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Numerical Analysis on Flow and Heat Transfer in an Axially Rotating Flow Passage with a Concentric Orifice

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This paper deals with numerical analysis on steady-state incompressible laminar flow transfer in an axially rotating passage with a concentric orifice. Consideration is given to rotational effects on secondary flow and heat transfer in rotating flow passages. The governing equations are discretized by means of a finite-difference technique and numerically solved for the distributions of velocity vector and fluid temperature subject to constant wall temperature and uniform inlet velocity and fluid temperature. The Reynolds number and rotation rate are varied to determine their effects on both the formation of vena contracta and the heat-transfer performance behind the orifice. It is disclosed that: (i) for a laminar flow through a concentric orifice in a pipe, axial rotation causes the vena contracta in the orifice to stretch and amplifies the heat-transfer performance in the downstream region, and (ii) the elongation of the vena contracta and heat-transfer performance are enhanced with an increase in both the rotation rate and the Reynolds number. Results may find applications in automotive and rotating hydraulic transmission lines.

Keywords: Secondary flow; Rotational effect; Vena contracta; Numerical analysis; Heat transfer

If an inlet to a pipe has sharp corners, flow separation occurs at the corners, and a vena contracta is formed. The fluid must accelerate locally to pass through the reduced flow area at the vena contracta. Losses in mechanical energy result from the unconfined mixing as the stream decelerates again to fill the pipe. This phenomenon is observed in a typical orifice meter which has been popularly used to measure the instantaneous flow rate in pipes. The other two most common devices for flow-rate measurement are the nozzle meter and the venturi meter. This paper investigates the effects of two factors, namely rotation and heating, on the secondary flow and heat transfer performance in a pipe with a concentric orifice.

In many practical engineering applications, the working fluid in rotating machinery flows in sudden expansion or contraction passages or in passage with an orifice. Torii and Yang (1998a and b) investigate the thermal and fluid-flow transport phenomena in axially rotating pipes with sudden expansion or contraction. They reported that (i) the pipe rotation causes the stretching of the secondary flows formed in the expansion region, resulting in the suppression of the thermal boundary layer thickness and the amplification of heat transfer performance, and (ii) in contrast, the secondary flow zone in the contraction region is suppressed due to pipe rotation. On the other hand, to the authors' knowledge, there have been no experimental or numerical results for an axially rotating pipe with a concentric orifice. To understand the transport phenomena in such passages, it is necessary to investigate the area of vena contracta, the minimum area. This minimum area results when the converging streamlines begin to expand to fill the downstream region. Detailed information on the flow, heat and mass transfer in the vena contracta is of great importance to many engineering applications.

This paper treats the thermal and fluid-flow transport phenomena in axially rotating pipes with a concentric orifice. Emphasis is placed on the effects of Reynolds number and rotation rate on both the formation of vena contracta and the heat-transfer rate in the axially rotating

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passage. A numerical method is employed to determine velocity and temperature profiles.

GOVERNING EQUATIONS AND NUMERICAL METHOD

Consider a forced flow through an axially rotating pipe with a concentric orifice, which is heated with uniform wall temperature. The physical configuration and the cylindrical coordinate system of the flow are shown in Figure 1. The following assumptions are imposed in the formulation of the problem based on the characteristics of the flow: it is an incompressible, laminar, steady flow with constant fluid properties; there is constant wall temperature, uniform inlet velocity, and uniform inlet fluid temperature and negligible axial conduction (due to the high Peclet number). Then, the governing differential equations for mass, momentum and energy can be expressed as:

Continuity equation:

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad [1]$$

Momentum equation:

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j}\right) \qquad [2]$$

Energy equation:

$$U_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) \qquad [3]$$

An isothermal laminar flow in the absence of rotation is assumed as the inlet condition. Only one-half of the pipe cross section is treated because of the symmetry of the fluid flow. The boundary conditions in the axially rotating pipe are specified as:

$$U = V = 0, W = W_w \quad \text{at wall}$$
$$U = U_{in}, V = 0, W = 0 \quad \text{at the inlet, } i.e., \ x = 0$$
$$\frac{\partial U}{\partial r} = 0, V = W = 0 \quad \text{at center } (r = 0)$$

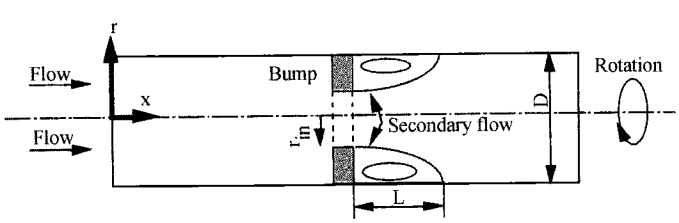

FIGURE 1 Schematic diagrams of axially rotating pipe flow systems with an orifice and their coordinates.

To discretize the governing equations, the finite difference method is employed here. Note that this method is based on the conservative formulation, although the governing equations (Eqs. [1]–[3]) are written in non-conservative form. The system variables, P, T, U, V and W are calculated with a staggered grid. Computation reveals only a slight difference when the grid system is properly changed from $20 \times 10 \times 50$ (r, θ, x) to $40 \times 20 \times 100$, resulting in a grid-independent solution. Hence, a grid system of $20 \times 10 \times 50$ nodal points with uniformly distributed nodal points is employed here to also save computation time.

The numerical computation was performed on a personal computer (Pentium II, CPU 300 Mz), and consumed nearly 78 CPU hours, using water as the working fluid ($Pr = 7.1$). The parameters used in the present study are Reynolds numbers $Re = 240 \sim 1{,}000$, velocity ratios $N = 0 \sim 5$ and diameter ratio of the orifice to the pipe, $r^* = 0.75$.

RESULTS AND DISCUSSION

Figures 2(a), (b) and (c), for $Re = 480$, depict the flow pattern over the $r-x$ cross-section in an axially rotating pipe with a concentric orifice at different velocity ratios N. Note that the passage size is substantially increased to make the thermal and velocity fields clearer. $N = 0$ corresponds to no rotation case. An increase in N means an increase in the rotational speed at a given axial velocity. It is observed that a recirculation zone appears behind the orifice region of the flow passage and is extended with the increase of the velocity ratio of rotation to fluid flow N. The corresponding temperature distributions over the $r-x$ cross-section in an axially rotating pipe with a concentric orifice are depicted in Figures 3(a), (b) and (c) with velocity ratio N, as the parameter. Here, $\theta = 1$ and 0 in Figure 3 correspond to the heated wall temperature and the fluid temperature at the inlet of the passage, respectively. It is observed that as the flow goes downstream, a thermal boundary layer develops in the sudden expansion region behind the orifice but its thickness is suppressed with an increase in the velocity ratio of rotation to fluid flow N. The changes in the vena contracta, *i.e.*, the secondary flow zone and in the thermal boundary layer thickness due to pipe rotation become clearer for axial movement of the reattachment point and local heat-transfer rate.

Figure 4 illustrates the distance L of the expansion location to the reattachment point, as a function of Reynolds number Re. One observes that with Re fixed, the reattachment point is moved in the downstream direction with an increase in N. Thus the streamwise movement of the reattachment point is ascribed to the pipe rotation. In other words, the vena contracta is stretched in the streamwise direction due to the pipe rotation. Imao et al. (1989)

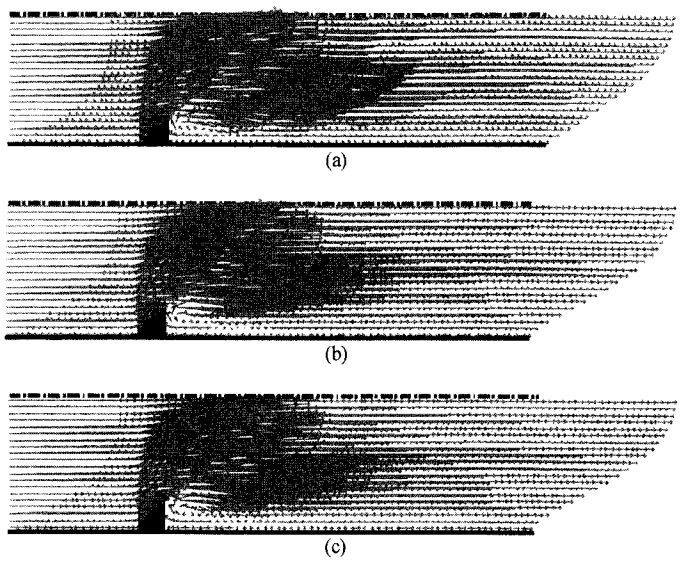

FIGURE 2 Velocity vector in an axially rotating pipe with a concentric orifice at Re = 480, (a) N = 0, (b) N = 2, and (c) N = 5.

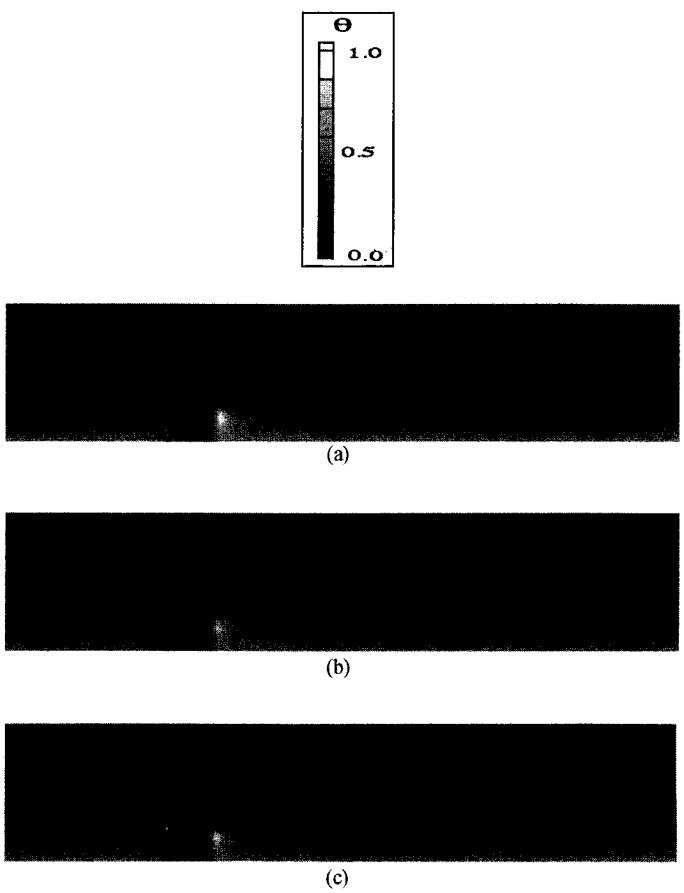

FIGURE 3 Temperature distribution in an axially rotating pipe with expansion at Re = 240, (a) N = 0, (b) N = 2, and (c) N = 5.

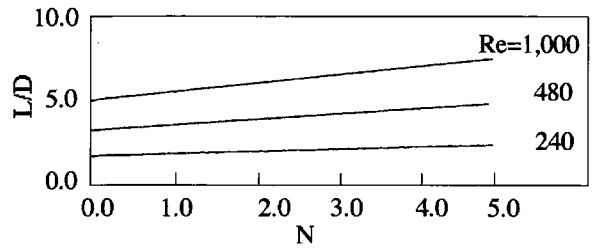

FIGURE 4 Effects of Reynolds number and rotation rate on L for axially rotating flow case.

reported that when laminar flow passes in an axially rotating pipe, an increase in its speed makes the streamwise velocity approach that of turbulence. It is postulated, therefore, that in the swirling case, the velocity in the vicinity of the wall is stimulated more than that without pipe rotation, resulting in a stretch of the vena contracta. This tendency is found to be amplified with an increase in the Reynolds number, as seen in Figure 4. Similar phenomenon is reported for the swirling flow in the axially rotating pipe with expansion, although a substantial stretch of the vena contracta due to pipe rotation occurs in the rotating pipe with expansion than with bump.

Figure 5 illustrates the local Nusselt numbers at Re = 480 with rotation rate N, as the parameter. For comparison, the well-known correlation (Kays & Crawford, 1983) of laminar heat transfer in the hydrodynamically and thermally fully-developed pipe flow is also shown in the figure with solid straight lines. It is observed that the local Nusselt number at N = 0 is higher than the heat-transfer coefficient in the thermally and hydrodynamically full developed region because of the thermal entrance effect and subsequently, approaches the laminar correlation in the downstream section. One observes that (i) a substantial enhancement in heat transfer performance is caused due to the presence of the orifice and (ii) the local Nusselt number is diminished in the vena contracta and is gradually

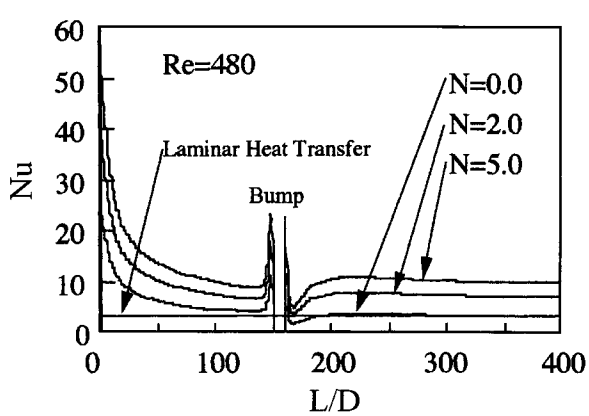

FIGURE 5 Effects of rotation rate on local Nusselt numbers at Re = 480.

increased along the flow. This trend is intensified with an increase in the velocity ratio. The substantial reduction in the vena contracta behind the orifice diminishes due to pipe rotation. In other words, heat-transfer performance in the vena contracta is induced with an increase in an axial rotation of the pipe. This behavior accords with the suppression of the thermal boundary layer thickness in the recirculation zone, as seen in Figure 3. It is seen that an enhancement of heat transfer performance occurs due to the pipe rotation in the further downstream region. This phenomenon is in accordance with the experimental result reported by Imao et al. (1989), as mentioned above. That is, an increase in a pipe rotation speed induces a substantial amplification of the streamwise velocity gradient so that heat transfer performance is intensified.

In summary, when laminar flow is introduced into an axially rotating pipe with a concentric orifice, the vena contracta region is stretched by the axial rotation of the pipe and heat-transfer performance is increased in this region. This trend is amplified with an increase in the rotation rate and the Reynolds number.

SUMMARY

Numerical simulation has been employed to investigate the thermal fluid flow in an axially rotating pipe with a concentric orifice. Consideration is given to the influence of rotation ratio and Reynolds number on the formation of a vena contracta and the heat-transfer performance behind the orifice. The results are summarized as follows:

When laminar flow passes in an axially rotating pipe with the orifice, the pipe rotation causes the stretching of the vena contracta formed in the expansion region, resulting in the suppression of the thermal boundary layer thickness. Consequently, heat-transfer performance is amplified in the recirculation region. This trend is amplified with an increase in the rotation rate and the Reynolds number.

NOMENCLATURE

D	pipe diameter, m
h	heat transfer coefficient, $W/m^2/K$, $h = (\lambda \partial T/\partial r|_{r=R})/(T_w - T_b)$
L	distance from the orifice, m
N	velocity ratio, W_w/U_m
Nu	Nusselt number, hD/λ
P	time-averaged pressure, Pa
Pr	Prandtl number
r	radial coordinate, m
r_{in}	radius of concentric orifice, m
r^*	radius or diameter ratio, r_{in}/R
R	pipe radius, D/2, m
Re	Reynolds number, $U_m D/\nu$
T	temperature, K
U, V, W	time-averaged velocity components in axial, radial, and tangential directions, respectively, m/s
U_m	axial mean velocity over tube cross section at the inlet, m/s
U_i	velocity component in the x_i direction, m/s
W_w	tangential velocity on pipe wall, m/s
x	axial coordinate, m
x_i	coordinates, m

Greek Letters

α	thermal diffusivity, m^2/s
ν	molecular viscosity, m^2/s
ρ	density of fluid, kg/m^3
λ	thermal conductivity, W/m/K
θ	dimensionless temperature or tangential direction

$$\theta = \frac{T - T_{\text{inlet}}}{T_w - T_{\text{inlet}}}$$

Subscripts

b	bulk
inlet	inlet
m	mean
w	wall

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