Research Article

Suppression of Base Excitation of Rotors on Magnetic Bearings

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This paper deals with rotor systems that suffer harmonic base excitation when supported on magnetic bearings. Magnetic bearings using conventional control techniques perform poorly in such situations mainly due to their highly nonlinear characteristics. The compensation method presented here is a novel optimal control procedure with a combination of conventional, proportional, and differential feedback control. A four-degree-of-freedom model is used for the rotor system, and the bearings are modeled by nonlinear expressions. Each disturbance frequency is expected to produce a multiharmonic system response, a characteristic of nonlinear systems. We apply optimal control choosing to minimize a performance index, which leads to the optimization of the trigonometric coefficients in the correction current function. Results show that the control technique suppresses rotor vibration to amplitudes that were significantly smaller than the disturbance amplitudes for the entire range of disturbance frequencies applied. The control technique explored in this paper is a promising step towards the successful application of magnetic bearings to systems mounted on moving platforms.

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1. INTRODUCTION

Magnetic bearings suspend a rotor between electromagnets and thus are noncontacting. They can significantly reduce machinery wear because the bearings themselves accumulate nearly no wear; they can actively minimize rotor vibration and, if widely used, they would reduce the amount of oily waste generated. However, magnetic bearings' poor performance when exposed to disturbances has prevented their use in many applications, particularly in transportation systems. Magnetic bearings are naturally unstable and require feedback control. Conventional control techniques work well when the bearings are exposed only to small disturbances, but quite poorly when exposed to more significant disturbances from rotor unbalance or base motion. The disturbances that may be easily handled in other similar applications by control techniques fail when applied to magnetic bearings. Magnetic bearings are very nonlinear and compensation techniques that are based on linearized system models fail when disturbances cause the rotor to deviate too far from its centered position. This fact has hence generally limited their application to machinery that is not exposed to significant disturbance motion.

The aviation industry in particular has been interested in applying magnetic bearings to gas turbine engines. Storace et al. [1] provided an overview of the possible benefits of adapting magnetic bearings to gas turbines. Eventually, magnetic bearings could substantially improve the power-to-weight ratio of gas turbines. Significant weight reductions would be realized by the removal of the lube oil system, which includes lube oil tanks, pumps, coolers, and a variety of tubing, valves, and fittings. Reduced life cycle costs would accompany the removal of the lube oil system due to increased bearing service life and reduced maintenance. Magnetic bearings would also enable new engine designs that spin faster and operate at higher temperatures to achieve greater thermal efficiency, further increasing the power-to-weight ratio [2].

The US Navy also had an interest in magnetic bearings as a way to attenuate machinery noise. One study by the Navy [3] explored a feedback noise control method for a pump with magnetic bearings. The bearings would actively move the pump impeller to suppress fluidborne noise. Nataraj [4, 5]; Nataraj and Calvert [6] studied the dynamic response of rotating machinery with magnetic bearings mounted on a moving platform. Each identified significant technical

challenges that must be addressed prior to a magnetic bearing's application on board Navy ships.

There are several sources of nonlinearity; the magnetic force is inversely proportional to the square of gap length, and it is proportional to the square of coil current. Other, less important, sources of nonlinearity are electromagnetic hysteresis, eddy currents, and skewed force lines from geometric force coupling between degrees of freedom [7]. Skewed force lines become a concern only when air gaps are enlarged as a possible way to limit the effects of vibration [8].

Some studies tried to define more generalized characteristics of magnetic bearings. Mohamed and Fawzi [9] examined magnetic bearings' response characteristics due to the nonlinearity of the electromagnetic force. The authors constructed a five-degree-of-freedom model with a magnetic thrust bearing and two radial magnetic bearings. They controlled the model with a linear feedback control system to study the nonlinear oscillations that occur due to gyroscopic effects. The authors found that the model exhibited bifurcation effects and was unstable around critical speeds. They also proposed a nonlinear feedback control method. Chinta and Palazzolo [10] characterized the nonlinear responses of a two-degree-of-freedom model with PD control in which the rotor speed, an excitation force, the static load, and the amount of coupling between the two axes were varied. They simulated the system using the trigonometric collocation method and were able to identify the conditions in which bifurcation and changes in stability would occur. Bifurcations in some conditions produced superharmonic response frequencies of one half and one quarter the forcing frequency.

A rotor's responses to vibrations due to rotor unbalance and base motion have attracted the most attention. Of the two kinds of disturbances, more papers explored methods intended to manage the effects of rotor unbalance. Petela and Botros [11] considered both linear and nonlinear models for their method using multiaccess displacement and velocity feedback to suppress vibration from a flexible shaft rotor. Results for the nonlinear model, however, were inconclusive, and they acknowledged that ranges of parameters, such as rotor unbalance and rotation speed, which would have no effect on conventional bearings would cause instability with magnetic bearings.

Lindlau and Knopse [12] applied feedback linearization, which linearizes the system at all operating points rather than just one neutral point. The response remained stable on a single-degree-of-freedom model, even when exposed to a disturbance with an amplitude of 90 percent of the gap width, and the simulated responses were similar to those measured from a test rig. An approach was developed by Knopse et al. [13] to suppress vibration in a linear system due to unbalance-combined adaptive open-loop control with feedback control. The adaptive open loop control system adjusted the control signal's magnitude and phase periodically in response to changes in the rotor's operating conditions. The controller optimized the signal at each update by calculating the minimum-expected rotor response. The controller used an estimated influence coefficient matrix for the calculation, which was estimated offline, prior to implementation,

by applying a known disturbance to the system and measuring the rotor's response. The authors demonstrated that the method could suppress vibrations due to unbalance. In another approach, Cole et al. [14] employed PID feedback control combined with H_∞ control, which used weights that were selected by optimizing a linearized model. The results demonstrated that the control method suppressed vibration more effectively than using PID control alone; however, the paper did not indicate the relative magnitude of the applied disturbance.

Beale et al. [15, 16], Hisatani and Koizumi [17], and Mizumo and Higuchi [18] considered methods that estimate the forces of unbalance so that counter forces can be generated by the magnetic bearing to induce the rotor to rotate about its center of mass. None of the techniques used a nonlinear model. Beale et al. and Hisatani and Koizumi used adaptive filtering to estimate the disturbance and to generate counter force control signals. Beale et al. [16] noted some disturbance amplitude and bandwidth limitations, outside of which the technique will not be able to suppress vibration.

There has also been some interest in considering base motion of a rotor supported on a magnetic bearing. Cole et al. [14] was among a few efforts that studied techniques that could limit the effects of base motion. They employed PID control in parallel with H_∞ control of bearings in systems subject to horizontal base impact (in addition to their experiments with rotor vibration mentioned above). The control technique enabled the system to recover from the disturbance when the control function weights were optimized for the base motion disturbance and rotor vibration. Kasarda et al. [19] tried a variety of gains in a PID controller on a one-degree-of-freedom test stand mounted on a shaker table that generated a sinusoidal base motion. They ran tests designed to find the limits of stability for several combinations of controller gains. They also found that some stable responses were linear and others were nonlinear. Nataraj [5]; Nataraj and Calvert [6] demonstrated that a feedforward control method that would work well with a linearized model would not work for the nonlinear model subject to a sinusoidal base motion. Somewhat similar techniques were used by Knopse et al. [13] and Saunders et al. [20], who used a hybrid control approach. In their paper, the feedback gain, optimized with a linear quadratic Gaussian regulator, was added to a feedforward signal designed to cancel the effects of steady, oscillating disturbances.

This paper examines a novel method designed to suppress rotor motion due to base excitation with an emphasis on the nonlinear model for the magnetic bearings. The method combines PD feedback with feedforward optimal control, where a known base motion is used to select a set of predetermined frequencies and their amplitudes for a control signal optimized to suppress the rotor response.

2. MATHEMATICAL MODEL

A typical radial magnetic bearing arrangement (see Figure 1) is four electromagnets with two opposing electromagnets aligned along each radial axis where each electromagnet has

FIGURE 1: An 8-pole radial magnetic bearing.

FIGURE 2: The four-degree-of-freedom model.

two poles. We consider a four-degree-of-freedom model of a rigid rotor that is supported at each end by such a magnetic bearing. The rotor is modeled as a shaft with a saddle-mounted disk perpendicular to the shaft (see Figure 2). Each end is free to move radially with the bearing forces being applied along the y and z axes. Base motion and rotor unbalance disturbance forces could also be applied to both directions at each end. This model also includes the gyroscopic effect. The gyroscopic moments applied to each direction of each bearing depend on the angular position and velocity of the disk in all three directions, which couples the xy and xz motion.

Using standard techniques from analytical dynamics with Euler angles and Lagrange equations, and assuming small displacements, the equations of motion can be derived. The

term F_{rc} represents the forces and moments applied to the rotor at its center of gravity, which include the magnetic bearing forces (F_m) and disturbance forces such as unbalance. The resulting equation of motion is

$$\begin{bmatrix} m_r & 0 & 0 & 0 \\ 0 & m_r & 0 & 0 \\ 0 & 0 & J_d & 0 \\ 0 & 0 & 0 & J_d \end{bmatrix} \begin{bmatrix} \ddot{V} \\ \ddot{W} \\ \ddot{\beta} \\ \ddot{\Gamma} \end{bmatrix} - \Omega_r \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -J_p \\ 0 & 0 & J_p & 0 \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{W} \\ \dot{\beta} \\ \dot{\Gamma} \end{bmatrix} = F_{rc}, \quad (1)$$

where J_d and J_p are the diametral and polar mass moments of inertia. The equation also can be represented as

$$M_r \ddot{\bar{p}} - \Omega_r G_r \dot{\bar{p}} = F_{rc}, \quad (2)$$

where $\bar{p}$ is a displacement vector of the rotor's positions and angles at the rotor's center.

These equations can be then converted to the horizontal and vertical positions at each bearing, $q = T_q \bar{p}$, using the transformation matrix

$$T_q = \begin{bmatrix} 1 & 0 & 0 & -L_1 \\ 0 & 1 & L_1 & 0 \\ 1 & 0 & 0 & L_2 \\ 0 & 1 & -L_2 & 0 \end{bmatrix}. \quad (3)$$

The terms L_1 and L_2 are the distances from the center of gravity to the bearings. Equation (2) then becomes

$$M_r T_q^{-1} \ddot{\bar{q}} - \Omega_r G_r T_q^{-1} \dot{\bar{q}} = F_r, \quad (4)$$

where F_r are the forces applied at the bearings and $\bar{p}$ is the vector of state variables of the rotor's positions, and their derivatives, at each bearing. When nondimensionalized, (4) can be restated as

$$A_m \ddot{\bar{x}} - A_g \dot{\bar{x}} = f_r. \quad (5)$$

The equation is then rearranged into the form

$$\begin{bmatrix} I & 0 \\ 0 & A_m \end{bmatrix} \begin{bmatrix} \dot{\bar{x}} \\ \ddot{\bar{x}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & A_g \end{bmatrix} \begin{bmatrix} \bar{x} \\ \dot{\bar{x}} \end{bmatrix} + \begin{bmatrix} c0 \\ F_m \end{bmatrix} + \begin{bmatrix} 0 \\ -F_s \end{bmatrix} \quad (6)$$

in which F_r is separated into the magnetic bearing forces (F_m) and the static load on the rotor (F_s). Note that these forces are the sum of the the individual forces at the bearings transformed appropriately. The next section discusses the force model for each axis at each bearing.

2.1. Magnetic bearing model

The magnetic bearing forces can be shown to be the following; detailed derivations can be found in Schweitzer et al. [21] and Nataraj [4]. We directly present nondimensional forces here:

$$f_m = f_{m0} \left(\frac{i_1^2}{4g_1^2} \right) - f_{m0} \left(\frac{i_2^2}{4g_2^2} \right), \quad (7)$$

FIGURE 3: Control scheme.

where the magnetic bearing constant (f_{m0}) is a function of the bearing and rotor parameters. The currents applied (i) and the rotor gaps (g) are

$$\begin{aligned} i_1 &= 1 + i_s - i_c, & i_2 &= 1 - i_s + i_c, \\ g_1 &= 1 - (x - u), & g_2 &= 1 + (x - u). \end{aligned} \quad (8)$$

The magnetic bearing force is then reduced to

$$f_m = \frac{f_{m0}}{4} \left[\left(\frac{(1 + i_s - i_c)^2}{(1 - (x - u))^2} \right) - \left(\frac{(1 - i_s + i_c)^2}{(1 + (x - u))^2} \right) \right], \quad (9)$$

where x is the appropriate displacement component, $u(t)$ is the base excitation in that direction, and i_c is the control current.

Finally, i_s , the current to compensate for a static load, is determined from the relation $i_s = f_s/f_{m0}$, which is derived by assuming that the sum of the magnetic bearing forces (f_1 and f_2) equals the static load (f_s), or

$$f_s = \frac{f_{m0}}{4} ((1 + i_s)^2 - (1 - i_s)^2). \quad (10)$$

3. CONTROL

The control system configuration proposed in this paper has a feedback loop and a feedforward loop (see Figure 3). The feedback loop is a conventional PD controller with proportional and derivative gain (K_p and K_d) that yields a current $i_{fb} = K_p x + K_d \dot{x}$.

The feedforward loop is based on the fundamental observation that nonlinear systems exhibit sub and super harmonics when subjected to harmonic excitation. Hence, we postulate a control scheme that compensates for these frequency terms by supplying correction currents at the appropriate frequencies

$$i_{cc} = \sum_{j=1}^N a_j \cos j\omega_b t + \sum_{j=1}^N b_j \sin j\omega_b t, \quad (11)$$

where the frequencies of the correction current are multiples and fractions of the base motion frequency (ω_b). The final

control current is then the sum of the feedback current and the feedforward correction current $i_c = i_{fb} + i_{cc}$.

We next seek to minimize a performance index

$$J = \phi(t_f) + \int_0^{t_f} (\bar{x}^T Q \bar{x} + \bar{i}_{cc}^T R \bar{i}_{cc}) dt, \quad (12)$$

where Q and R are weighting matrices. We add a final state penalty that was a function of the final time of the simulation period just completed to ensure convergence to the optimal solution; this is a Bolza type of cost function [22]. If the system were linear, we would be able to use conventional linear optimal control theory. In addition, since the linear response would be harmonic, it is possible to derive simpler versions of the Riccati equation [23]. In fact, that is the procedure we used with the linearized versions of the above equations to provide a starting solution for the nonlinear optimal control process.

Being nonlinear, the response of a system suspended by magnetic bearings due to any particular disturbance is not possible to predict analytically and a single optimal solution cannot be derived. Hence, we implement the scheme numerically. In order to make the problem more insightful, and since we are only dealing with harmonic disturbances, we can now choose to optimally determine the coefficients in the series (11), rather than elaborate functions of time as is done in conventional optimal control.

The intended application of the results is then as follows. A table of optimal parameters is obtained prior to implementing the control method by measuring the responses of a simulated magnetic bearing system to a range of expected disturbances. A correction current function selector is then used to select the optimal control current parameters from a lookup table, based on the measured disturbance. Note that, in many practical situations (such as ship-board machinery which was the motivation for this paper), the disturbance frequencies are often known a priori to some extent.

4. NUMERICAL RESULTS

In order to generate an initial guess of the correction current coefficients, we used a linearized system model with optimal

control. A valid guess had to produce a stable solution, otherwise the iterative selection of coefficients while trying to find an optimized set would not produce meaningful solutions. To start with, guesses were based on a solution using linear optimal control (with linearized magnetic bearing expressions). The details of the linear optimal control are not shown here in the interest of brevity. The optimized coefficients found for the correction current to compensate for a base motion that is near the base motion for which coefficients are being searched, are used as the initial guess for subsequent runs.

In this case, a least-squares-fit curve-fitting technique was used to identify the trigonometric series coefficients for the correction current. The first guess would have coefficients for only one correction current frequency (the base motion frequency) although many more are needed for the nonlinear closed-loop model.

The optimization process was iterative, repeating the simulation of the nonlinear closed-loop system. The process was started using the initial guess coefficients, generating a response curve and calculating the initial performance index (12) for the response curve. The coefficients were then altered for all three frequencies prior to the next iteration, the response was found again, and the performance index was recalculated. The coefficients were altered after each iteration using the simplex search method to select successive sets of variables to find values that approach a minimum. The process was repeated until the performance indices converged to some minimum value within a specified tolerance. Note that the feedback control parameters are not part of the optimization process; although we have investigated that aspect (and found it to be not so efficacious), it is not included in this paper as the focus of the paper is on the feedforward controller. Also note that the transient solutions are completely dropped from this analysis as the steady solution is the main focus of this paper. We do believe that the transient solution is important especially for situations dealing with shock, but could be the subject of future research.

In order to illustrate the efficacy of the method, we use an example here of a rotor in a standard U.S. Navy fire pump. The air gap length is a typical one for magnetic bearings. It is saddle mounted, and can reasonably assumed to be a rigid rotor: shaft weight = 76 lbs; disk weight = 30 lbs; shaft diameter = 2.75 in; disk diameter = 20.9 in; shaft length = 45 in; disk position = 26 in; air gap = 0.020 in; $f_{m0} = 10$; nominal rotational speed = 30,000 rpm. The selection of control parameters for the model was based primarily on combinations of feedback gains that were selected based on the standard control techniques applied to the linearized system and predicted low responses from numerical simulations for the nonlinear system. The parameters selected were $K_p = 1.1$, and $K_d = 0.0200166$. The differential feedback gain produces a damping ratio of $\zeta = 0.1$.

All mechanical and control parameters were nondimensionalized so that the solution methods used here could be applied to a variety of rotor designs. We used nondimensional base motion frequencies (Ω_b) in the range of 0.1 to 3.0. The base motion amplitude used for each model and for

FIGURE 4: Effect of optimal FF controller for $\Omega_b = 0.6$.

all base motion frequencies was 0.9, or ninety percent of the air gap length.

The same three relative frequencies of one, two, and four times the base motion frequency were used for the correction current for each case and for all base motion frequencies. The correction current frequencies were selected by identifying what the dominant response frequencies were, generally, for the entire range of base motion frequencies prior to optimization. The use of three correction current frequencies required at least six subharmonics. Six subharmonics could have multiples of two, three, four, five, and six times the base motion frequency. Two superharmonic frequencies, one half and one third of the base motion frequency, were used because either one may contribute to a response.

First we compare results from the optimal feedforward controlled system with a conventionally controlled system in order to demonstrate the need for our approach. Examples are shown at two speeds, (see Figures 4 and 5). Note that the response with the conventional PD control is too large to be shown in its entirety on the same graph as the optimal feedforward controlled system. Response at other speeds is similar.

All the vertical motion response plots resulting from the application of optimized correction current coefficients had amplitudes that were significantly less than the amplitude of the base motion. In fact, the response amplitudes when just the correction current was optimized for the linearized model (labeled as “w/o opt. coeff.”) were typically less than ten percent of the amplitude of the base motion. *It should be noted that the comparison in all the subsequent figures is between an optimal linear system and an optimal nonlinear system both with the feedforward controller proposed in this paper, and investigates and emphasizes the need for the nonlinear model.* Response without the FF controller is at least an order of magnitude worse and is not compared any more.

FIGURE 5: Effect of optimal FF controller for $\Omega_b = 1.3$. FIGURE 7: Response plots at $\Omega_b = 0.6$, bearing 2. FIGURE 6: Response plots at $\Omega_b = 0.1$, bearing 1. FIGURE 8: Response plots at $\Omega_b = 1.0$, bearing 1.

Figures 6, 7, 8, 9, 10, and 11 are sample plots of responses of the four-degree-of-freedom model. In fact, because two most dominant response frequencies (see Figures 12 and 13) were usually one and two times the base motion frequency, it is possible that using only those frequencies as the correction current frequencies would have been sufficient to suppress most of the motion.

For horizontal motion, however, the dominant frequencies were more variable. Figures 14 and 15 have magnitude response profiles where one of the two most dominant frequencies were three times the base motion frequencies. Therefore, the indirect effects on horizontal motion by the gyroscopic effects of the four-degree-of-freedom model

could require the addition of three times the vertical motion frequency to the horizontal motion correction current.

There is one obvious overall trend, which is a general decline in the overall magnitudes of the responses with increasing base motion frequency (see Figure 16). The one exception is the root mean square plot of the solutions from the numerical method for the two-degree-of-freedom model. When the base motion frequency of $\Omega_b = 1.6$ was applied, the initial simulation failed when starting with the optimized control current coefficients found for $\Omega_b = 1.5$. Therefore, the optimization cycle had to start with the optimized coefficients for the linearized model. Because the coefficients for the linearized model generated responses with higher

FIGURE 9: Response plots at $\Omega_b = 1.3$, bearing 1. FIGURE 11: Response plots at $\Omega_b = 3.0$, bearing 1. FIGURE 10: Response plots at $\Omega_b = 2.2$, bearing 2. FIGURE 12: Vert. response mag. profiles at $\Omega_b = 1.0$, bearing 2.

magnitudes, the RMS plot jumped up at $\Omega = 1.6$. This may be an indication of the model's sensitivity to deviations from the expected base motion frequency. This sensitivity should be explored because high sensitivity would require smaller frequency intervals in a feedforward control lookup table.

Another noteworthy feature of the vertical response plots are the static offsets, with the neutral points for the oscillations occurring around some point below the bearings' centers. This is likely due to the effect of the static load on the nonlinear system, even though a compensating current (i_s) was applied. Flowers et al. [24] explored this problem and proposed an integrally augmented state feedback control method as a remedy. In none of the cases, however, was

the static offset any larger than ten percent of the nominal gap length.

The optimal control algorithm actually added amplitude to the horizontal motion. The response plot model would show no motion in the undisturbed horizontal direction prior to optimization, but as the optimization method iteratively tried different sets of control current coefficients, motion was added even though the performance index was reduced. Without a disturbance applied, the optimal solution for horizontal motion was the correction current coefficients found for the linearized model. Figures 17 and 18 are plots where the horizontal motion increased by a small amount. More details of all the studies can be found in Marx [25].

FIGURE 13: Vert. response mag. profiles at $\Omega_b = 2.2$, bearing 2. FIGURE 15: Horiz. response mag. profiles at $\Omega_b = 1.5$, bearing 2. FIGURE 14: Horiz. response mag. profiles at $\Omega_b = 1.4$, bearing 1.

FIGURE 16: RMS plots for 4 DOF model.

5. CONCLUSION

This paper developed a combination of PD feedback and an optimal feedforward control procedure that can effectively limit the deleterious influence of harmonic base motion vibration on a rotor suspended by magnetic bearings. We considered a four-degree-of-freedom rigid rotor model with two magnetic bearings, and derived a procedure to compute correction currents based on the sub and super harmonic responses of nonlinear systems.

Three correction current frequencies were used for all base motion frequencies applied, which were one, two, and four times the base motion frequency. The same three relative

correction current frequencies were used exclusively so that their coefficients could be used to initiate optimization of coefficients for nearby base motion frequencies. Although the combination of correction current frequencies was not necessarily the dominant response frequencies for all base motion frequencies, they were sufficient to consistently produce satisfactory results.

In summary, the response with the proposed optimal feedforward controller is orders of magnitude better than a system without one. This is so even when the response is in the nonlinear regime. While the results from this work were satisfactory, we believe that much needs to be done before magnetic bearings can be used successfully and reliably in

FIGURE 17: Horiz. response mag. profiles at $\Omega_b = 0.1$, bearing 1. FIGURE 18: Horiz. response mag. profiles at $\Omega_b = 0.6$, bearing 2.

very demanding scenarios. Some initial thoughts in this regard follow.

Additional sets of correction current coefficients should be found for each base motion frequency, with each set selected to compensate for a different base motion magnitude, with the magnitudes separated by some fixed intervals. A lookup table should then be assembled. How well the coefficients suppress vibrations that have frequencies or amplitudes between those for which optimized coefficients were found should be determined. And, the technique should be applied to a variety of rotor configurations, including rotors with flexible shafts. The technique developed (and applied for rigid rotors here) is quite general; however, it can be anticipated that flexible rotor models would introduce ad-

ditional complexities that would challenge the current algorithm.

Eventually, the control method could be enhanced by introducing algorithms that would estimate the actual disturbance and periodically adjust the correction current parameters to adapt them to changing rotor characteristics. However, the control technique studied in this effort cannot be ultimately regarded as successful until it is proven on a test stand and then on in-service machinery. Even if successful, it is still only one part of a more complete solution. We focused on steady harmonic excitation and steady response. However, a magnetic bearing must also be able to limit the transient response due to step or impulse disturbances, which could include severe momentary disturbances such as shock from a nearby underwater explosion or a hard aircraft landing. For example, an additional feedback loop can be added that uses linear quadratic regulator (LQR) control, feedback linearization, or H_∞ control. The added feedback loop could be engaged temporarily, for at least as long as the expected transient response, after being triggered by a signal from a platform mounted sensor. These techniques are being explored in our current work.

The numerical method used here was indeed successful; however, with harmonic excitations, much better insight can be gleaned from a seminumerical approach (such as the trigonometric collocation method, [26]), and is also the subject of our current work.

REFERENCES

- [1] A. F. Storace, D. Sood, J. P. Lyons, and M. A. Preston, "Integration of magnetic bearings in the design of advanced gas turbine engines," *Transactions of the ASME, Journal of Engineering for Gas Turbines and Power*, vol. 117, no. 4, pp. 655–665, 1995.
- [2] G. Montague, M. Jansen, R. Jansen, B. Ebihara, A. Provenza, and A. Palazzolo, "Experimental high temperature characterization of a magnetic bearing for turbomachinery," in *Proceedings of the 59th Annual Forum of the American Helicopter Society*, pp. 6–8, Phoenix, Arizona, May 2003.
- [3] G. E. Piper, "Active feedback noise control of a magnetic bearing pump," *Noise Control Engineering Journal*, vol. 45, no. 2, pp. 78–84, 1997.
- [4] C. Nataraj, "Design of magnetic bearings for propulsion drive-shafts: a feasibility study," Tech. Rep., Naval Surface Warfare Center, Carderock Division, Annapolis, Md, USA, 1997.
- [5] C. Nataraj, "Dynamic response of rotating machinery on magnetic bearings," Tech. Rep., Naval Surface Warfare Center, Carderock Division, Annapolis, Md, USA, 1998.
- [6] C. Nataraj and T. E. Calvert, "Compensation of base motion in magnetic-bearing supported rotors for Navy applications," in *Proceedings of ASNE Intelligent Ships Symposium*, Philadelphia, Pa, USA, May 2003.
- [7] L. N. Virgin, T. F. Walsh, and J. D. Knight, "Nonlinear behavior of a magnetic bearing system," *Transactions of the ASME: Journal of Engineering for Gas Turbines and Power*, vol. 117, no. 3, pp. 582–588, 1995.
- [8] N. Motee and M. S. de Queiroz, "Control of active magnetic bearings with a smart bias," in *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 1, pp. 860–865, Las Vegas, Nev, USA, December 2002.

- [9] A. M. Mohamed and P. E. Fawzi, "Nonlinear oscillations in magnetic bearing systems," *IEEE Transactions on Automatic Control*, vol. 38, no. 8, pp. 1242–1245, 1993.
- [10] M. Chinta and A. B. Palazzolo, "Stability and bifurcation of rotor motion in a magnetic bearing," *Journal of Sound and Vibration*, vol. 214, no. 5, pp. 793–803, 1998.
- [11] G. Petela and K. K. Botros, "Magnetic bearing control of flexible shaft vibrations based on multi-access velocity-displacement feedback," *Transactions of the ASME, Journal of Engineering for Gas Turbines and Power*, vol. 117, no. 1, pp. 188–197, 1995.
- [12] J. D. Lindlau and C. R. Knopse, "Feedback linearization of an active magnetic bearing with voltage control," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 1, pp. 21–31, 2002.
- [13] C. R. Knopse, R. W. Hope, S. M. Tamer, and S. J. Fedigan, "Robustness of adaptive unbalance control of rotors with magnetic bearings," *Journal of Vibration and Control*, vol. 2, no. 1, pp. 33–52, 1996.
- [14] M. O. T. Cole, P. S. Kneogh, and C. R. Burrows, "Vibration control of a flexible rotor/magnetic bearing system subject to direct forcing and base motion disturbances," *Proceedings of the Institution of Mechanical Engineers—Part C: Journal of Mechanical Engineering Science*, vol. 212, no. 7, pp. 535–546, 1998.
- [15] S. Beale, B. Shafai, P. LaRocca, and E. Cusson, "Adaptive forced balancing for magnetic bearing control systems," in *Proceedings of the 31st Conference on Decision and Control*, pp. 3535–3539, Tucson, Ariz, USA, December 1992.
- [16] S. Beale, B. Shafai, P. LaRocca, and E. Cusson, "Adaptive forced balancing for multivariable systems," *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, vol. 117, no. 4, pp. 496–502, 1995.
- [17] M. Hisatani and T. Koizumi, "Adaptive filtering for unbalanced vibration suppression," in *Proceedings of the 4th International Symposium on Magnetic Bearings (ISMB '94)*, pp. 125–130, Zürich, Switzerland, August 1994.
- [18] T. Mizumo and T. Higuchi, "Design of magnetic bearing controllers based on disturbance estimation," in *Proceedings of the 2nd International Symposium on Magnetic Bearings*, pp. 281–288, Tokyo, Japan, July 1990.
- [19] M. E. Kasarda, J. Clements, A. L. Wicks, C. D. Hall, and R. G. Kirk, "Effect of sinusoidal base motion on a magnetic bearing," in *Proceedings of the IEEE International Conference on Control Applications*, vol. 1, pp. 144–149, Anchorage, Alaska, USA, September 2000.
- [20] W. R. Saunders, H. H. Robertshaw, and R. A. Burdisso, "A hybrid structural control approach for narrow-band and impulsive disturbance rejection," *Noise Control Engineering Journal*, vol. 44, no. 1, pp. 11–21, 1996.
- [21] G. Schweitzer, H. Bleuler, and A. Traxler, *Active Magnetic Bearings*, vdf Hochschulverlag an der ETH, Zürich, Germany, 1994.
- [22] R. F. Stengel, *Optimal Control and Estimation*, Dover, New York, NY, USA, 1994.
- [23] C. Nataraj and S. Marx, "An efficient linear optimal control algorithm for harmonic base excitation of flexible rotor systems," submitted to *Journal of Vibration and Control*.
- [24] G. T. Flowers, G. Szász, V. S. Trent, and M. E. Greene, "A study of integrally augmented state feedback control for an active magnetic bearing supported rotor system," *Transactions of the ASME, Journal of Engineering for Gas Turbines and Power*, vol. 123, no. 2, pp. 377–382, 2001.
- [25] S. Marx, "Compensation for base motion in magnetic bearing rotors," Master's thesis, Villanova University, Villanova, Pa, USA, 2005.
- [26] C. Nataraj and H. D. Nelson, "Periodic oscillations in rotor dynamic systems with nonlinear supports: a general approach," *Transactions of the ASME, Journal of Vibration, Acoustics Stress and Reliability in Design*, vol. 111, pp. 187–193, 1989.