vibration modes below the maximum rotating speed reached during the test. The correction of the mode responsible for the high vibration, however, will probably need one or more trial runs, unless an accurate estimate of the corresponding modal parameters can be made.

Some of the parameters in Eq. [6] may be determined from the vibration signals initially registered by the transducers and some of them need to be calculated using a computer model of the rotor. In any case, it is convenient to consider Eq. [6] when the rotating frequency is the same as the natural frequency.

One of the most important characteristics of unbalanced rotors is that the angular position of the unbalance force leads the displacement response by 90° when the rotating frequency is the same as the natural frequency. Therefore, identification of the phase lag of the response at the resonance will define the angular position of both, the modal component of the unbalance force and the corresponding correction masses.

It is important to remember, however, that the phase angle of the displacement response must be that of the modal component of vibration. Therefore, it is necessary to extract that modal component of vibration υ_{ri} from the signal registered by the transducer.

Extraction of amplitude and phase of the modal component of vibration may be done applying a technique similar to that proposed by Kennedy and Pancu (1947). This was already mentioned by Bishop and Parkinson (1963), but their technique was based on a subjective and, at the same time, accurate identification of the points with a maximum frequency spacing, which defines the position of the natural frequency. This characteristic, however, is only strictly true when the response contains the influence of a single mode of vibration, or is approximately true when the vibration modes are well separated from each other. Therefore, serious errors of appreciation may be faced when the rotor exhibits mixed modal characteristics in its response.

Fortunately, the latest progress in the theory of modal analysis has produced a good number of specialised computer programs that extract modal parameters from the vibration response considering the interaction of many degrees of freedom. Unfortunately, however, the majority of computer programs developed for extraction of modal parameters have been designed for their application to structural analysis, such that their use in rotor dynamics is not straightforward.

No commercial computer programs for extraction of modal parameters in rotor-bearing systems were available at the beginning of this work. Therefore, an experimental program had to be used. This program provides the natural frequency, the modal damping ratio, and the amplitude and phase of the modal components of vibration.

Now, returning to the analysis of Eq. [6], the dynamic amplification factor for the resonance (that is for $\Omega_r = 1$) reduces to

$$A_r(\omega) = \frac{1}{2\zeta_r} \qquad [7]$$

The above equation shows that the amplification factor is completely defined for the resonance frequency, because the damping ratio is one of the modal parameters extracted from the unbalance response. Substituting Eq. [7] in Eq. [6] transforms the expression for the correction mass into

$$U_{rj} = -\frac{2 \cdot \zeta_r \cdot m_{ri}}{R_j \cdot (\varphi_r)_{ji}} \cdot \upsilon_{ri} \qquad [8]$$

which is only valid when the rotating frequency is the same as the natural frequency.

Now consider the value of the modal mass. This parameter is a function of the axial position from which the behaviour of the rotor is observed, and it is given by

$$m_r = \int_0^l \rho A(z)[\varphi_r(z)]^2 dz \qquad [9]$$

The determination of the above integral is very difficult for systems other than simple beams. This is the point where most authors suggest the use of a computer model to determine the characteristic functions of the rotor. The use of a discrete parameter model transforms Eq. [9] into

$$m_r = \sum_{j=1}^{m} m_j \cdot [\varphi_r(z_j)]^2 \qquad [10]$$

where the series limit m represents the number of elements in the model and m_j is the mass of the jth model element. The rth modal mass in Eq. [8] is that observed at the position of the vibration transducer used as reference. Therefore, the modal mass observed at the transducer position is given by

$$m_{ri} = \sum_{j=1}^{m} m_j \cdot [(\varphi_r)_{ji}]^2 \qquad [11]$$

The modal mass may be calculated using a computer model of the rotor to determine the mode shapes. When the use of a computer model does not provide satisfactory results, the mode shape factors must be found using an experimental procedure, such as the mass traversing technique proposed by Lindley and Bishop (1963). In such a case, however, we are not talking about balancing without trial runs any longer.

Summarising, the correction mass required to compensate the rth modal component of unbalance can only be found without trial runs if all terms in the right hand side of Eq. [8] may be determined beforehand. In this equation, the radius of the balancing plane is a known geometric parameter. The dynamic amplification factor at the resonance is defined by Eq. [7] and requires the determination of the modal damping ratio. The modal damping ratio results from the extraction of modal parameters, which also provides the magnitude and phase angle of the modal component of vibration for the resonance. The phase angle defines the angular position required for the correction mass. The modal mass may be determined using a computer model of the rotor to determine the mode shape factors. Finally, the mode shape factors allow transforming the single correction mass into a set of masses that produce no effect on other modes.

Therefore, the main difficulty seems to lie on the development of a suitable mathematical model able to provide a fair approximation of the characteristic functions. All the other parameters are obtained from the initial vibration readings.

This has been analysed in some way by different authors. There is, however, an additional problem that has not been identified before. This problem is related to the angular position of the vibration transducers with respect to the principal axes of stiffness for a rotor supported on asymmetric bearings.

Parkinson (1965) analysed the behaviour of a symmetric flexible shaft rotating in asymmetric bearings and, from the results of his analysis, he proposed a modified balancing procedure that considered the differences in mode shapes of a pair of modes produced by the asymmetric characteristics of the bearings. His procedure considered balancing the rotor in two planes and he recommended using the principal planes of the bearings, although he neither justified this recommendation nor mentioned how these planes could be identified in practice. Furthermore, he considered the principal planes to be perpendicular to each other, which is not necessarily the case for real bearings.

Analysis of the rotor response equations shows that errors are introduced in the determination of the magnitude and phase of the resonance vibration vector if the transducer is not aligned with the principal axis of stiffness of the mode to be corrected. This is a serious limitation because the positions of the principal axes of stiffness are not know and they are not necessarily the same for all modes. This problem was analysed in detail by Preciado (1998). Some considerations on this subject are included in the following section.

3. EFFECTS OF TRANSDUCERS ORIENTATION

According to modal theory, there is a phase lag of 90° between the displacement response and the unbalance force in a rotor operating at its natural frequency. The above statement also applies in the case of a rotor supported in asymmetric bearings, but only when the vibration transducer is located in the direction of one of the principal axes of stiffness. For any other direction the phase lag of the resonance vector is no longer 90° and errors are introduced when calculating the angular position of the correction masses (Preciado, 1998).

A principal axis of stiffness is the direction for which no cross coupling terms exist and the corresponding differential equation of motion uncouples from the other principal coordinate. There are two such axes for each mode of vibration, which means that the resonances are twice as many as in the case of symmetric bearings. The corresponding modes for these principal axes are usually

known as horizontal and vertical modes of the shaft, even when the directions of the principal axes are neither truly horizontal nor truly vertical.

In practice, however, there is no guarantee that the location of the transducers will coincide with the directions of the principal axes. In fact, practical experience shows that the transducers usually capture the influence of the principal modes corresponding to both principal axes.

Moreover, the characteristics of real bearings are such that the principal axes are not necessarily perpendicular to each other. Apart from the increased complexity of the vibration signals, the above considerations have consequences that require deeper analysis.

It is possible to find the Nyquist plot that would be generated by a transducer located in an arbitrary angular position q. For this, it is necessary to have the vibration signals registered by two transducers located at angles other than 0° and 180° between each other.

Consider two transducers located in the directions x and r, as shown in Figure 1. The two transducers are supposed to be at the same axial position along the rotor. The angle between the two transducers (τ) is usually, but not necessarily, equal to 90°. Consider also another transducer located at θ degrees from x. The direction of this new transducer is identified as q in Figure 1. The response of the rotor in terms of the reference coordinates $x-y$ is given by the following expressions.

$$\begin{aligned} x &= X_c \cos \omega t - X_s \sin \omega t \\ y &= Y_c \cos \omega t - Y_s \sin \omega t \end{aligned} \qquad [12]$$

Similarly for the directions r and q,

$$\begin{aligned} r &= R_c \cos \omega t - R_s \sin \omega t \\ q &= Q_c \cos \omega t - Q_s \sin \omega t \end{aligned} \qquad [13]$$

The amplitude of vibration registered by a transducer is the maximum value of displacement, which is obtained substituting the value of ωt that nullifies the first derivative of the displacement with respect to time. This value of ωt is also the relative phase angle ϕ of the vibration signal, as measured by a transducer that observes a fixed mark on the shaft. For example, for the x direction,

$$\frac{dx}{dt} = -\omega X_c \sin \omega t - \omega X_s \cos \omega t = 0 \qquad [14]$$

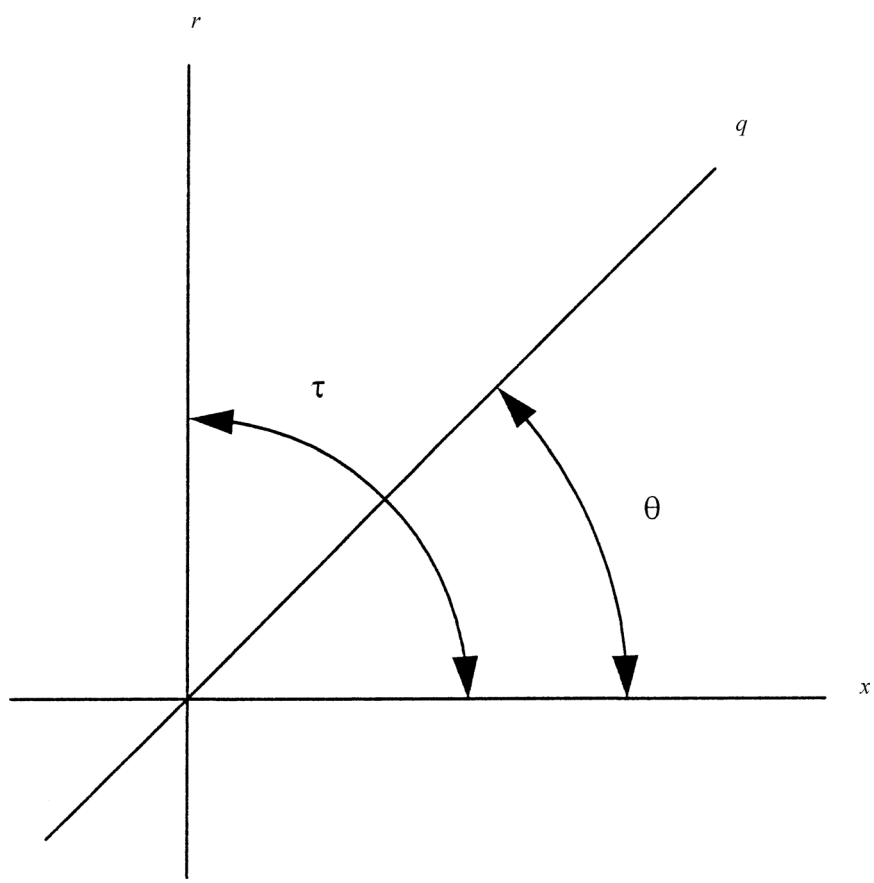

FIGURE 1 Reference system.

Therefore, the relative phase angle for the vibration measured by the transducer in the x direction is given by

$$\phi_x = \tan^{-1}\left(-\frac{X_s}{X_c}\right) \qquad [15]$$

Applying a coordinate transformation, it is possible to demonstrate that the amplitude of vibration for the transducer in the q direction is given by

$$q = \frac{x\sin(\tau-\theta) + r\sin\theta}{\sin\tau} \qquad [16]$$

which is a function of the displacements measured by the transducers in the x and r directions. Remembering from Eq. [8] that the correction mass is a function of the measured resonance vibration, it is clear that the magnitude of the correction mass will be affected by the angular position of the transducer.

Now, the relative phase angle for q is the value of ωt for which the first derivative of Eq. [16] with respect to time vanishes. It may be shown that

$$\phi_q = \tan^{-1}\left[-\frac{X_s\sin(\tau-\theta) + R_s\sin\theta}{X_c\sin(\tau-\theta) + R_c\sin\theta}\right] \qquad [17]$$

Equation [17] may be applied to a vibration mode orthogonal to the r direction. In this case, the sine and cosine components are such that $R_s = R_c = 0$. Therefore, Eq. [17] reduces to

$$\phi_q = \tan^{-1}\left(-\frac{X_s}{X_c}\right) \qquad [18]$$

which is identical to Eq. [15]. This demonstrates that the relative phase angle is independent of the angular position of the transducer. From the balancing point of view this is a rather surprising conclusion, because the angle of the correction masses is a function of both, the transducer location and the relative phase angle for the resonance. That is to say

$$\chi = \phi + \varsigma + 90^\circ \qquad [19]$$

where χ is the angle required for the correction mass and ς is the angular position of the transducer. These two angles are measured on the shaft with respect to the fixed mark used for the generation of the reference pulse.

Equation [19] indicates that a change in the angular position of the transducer should be accompanied by a change in the relative phase angle. That is if the correction angle is to be maintained. However, comparison of Eqs. [15] and [18] shows that the relative phase angle is independent of the transducer position. Therefore, we have to accept the fact that only one measuring direction provides the right angular location for the correction masses. This measuring direction should be that of the principal axis of stiffness, otherwise the error introduced will be equal to the angle between the transducer and the principal axis, as can be seen from the analysis of Eq. [19].

Of course, the above considerations are only relevant when trying to find the correction masses without performing any trial run. If trial runs are used as part of the balancing process, the correction masses are found by the principle of cause and effect and deviations of the rotor response from the theory of modal analysis, produced by improper alignment of vibration transducers, become irrelevant.

4. BALANCING OF A ROTOR RIG

The section describes step by step the balancing of the experimental rotor rig shown schematically in Figure 2.

The shaft rotates clockwise when seen from the driven end. The rotor passes only one pair of modes before

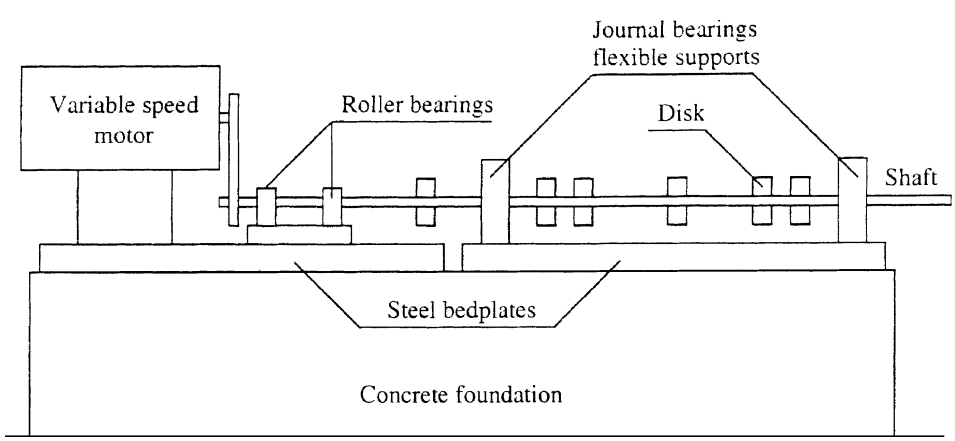

FIGURE 2 Schematic representation of the experimental rotor rig used to validate the balancing procedure without trial runs.

becoming unstable. Thus, only one balancing plane is necessary and no decomposition of correction masses into modal sets is required. The measured critical speeds were 2056 r/min and 2101 r/min.

Figure 3 shows the measuring positions used during the balancing. The vibration signals were recorded in horizontal and vertical directions in order to identify the location of the principal axes of stiffness. The measuring position located near the free end bearing was selected as the reference point for the determination of mode shapes and modal masses. It was decided to use the central balancing plane in order to maximise the influence of errors in the determination of mode shapes with a numerical model.

The correction mass necessary to compensate the first modal component of unbalance is given by Eq. [8]. The only known parameter in this equation is the radius of the balancing plane (100 mm). The first step in the balancing procedure consisted in measuring the rotor response during run-down. Figures 4 and 5 show the Nyquist plots for the free end obtained with the original unbalance. The plots for the driven end are not included. The Nyquist plot for the horizontal direction clearly shows the influence of two resonances. The plot for the vertical direction, on the other hand, shows almost no influence of the first resonance, which means that the corresponding principal axis is almost vertical.

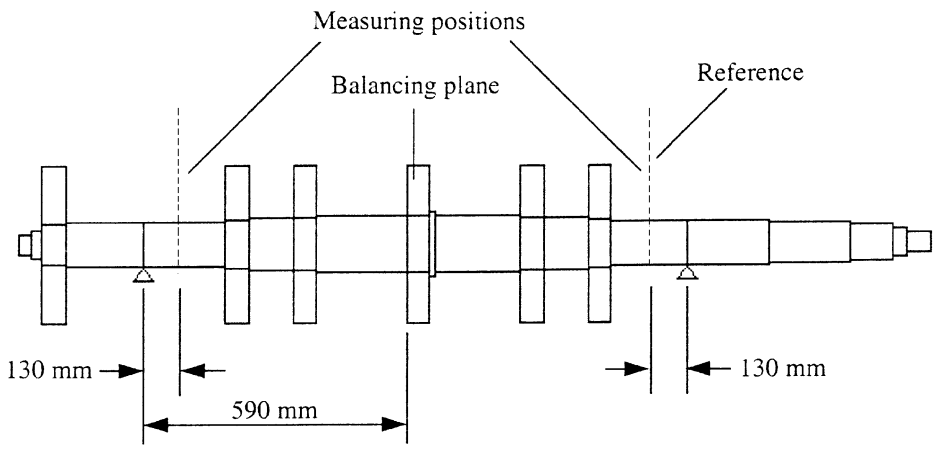

FIGURE 3 Measuring positions and balancing plane.

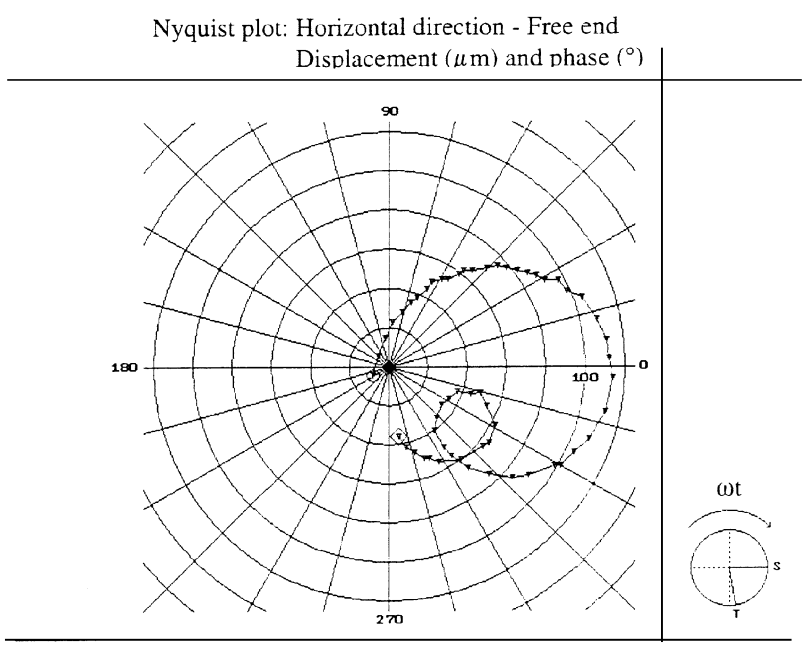

FIGURE 4 Rotor response with original unbalance.

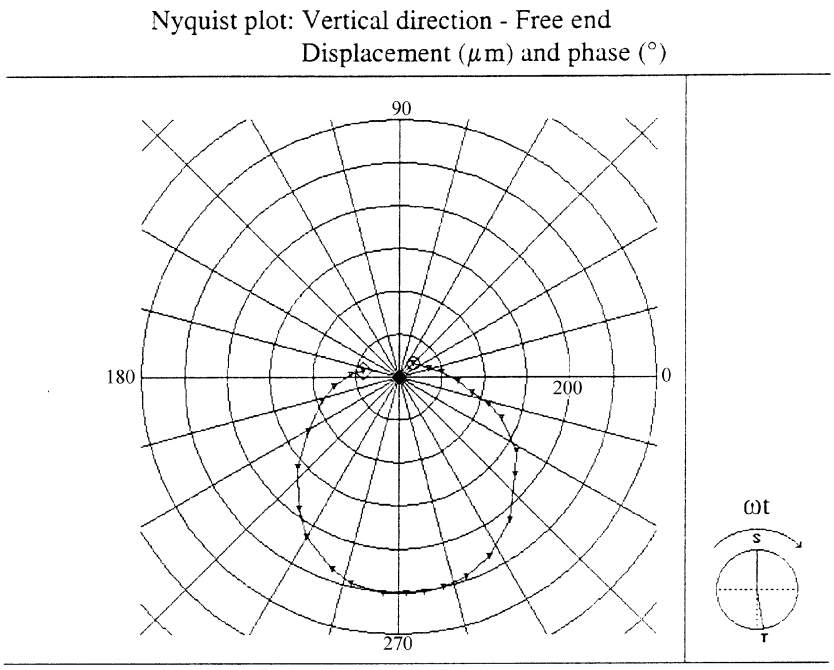

FIGURE 5 Rotor response with original unbalance.

The transducer observing the reference mark, identified with the letter T in the bottom right corner in each diagram, was located in the lower part of the rotor. The angle between this transducer and the horizontal transducer was 87° in the direction of rotation.

The next step consisted in the use of a program for extraction of the modal parameters and the determination of the principal axes of stiffness. The results are presented in Table I.

From the data for the higher resonance, the angular location of the correction mass with respect to the reference mark is given by Eq. [19]. Thus, for a transducer located at 87° with respect to the horizontal direction (i.e., $\varsigma = 174°$ with respect to the reference mark) we have that

$$\chi = -82° + 174° + 90° \qquad [20]$$

$$\boxed{\chi = 182°} \qquad [21]$$

measured anticlockwise from the reference mark on the shaft, when seen from the driven end (*i.e.*, against the shaft rotation).

TABLE I Modal parameters

Position of the principal axis	335°	87°
Natural frequency	2055.59 r/min	2101.37
Modal damping ratio	0.07964	0.02351
Resonance vector amplitude	110 μm p–p	263 μm p–p
Resonance vector phase lag	35°	–82°

Then, the mode shape of the rotor was obtained using a computer program based on the transfer matrix technique. The model was developed using the geometric data of the rotor. The supports stiffness values were adjusted such that the natural frequency calculated and the critical speed measured were the same. Also, the mode shape factor between the two vertical transducers was the same as the value found with the computer program.

The natural frequency calculated by the program using the above stiffness values was equal to 2101.32 r/min. The corresponding mode shape factors between the two measuring positions was calculated as 1.004, which compared against the measured value of 1.102 gives a difference of 8.9%. Also, from the program results, the mode shape factor between the central balancing plane and the free end measuring position was found to be:

$$(\varphi)_{ct} = 2.394 \qquad [22]$$

The modal mass observed at the transducer position was then calculated using Eq. [11] with the mode shape determined by the computer program. This resulted in

$$m_t = \sum_{j=1}^{m} m_j \cdot [(\varphi_r)_{jt}]^2 = 499.58\,\text{kg} \qquad [23]$$

Introducing numerical values into Eq. [8] gives the correction mass required at the central balancing plane.

$$U_c = \frac{(0.02351)(499.58\,\text{kg})}{(0.100\,\text{m})(2.394)}(236 \times 10^{-6}\,\text{m}) \qquad [24]$$

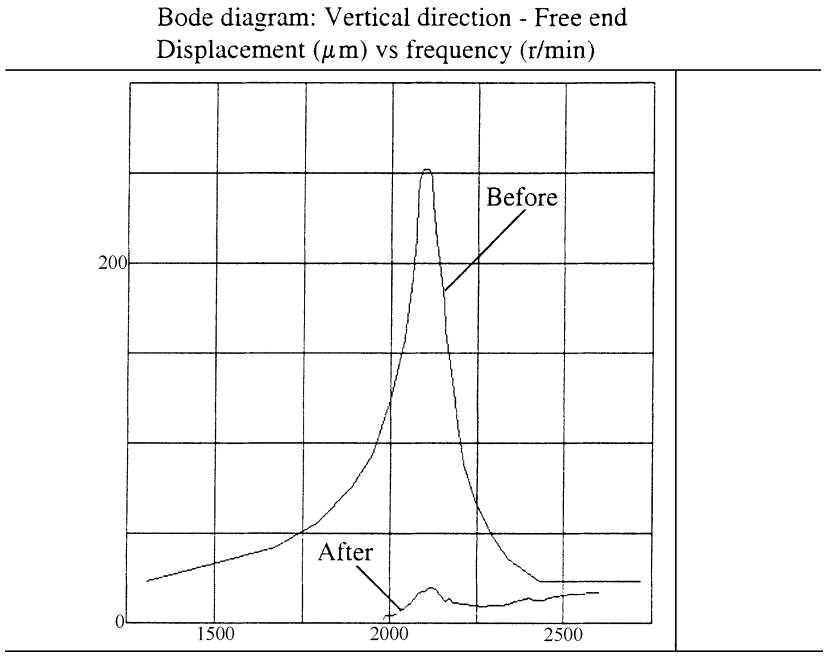

FIGURE 6 Vibration recorded at the free end bearing before and after the balancing.

$$\boxed{U_c = 12.9\,\mathrm{g}} \qquad [25]$$

The number two was dropped because the resonance vibration is expressed in peak to peak instead of zero to peak units. The negative sign was not considered in the calculations because it indicates only that the correction mass has to be placed in antiphase with the unbalance position.

Thus, the correction mass required to balance this rotor was found to be 12.9 grams at 182° from the reference mark on the shaft against the direction of rotation. There was no hole at that angular position in the balancing plane. Therefore, the correction mass was transformed into a pair of masses: 6.3 grams at 150° and 8.5 grams at 210°, which make a total of 12.86 grams at 185°.

Errors should be expected from the difference between the magnitude and phase of the calculated and the attached masses. Also, the differences observed between the calculated and the measured mode shapes should affect the results, as well as the possible errors introduced during the extraction of modal parameters.

Figure 6 shows the Bode diagram for the vertical direction in the free end, before and after the attachment of the correction mass. The residual horizontal signal is not shown because it had too much noise. Besides, it did not reach more than five microns.

The resonance vectors for the vertical direction before and after the addition of the correction masses are shown in Table II.

It is possible to use these results to determine a modified correction mass using an influence coefficient procedure.

TABLE II Resonance vectors for the vertical direction

Before the balancing	260.08 μm at −82°
After the balancing	16.22 μm at 153°

Comparison between this modified correction mass and the calculated correction mass should give a good estimation of the accuracy of the procedure described in the previous paragraphs.

The difference vector between the vibrations measured before and after the balancing is 269.71 μm at 101°. Therefore, according to the result of the balancing, the mass required to completely eliminate the vibration vector at the resonance is equal to 12.40 grams at 182°.

From here, the error in the magnitude of the correction mass calculated with the procedure previously described is of just 3.7%. There is, however, no difference in the required angular position. This assumes that the modified correction mass would in fact, completely eliminate the vibrations produced by the original unbalance.

Unfortunately, due to the limited capacity of the rotor rig, the experiment considered only one pair of modes. Nevertheless, the experiment shows that the proposed procedure works in practice.

5. CONCLUSION

Modal theory indicates the possibility of identifying the unbalance components from the rotor response eliminating

the necessity of trial runs. The process requires the mode shapes, the modal damping ratios, the modal masses and the amplitude and phase of the resonance vectors for each mode of vibration within the speed range of the rotor.

A single mass is required to correct a vibration mode, but it would excite other modes of vibration. Thus, a set of masses is required to correct a mode without upsetting the unbalance condition in other modes. The transformation of the individual correction masses into sets of masses requires the knowledge of the mode shapes. These mode shapes are also used to determine the modal masses of the rotor.

The parameters required to compensate each modal component. which include the modal damping and the amplitude and phase of the resonance vectors, are obtained directly from the measured vibrations.

The equation that gives the angle required for the correction mass is a function of the transducer angular position and the phase angle. However, the phase angle does not change even if the position of the transducer is modified. The analysis presented in Section 3 shows that a transducer will render the correct angular position for the balancing mass only if it coincides with the corresponding principal axis of stiffness.

Also, the magnitude of the correction mass is a direct function of the magnitude of the resonance vector, which changes for different observers around the shaft. The analysis presented in Section 3 demonstrates that the only measuring position that gives the right amplitude for the resonance vector is that of the corresponding principal axis of stiffness.

Balancing without trial runs is only possible if the mode shapes of the rotor are known and the rotor is able to travel through the lower critical speeds without reaching vibration amplitudes above the maximum tolerated values.

The described balancing procedure can only compensate the unbalance distribution of modes with natural frequencies located below the maximum rotating speed of the machine. Compensation of higher modes requires additional balancing runs.

The application of the procedure to the balancing of an experimental rotor shows that balancing without trial runs is not only a theoretical, but also a practical possibility. The dynamic characteristics of the rotor rig, however, did not cover all the situations possible to find in the field. Therefore, more work is required to verify the applicability of the proposed balancing procedure to rotors with more general characteristics.

NOMENCLATURE

$A(z)$	cross section area
$A_r(\omega)$	rth dynamic amplification factor
$e_r(z)$	mass eccentricity distribution
m_j	local mass at $z = z_j$
m_r	rth modal mass
$q_r(t)$	rth principal coordinate
R_j	shaft radius at $z = z_j$
U_{rj}	correction mass for the rth mode at $z = z_j$
$v(z, t)$	vibration response
$x - y$	reference coordinates
X_s, X_c	sine and cosine components of x
Y_s, Y_c	sine and cosine components of y
z	axial coordinate
ε_r	rth modal component of eccentricity
ζ_r	rth damping ratio
θ	angle of a transducer relative to the x axis
ρ	density
τ	angle between two transducers
ς	angular position of an arbitrary transducer
χ	angle required for the correction masses
ϕ	phase angle
$\varphi_r(z)$	rth mode shape for free undamped vibrations
$(\varphi_r)_{ji}$	rth mode shape factor at $z = z_j$
Ω_r	rth frequency ratio

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