Research Article

Effects of Centrifugal Buoyancy and Reynolds Number on Turbulent Heat Transfer in a Two-Pass Angled-RIB-Roughened Channel with Sharp 180° Turns Investigated by Using Large Eddy Simulation

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The effects of the centrifugal buoyancy and the Reynolds number on heat transfer in a rotating two-pass rib-roughened channel with 180° sharp turns were numerically investigated by using the large eddy simulation. The effect of the Reynolds number was seen in the finer flow structure. The effect of the aiding/opposing buoyancy contributions was seen more vigorously on the pressure surface than that on the suction surface, though the details depended on the Reynolds number, the rotation number, and the existence of the ribs. As the buoyancy increased, the friction factor dominated by the pressure loss of the sharp turn decreased, and the decreasing rate is smaller for the higher rotation speed case. The Colburn's j factor stayed almost constant irrespective of the rotation speed. As a result, the heat transfer efficiency index slightly increased by the buoyancy, and it became smaller for the higher rotation speed and higher Reynolds number cases.

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1. INTRODUCTION

The effective cooling of a gas turbine rotor blade is essential because the higher efficiency of the turbine requires a higher inlet gas temperature. Generally, this blade cooling is performed by film cooling at the external surface of the turbine blade and also by internal forced-convection cooling which uses winding flow passages inside the turbine blade. In the internal forced-convection cooling, the real phenomena are very complicated due to external forces: the Coriolis force and the buoyancy force in the centrifugal acceleration field. In addition to these external forces, the disturbances induced by turbulence promoters (ribs) and 180° sharp turns further complicate the phenomena [1].

As for the heat transfer in smooth and/or rib-roughened channels with the 180° sharp turn, several researchers investigated the detailed spatial variation of the local heat transfer in the stationary condition with various techniques:

wall temperature measurement by using hundreds of thermocouples [2, 3], naphthalene sublimation technique to measure the local mass transfer, which was transformed into heat transfer by using the analogy between heat and mass transfer [4, 5], unsteady wall temperature measurement by using temperature-sensitive liquid crystal [6, 7], and wall temperature measurement by using infrared thermography [8]. In these studies, the characteristic heat transfer variation induced by the ribs and the 180° sharp turn was captured; the high heat transfer areas observed in and after the sharp turn and on and between the ribs. As for the flow field in the stationary condition, Son et al. [9] applied the particle image velocimetry technique to the two-pass channel with the 180° sharp turn, and the detailed two-dimensional flow field was measured. For the rotating condition, however, both the flow velocity and wall temperature measurements become very difficult because of the following two reasons: the high centrifugal force preventing the data acquisition system

FIGURE 1: Schematic of a rotating two-pass rib-roughened square channel with sharp 180° turns (60° rib with NP arrangement).

from normal operation and the difficulty in transferring data from the rotating system to the stationary system. Nevertheless, some researchers have performed experiments in the rotating condition by measuring the wall temperature distribution using thermocouples for the smooth [10–12] and rib-roughened [13–15] wall two-pass channels with the 180° sharp turn. Liou et al. [16] conducted the flow velocity measurement of a rotating two-pass smooth channel with the 180° sharp turn by using the laser Doppler anemometer in addition to the detailed heat transfer measurement by using the transient liquid crystal method. However, it is extremely difficult to perform the experiments in the rotating condition that can identify both the flow structure and its influence on the heat transfer at the same time. Thus, further progress in experimental studies has so far been prevented, and the authors decided to investigate the phenomena by performing numerical analysis.

In previous numerical studies of the serpentine flow passage with the 180° sharp turn, the Reynolds-averaged Navier-Stokes equation with a turbulence model was adopted: Banhoff et al. [17] used the k - ϵ two-equation turbulence model or the Reynolds stress equation model with the wall function, and Lin et al. [18] used the low-Reynolds number two-equation turbulence model without the wall function. Although this approach using the Reynolds-averaged turbulence model could reproduce the heat transfer of blade cooling to a certain extent, even the Reynolds stress equation model has empirical constants and functions, and therefore the applicability of the model should carefully be examined. Recent advancement in computers enables us to numerically simulate the fluctuating components of the turbulent flow by using the large eddy simulation (LES) or the direct numerical simulation (DNS) [19]. Because LES and DNS directly resolve temporal variation of the fluctuating components, the results are more universal, in other words, more free from the empirical modeling than the Reynolds-averaged turbulence models. Although LES also has empirical constants and functions, the modeling of the

turbulence is confined to the subgrid-scale turbulence, and therefore the effect of the empirical modeling on the result is ideally less than that in the Reynolds stress equation model. Recently, the authors performed the numerical analyses by using a dynamic subgrid-scale model for the two-pass smooth [20] and rib-roughened [21] channels with the 180° sharp turn in the stationary and rotating conditions. The effect of the buoyancy force was also examined for the two-pass smooth [22] and rib-roughened [23] channels where the aiding and opposing contributions of the centrifugal buoyancy were clarified. However, the bulk Reynolds number of the authors' previous studies for the rib-roughened duct was confined to the low values among 3000–5000, though for the smooth duct [22], the bulk Reynolds number was increased up to 9851. The higher grid resolution needed for higher Reynolds number demands more computational resource, and it had prevented us from simulating high-Reynolds-number cases.

In this study, the bulk Reynolds number was increased to about 10^4 for the rotating two-pass angled-rib-roughened channel with 180° sharp turns. The extremely heavy computational load was managed to be within the feasible computational time by using the latest supercomputer and the parallel computing technique. How the centrifugal buoyancy force affects the heat transfer for different Reynolds number cases was examined. The computations were performed varying the Rayleigh number for an angled 60° rib arrangement.

2. NUMERICAL ANALYSIS

Figure 1 shows the computational domain and coordinate system used in this study. The duct had a square cross-section with a side length of D . The coordinate system was fixed to a rotating channel that had an angular velocity of ω with respect to the axis of rotation parallel to the x -axis. The axial direction of the channel straight pass was parallel to the z direction; the x and y directions were the parallel and perpendicular directions, respectively, to leading/trailing

walls. Round-type ribs as used in [17, 18] were installed with the in-line arrangement on the trailing and leading walls with the rib angle of 60° with respect to duct axis of the straight pass. The rib arrangement of this study gave $e/D = 0.1$ and $P/e = 10$, which was chosen because it was within the previously reported optimal range for straight ducts considering both the Nusselt number and the friction factor [24–26]. The rib angle, 60° , was chosen as the angle which gave the maximum heat transfer for straight ducts in Han et al. [27]. The 60° rib “NP” arrangement was investigated in this study. Here, “N” and “P” were from “negative” and “positive” considering relation between angled-rib induced and sharp-turn induced secondary flow directions. For example, in the “NP” arrangement the angled-rib induced secondary flow in the first straight pass circulated in the opposite direction to the turn A induced one, and on the other hand the angled-rib induced secondary flow in the second straight pass circulated in the same direction as the turn A induced one.

The present procedure of the numerical analysis was the same as our recent studies [20–23]. After applying a filtering operation to the incompressible Navier-Stokes equation with a filter width equal to the grid spacing, the dimensionless governing equations scaled by a length scale, ℓ ($=0.5D$), and a mean friction velocity, u_* , became a set of dimensionless governing equations with respect to grid resolvable components indicated by overbars as $(\bar{u}, \bar{v}, \bar{w})$ under the assumption of constant fluid properties. In order to simulate a fully developed situation, the pressure and temperature fields were decomposed into the steady, x -directionally linear component and the remaining component [28]. By this decomposition, the latter component of the pressure and temperature fields can be treated using a periodic boundary condition in the x direction.

The temperature was made dimensionless by using a linearly increasing component of temperature, T_{linear} , and a friction temperature, T_r , as $\theta = (T - T_{\text{linear}})/T_r$. Accordingly, the dimensionless energy equation was derived for the grid resolvable component, $\bar{\theta}$. The governing equations in the Cartesian coordinates (x, y, z) were transformed into generalized curvilinear coordinates (ξ, η, ζ) that were aligned to the structured grid coordinate of this study; ζ was the duct-axial direction. The governing equations in the curvilinear coordinate system were expressed as follows [29]:

$$\frac{1}{J} \frac{\partial J \bar{U}^j}{\partial \xi^j} = 0, \quad (1)$$

$$\begin{aligned} \frac{\partial \bar{u}_i}{\partial t} = & -\bar{U}^j \frac{\partial \bar{u}_i}{\partial \xi^j} - \frac{\partial \xi^j}{\partial x_i} \frac{\partial \bar{p}}{\partial \xi^j} + \frac{1}{\text{Re}_*} \frac{\partial \xi^j}{\partial x_\ell} \frac{\partial}{\partial \xi^j} \left(\frac{\partial \xi^k}{\partial x_\ell} \frac{\partial \bar{u}_i}{\partial \xi^k} \right) \\ & + \frac{\partial \xi^k}{\partial x_j} \frac{\partial \tau_{\text{SGS}_{ij}}}{\partial \xi^k} + F_i, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial t} = & -\bar{U}^j \frac{\partial \bar{\theta}}{\partial \xi^j} - C_\theta \frac{\bar{u}}{\bar{u}_m} + \frac{1}{\text{Re}_* \text{Pr}} \frac{\partial \xi^j}{\partial x_\ell} \frac{\partial}{\partial \xi^j} \left(\frac{\partial \xi^k}{\partial x_\ell} \frac{\partial \bar{\theta}}{\partial \xi^k} \right) \\ & + \frac{\partial \xi^k}{\partial x_j} \frac{\partial \alpha_{\text{SGS}_j}}{\partial \xi^k}, \end{aligned} \quad (3)$$

where, $\bar{U}^j$ was a contravariant component of velocity, and the following expressions were assumed: $J = \partial(x, y, z)/\partial(\xi, \eta, \zeta)$, $(x_1, x_2, x_3) = (x, y, z)$, and $(\xi^1, \xi^2, \xi^3) = (\xi, \eta, \zeta)$.

An external force term, F_i , in the momentum equations (2) had the Coriolis force, the centrifugal buoyancy force with the Boussinesq approximation, and the mean pressure gradient term with a value of two as shown in the following equation:

$$F_i = \begin{pmatrix} 2 \\ 2\text{Ro}_* \bar{w} - \frac{\text{Gr}_*}{\text{Re}_*^2} \frac{(\bar{\theta} - \bar{\theta}_\infty) \gamma}{R_m} \\ -2\text{Ro}_* \bar{v} - \frac{\text{Gr}_*}{\text{Re}_*^2} \frac{(\bar{\theta} - \bar{\theta}_\infty) \gamma}{R_m} (H_0 + z) \end{pmatrix}. \quad (4)$$

Because the dimensionless mean-rotation radius, R_m , is much larger than y and z in the real gas turbines, the following approximations can be used: $y/R_m \doteq 0$ and $(H_0 + z)/R_m \doteq 1$. In the dimensionless reference temperature of $\bar{\theta}_\infty = (T_\infty - T_{\text{linear}})/T_r$, T_{linear} changes linearly in the x direction, and therefore $\bar{\theta}_\infty$ also changes linearly in the x direction. From the energy balance, the following equation holds:

$$\bar{\theta}_\infty = -C_\theta \frac{x}{\bar{u}_m}. \quad (5)$$

Consequently (4) was approximated as follows:

$$F_i = \begin{pmatrix} 2 \\ 2\text{Ro}_* \bar{w} \\ -2\text{Ro}_* \bar{v} - \frac{\text{Gr}_*}{\text{Re}_*^2} (\bar{\theta} - \bar{\theta}_\infty) \end{pmatrix}. \quad (6)$$

In this study, the friction velocity, u_* , was calculated from the force balance between the mean pressure gradient in the x direction and the wall shear stress as follows:

$$\rho u_*^2 = \frac{\ell}{2} \left| \frac{dp}{dx} \right|_{m, \text{dim}}. \quad (7)$$

Here, the mean pressure gradient with the subscript, “dim,” meant the value with dimension, and it drove the flow in the x direction. By nondimensionalizing (7), the dimensionless mean pressure gradient value of 2 came out. Because of the constant mean pressure gradient, the flow rate varied depending on the Rayleigh number; therefore, the flow rate was not known a priori, and it was calculated from the resultant computed flow field after the statistical steadiness was attained.

Subgrid-scale components of stress, $\tau_{\text{SGS}_{ij}}$, and energy flux, α_{SGS_j} , were expressed as follows:

$$\begin{aligned} \tau_{\text{SGS}_{ij}} &= 2\nu_{\text{SGS}} \bar{S}_{ij}, \\ \alpha_{\text{SGS}_j} &= \frac{\nu_{\text{SGS}}}{\text{Pr}_{\text{SGS}}} \frac{\partial \xi^k}{\partial x_j} \frac{\partial \bar{\theta}}{\partial \xi^k}, \end{aligned} \quad (8)$$

TABLE 1: Dimensionless number range of this study and the duct-averaged values of f and Nu_m (the values in parentheses in the second line are for higher grid resolution of $101 \times 101 \times 1569$. The values with asterisks are for quasi-DNS).

Rib arrangement	Re_*	Ro_*	Ra_*	Re_m	Ro_m	$Gr_{m,q}$	$\frac{Gr_{m,q}}{Re_m^2}$	$\frac{Gr_{m,\Delta T}}{Re_m^2}$	f/f_B	$Nu_m/Nu_\infty (= j/j_\infty)$
60° NP	3000 (2500)	1	0	11595	1.03	0	0	0	12.79	2.40
			1×10^4	12343	0.97	4.8×10^8	3.15	0.04	11.47	2.37
			2×10^4	12966	0.93	9.6×10^8	5.71	0.07	10.52	2.35
				14189	0.85				8.98	2.29
			4×10^4	(14100)	(0.85)	1.9×10^9	9.54	0.11	(9.08)	(2.38)
				15040*	0.80*				8.11*	2.23*
	4000	2	1×10^5	13199	0.76	4.0×10^9	22.96	0.26	7.08	2.38
			0	9985	3.21	0	0	0	29.55	2.57
			1×10^4	10124	3.16	6.4×10^8	6.24	0.08	28.84	2.59
			2×10^4	10213	3.13	1.3×10^9	12.27	0.16	28.39	2.60
				(10350)	(3.09)				(27.74)	(2.62)
			0	3447	1.16	0	0	0	11.87	2.67
	1000 [23]	1		(3501)	(1.14)				(11.56)	(2.70)
			5×10^3	3900	1.03	8.0×10^7	5.26	0.14	9.57	2.66
			1×10^4	4237	0.94	1.6×10^8	8.91	0.22	8.28	2.69
			2×10^4	4779	0.84	3.2×10^8	14.01	0.32	6.70	2.68
				(4736)	(0.84)				(6.81)	(2.72)
Smooth [22]	3500	2	0	9416	2.97	0	0	0	25.07	2.53
			1×10^4	9545	2.93	5.6×10^8	6.15	0.09	24.47	2.48
			3×10^4	9851	2.84	1.7×10^9	17.52	0.24	23.18	2.55

where,

$$\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \xi^k}{\partial x_j} \frac{\partial \bar{u}_i}{\partial \xi^k} + \frac{\partial \xi^k}{\partial x_i} \frac{\partial \bar{u}_j}{\partial \xi^k} \right), \quad (9)$$

$$\nu_{SGS} = C_S^2 (\Delta_1 \Delta_2 \Delta_3)^{2/3} \sqrt{2 \bar{S}_{ij} \bar{S}_{ij}}.$$

Because the flow field of this study has no homogeneous direction, we adopted the Lagrangian dynamic subgrid-scale model of Meneveau et al. [30] that averaged the value of C_s along the path-line for a certain distance. As for the coefficient of the Lagrangian averaging time scale, the same value of 1.5 as that in [30] was used. The turbulent Prandtl number for the subgrid-scale component, Pr_{SGS} , was set to 0.5 [31]. The width of the test filter was double the grid spacing.

Discretization was performed by a finite difference method using the collocated grid system [29]. The spatial and temporal discretization schemes were similar to those of Gavrilakis [19]: the second-order central differencing method and the Crank-Nicolson method for the viscous term, and the second-order differencing method satisfying the conservative property [29] and the second-order Adams-Bashforth method for the convective term. The external force term was also treated by the second-order Adams-Bashforth method. The pressure field was treated following the MAC method [32]. At the wall boundary, no-slip and constant heat flux conditions were imposed. The boundary conditions of the intermediate velocities and pressure were set following the procedure of [33, 34].

The local Nusselt number, Nu_L , was calculated from the wall temperature as follows:

$$Nu_L = \frac{2Re_* Pr}{\theta_w - \theta_b}. \quad (10)$$

The averaged Nusselt number was calculated by using the integrally averaged temperature difference for the area in question.

In this study, the values of Re_* ($= u_* \ell / \nu$) and Ro_* ($= \omega \ell / u_*$) were varied in $(Re_*, Ro_*) = (3000, 1)$ and $(4000, 2)$, and the Rayleigh number, Ra_* , was varied from 0 to 1×10^5 . For the case of $Ra_* = 1 \times 10^5$, Re_* was reduced from 3000 to 2500 in order to maintain the bulk Reynolds number to be 10^4 . This was needed because the increase in Ra_* reduced the friction factor and, as a result, increased the bulk Reynolds number. Hereafter, this case of $(Re_*, Ro_*) = (2500, 1)$ is also referred to as $(3000, 1)$ in this study for the simplicity. In correlating the experimental results, the effect of the buoyancy is often expressed by using the Grashof number, $Gr_{m,q}$, which is defined with the wall heat flux. The following relation holds due to the definition of $Gr_{m,q}$:

$$Gr_{m,q} = 16Re_* Pr Gr_* = 16Re_* Ra_*. \quad (11)$$

The conversion of the dimensionless numbers of this study (Re_*, Ro_*, Ra_*) defined by the mean friction velocity, the friction temperature, and the length scale of $0.5D$ into those of $(Re_m, Ro_m, Gr_{m,q})$ defined by the bulk mean velocity, the wall heat flux, and the hydraulic diameter, D , was summarized in Table 1. In Table 1, our previous numerical

results for the smooth [22] and rib-roughened [23] two-pass channels are also included for comparison. In the real aircraft gas turbine engines, the central region of the operating range is in the order of $Re_m \sim 10^4$, $Ro_m \sim 10^{-1}$, and $Gr_{m,\Delta T}/Re_m^2 \sim 10^{-1}$ [11]. From the definition of Grashof numbers, the following relation holds between $Gr_{m,q}$ and $Gr_{m,\Delta T}$:

$$Gr_{m,q} = Nu_m Gr_{m,\Delta T}. \quad (12)$$

Note that the computational conditions of this study are within the real operating range. It should also be noted that $Gr_{m,\Delta T}/Re_m^2$ is identical to the buoyancy parameter often used in experiments.

As shown in Figure 1, the structured grid system was generated by using Gridgen version 15 (Pointwise Inc., Tex, USA). The grids in the physical domain were contracted to the walls and the corners. On the rib-roughened surfaces, the grids were aligned to the ribs. The grid number was mainly $71 \times 71 \times 1207$ in ξ, η, ζ directions, and this grid configuration, for example, gave a grid spacing of $\Delta_1^+ = 0.59\text{--}70.0$, $\Delta_2^+ = 0.58\text{--}60.1$, and $\Delta_3^+ = 15.7\text{--}85.9$ ($Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 1 \times 10^5$). Here, the inner length scale of ν/u_{*mod} was used, because the friction velocity, u_* , defined in (7) overestimated the value. In (7), the streamwise direction was assumed to be only in the x direction, and therefore it cannot account for the longer total flow distance caused by the change of the streamwise direction in the two-pass channel. In order to estimate appropriate inner length and time scales, the friction velocity, u_{*mod} , was calculated by using the resultant flow rate and friction factor for each condition. The effect of the grid spacing on the computed result was checked by increasing the grid number to $101 \times 101 \times 1569$ for several cases, and no major difference was observed as shown in Table 1. The time step interval was $\Delta t = 1.0 \times 10^{-4}$, which can be expressed as $\Delta t^+ = 0.046$ when made dimensionless by an inner time scale, ν/u_{*mod}^2 , for $Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 1 \times 10^5$.

In order to deal with the very high computational load of this study, the computational domain was decomposed into 32 subdomains in the duct-axial (ζ) direction, and the parallel computing technique was applied. Each subdomain's computation was performed on a different CPU on HITACHI SR11000J (Information Technology Center, The University of Tokyo). When the algebraic equation for each variable was solved by using the SOR method, the values at the subdomain boundaries were transferred to the neighboring subdomains by using MPI functions in each iteration step.

The computation was started using the result of the similar condition as an initial condition. At first, the calculations were carried out till the statistically steady flow condition was attained. After that, additional 60 000 steps ($t = 6$ or $t^+ = 2760$ for $Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 1 \times 10^5$) were performed for computing the statistical values. This 60 000 step computation needed about 12.1×32 CPU hours for the $71 \times 71 \times 1207$ grid configuration.

3. RESULTS AND DISCUSSION

At first, the verification of the present numerical procedure is explained. In our previous studies of smooth [20] and rib-roughened [21] two-pass channels, the numerical results were compared with the experimental results of the stationary condition in the local and transversely averaged Nusselt numbers and also in the channel averaged f and j factors. The agreement between the numerical and experimental results was good, and the present numerical procedure was able to reproduce the heat transfer enhancement in and after the sharp turn and the quick development of the swatooth-profile heat transfer augmentation due to the periodically installed ribs in the straight pass [20, 21]. In order to examine the subgrid-scale model contribution on the flow and heat transfer, the quasi-DNS was performed for one condition of $Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 4 \times 10^4$. As shown in Table 1, the quasi-DNS gave the lower friction factor and heat transfer than the LES results, and this tendency is the same as the other researcher's result [35]. All the main features of the flow and heat transfer were reproduced by the quasi-DNS, and they confirmed the reliability of the present results.

Figures 2(a) and 2(b) show the time-averaged local Nusselt number (see Figure 2(a)) and the streamwise component of the wall shear stress (see Figure 2(b)) on all four walls by viewing the flow channel from six different directions. The Nusselt number of this study was normalized using the following empirical correlation for a fully developed pipe flow [36]:

$$Nu_\infty = 0.022 Re_m^{0.8} Pr^{0.5}. \quad (13)$$

It should be noted that the lower part of "outer wall" in the figure shows the inner wall values because it is visible through the inlet and outlet of the channel. In Figure 2(b), zero shear stress boundary is indicated by a white line. In the calculation of the streamwise component of the wall share stress, the streamwise direction was approximated by the ζ direction. In this study, the trailing and leading walls of the first straight pass correspond to the pressure and suction surfaces, respectively, and the opposite relation holds in the second straight pass. The pressure and suction surfaces are defined with respect to the secondary flow induced by the Coriolis force, which impinges onto the pressure surface. As a comparison, the result of the lower Reynolds number [23] is shown in Figure 3. When Figures 2 and 3 are compared, the Nusselt numbers of the lower Reynolds number case (Figure 3(a)) are more enhanced than the higher Reynolds number case (Figure 2(a)) on the rib-roughened pressure surfaces and in the turn section, though the approximate profiles are similar to each other. Here, it should be noted that the Nusselt numbers in the figures are normalized by Nu_∞ being the increasing function of the Reynolds number, and therefore the larger Nu_L/Nu_∞ does not necessarily mean the higher Nusselt number. When the instantaneous local Nusselt numbers are examined in Figure 4, the Reynolds number effect is seen in the finer flow structure and, as a result, the finer heat transfer distribution in Figure 4(a) for $Re_* = 3000$ as compared to that in Figure 4(b) for $Re_* = 1000$.

FIGURE 2: Time-averaged profiles of Nusselt number and streamwise component of wall shear stress (60° rib NP, $Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 0$).

FIGURE 3: Time-averaged profiles of Nusselt number and streamwise component of wall shear stress (60° rib NP, $Re_* = 1000$, $Ro_* = 1$, and $Ra_* = 0$).

As discussed in [22, 23], because of the secondary flow induced by the Coriolis force, the fluid temperature on the trailing (pressure) side of the first straight pass becomes lower than that on the leading (suction) side; therefore, on the pressure and suction surfaces of the first straight pass (radially outward flow), the buoyancy works

in the aiding and opposing directions to the main flow, respectively (see Figure 5(a)). On the other hand, in the second straight pass (radially inward flow), the situation is reversed to the first straight pass: on the pressure and suction surfaces of the second straight pass, the buoyancy works in the opposing and aiding directions to the main

FIGURE 4: Instantaneous profiles of Nusselt number (60° rib NP, $Re_* =$ (a) 3000 and (b) 1000, $Ro_* = 1$, and $Ra_* = 0$).

flow, respectively (see Figure 5(b)). In this way, when the buoyancy works, the radial flow direction becomes an important parameter which controls the flow and heat transfer in an orthogonally rotating channel. In the previous study for the lower Reynolds number case [23], when the centrifugal buoyancy was introduced, the effect of the aiding buoyancy contribution was seen in the larger variation of

FIGURE 5: Explanation of aiding and opposing buoyancy contributions in first (radially outward flow) and second (radially inward flow) straight passes.

the streamwise shear stress on the pressure surface of the first straight pass, and in the reduced area of the reverse flow between the ribs on the suction surface of the second straight pass. These changes in the flow field caused the heat transfer on the corresponding surfaces larger. The effect of the opposing buoyancy contribution was seen in the intensified and extended reverse flow region on the suction surface of the first straight pass, but the Nusselt number there showed a slight increase in the downstream part. As seen in Figures 6 and 7 by being compared with Figures 2 and 3, respectively, these effects are observed in the higher Reynolds number case (see Figure 6) but less clear than the lower Reynolds number case (see Figure 7).

Figure 8 shows the higher rotation number case of $Ro_* = 2$ ($Re_* = 4000$). When the rotation speed increases, the heat transfer on the pressure surface is deteriorated especially at the downstream region of the straight pass. This tendency was also observed in the low-Reynolds-number case in [23]. It should be noted that this deterioration of the heat transfer is not characteristic to the fully developed region of an infinitely long straight duct, because the fully developed results of smooth [37] and rib-roughened [38] straight ducts do not show the deterioration of the heat transfer even when the rotation speed is higher. The interaction between the sharp-turn induced and Coriolis-induced flows must be affecting the deterioration of the heat transfer. The buoyancy

FIGURE 6: Time-averaged profiles of Nusselt number and streamwise component of wall shear stress (60° rib NP, $Re_* = 3000$, $Ro_* = 1$, and $Ra_* = 1 \times 10^5$).

FIGURE 7: Time-averaged profiles of Nusselt number and streamwise component of wall shear stress (60° rib NP, $Re_* = 1000$, $Ro_* = 1$, and $Ra_* = 2 \times 10^4$).

effect on the heat transfer and the shear stress was minor for this high rotation speed case (figures not shown).

In order to examine the buoyancy effect in the most developed region in the present straight passes, the area-averaged Nusselt number at the downstream part of the straight pass is shown in Figure 9. The averaging area

between 5th and 6th ribs from the straight pass entrance (between 2nd and 3rd ribs from the downstream turn) was chosen so as for the flow and temperature fields to be most developed and at the same time to be still without the downstream turn effect. In Figure 9, the buoyancy parameter, $Gr_{m,q}/Re_m^2$, is used as the horizontal axis. On

FIGURE 8: Time-averaged profiles of Nusselt number and streamwise component of wall shear stress (60° rib NP, $Re_* = 4000$, $Ro_* = 2$, and $Ra_* = 0$).

the pressure surface (triangle symbols), the buoyancy effect on Nu_{area} is larger than that on the suction surface (square symbols), and Nu_{area} of the aiding contribution cases (solid lines) is larger than those of the opposing contribution cases (broken lines). This tendency is supported by the experimental results of the rib-roughened channel [14],

FIGURE 9: Buoyancy effect on area-averaged Nusselt number at developed region in straight passes (Averaging area was between 5th and 6th ribs from straight pass entrance. Open, filled, half-filled, and concentric symbols are for $(Re_*, Ro_*, \text{smooth/rib}) = (3500, 2, \text{smooth})$, $(1000, 1, \text{rib})$, $(3000, 1, \text{rib})$, and $(4000, 2, \text{rib})$, resp. Symbols with solid and broken lines are for aiding and opposing buoyancy contributions, resp.).

although the entrance condition for the first straight pass in [14] was not disturbed by the upstream sharp turn. The aiding contribution on the pressure surface for the increase of $Gr_{m,q}/Re_m^2$ differs depending on rib existence, Re_* , and Ro_* . As seen in the figure, the smooth case for $Re_* = 3500$ once increases and then decreases as $Gr_{m,q}/Re_m^2$ increases. For the rib-roughened case of $Ro_* = 1$ ($Re_* = 3000$), Nu_{area} on the pressure surface stays almost constant for the increase of $Gr_{m,q}/Re_m^2$. When Ro_* is increased to $Ro_* = 2$ ($Re_* = 4000$), Nu_{area} on the pressure surface becomes lower due to the above-mentioned deterioration of the heat transfer, and the clear buoyancy effect is only seen for the highest buoyancy case for the aiding contribution side. Nu_{area} on the suction surface shows much smaller variation due to $Gr_{m,q}/Re_m^2$, and a slight increase is observed due to the increase of $Gr_{m,q}/Re_m^2$.

In order to view the time-averaged flow structure of the whole two-pass channel, the isosurface (surface with the same value) of the second invariant, Q , of the deformation tensor, $\partial u_i/\partial x_j$, [39] is shown in Figure 10. In the figure, the time-averaged temperature is also shown as the color contour on the isosurface of Q . The value of Q is calculated by (14) for incompressible fluids, and it is often used to identify vortices because the positive value of Q means that the vorticity exceeds the strain,

$$Q = \frac{1}{2} \left\{ \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \right]^2 - \left[\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]^2 \right\}. \quad (14)$$

As seen in Figure 10(a), in and around the turn the strong vortices are produced. On the pressure surfaces, the stronger vortices are shed behind the angled ribs as compared to the suction surfaces. In Figure 10(b), the effect of the buoyancy is seen in the extended and intensified vortex region near the trailing surface which gives the aiding contribution to

FIGURE 10: Time-averaged vortex structure visualized as isosurface of $Q = 10$ with the color contour of fluid temperature (60° rib NP, $Re_* = 3000$, and $Ro_* = 1$).

the fluid motion. The aiding contribution enhanced the turbulent kinetic energy near the trailing surface (figures not shown). The high rotation speed case ($Ro_* = 2$) gave the drastic decay of both vortices and turbulent kinetic energy in the straight pass section (figures not shown).

Figure 11 shows the friction factor, f , normalized by using f_B in the following Blasius equation:

$$f_B = 0.079 Re_m^{-0.25}. \quad (15)$$

The precise values of f/f_B are shown in Table 1. In order to decompose the total pressure loss into the sharp-turn and straight-pass contributions, the following pressure loss coefficients, K_{turn} and $K_{straight}$, are also plotted in

Figure 11 [40]:

$$K_{turn} = \frac{\Delta p_{turn}}{(1/2)\rho U_m^2}, \quad K_{straight} = \frac{\Delta p_{straight}}{(1/2)\rho U_m^2}. \quad (16)$$

Here, Δp_{turn} is the pressure loss associated with the sharp turn, and it is calculated by linearly extrapolating the area-averaged wall-pressure profile at the central region of the first and second straight passes in the direction toward the turn inlet and outlet locations at $z = 17$ for the turn A and at $z = 1$ for the turn B [40]. The area-average was taken with the streamwise pitch of D in the straight pass. The straight pass component, $\Delta p_{straight}$, was calculated from the linear pressure profile at the central region of the straight pass. A decrease of the f factor by the buoyancy force is observed in Figure 11. Without the buoyancy, K_{turn}

FIGURE 11: Buoyancy effect on pressure loss (Legend for the symbols are the same as that in Figure 9. Symbols for K_{turn} and K_{straight} with solid lines are for turn A or first straight pass, and those with broken lines are for turn B or second straight pass).

FIGURE 12: Buoyancy effect on j factor and heat transfer efficiency index (legend for the symbols are the same as that in Figure 9).

is larger than K_{straight} . The buoyancy makes K_{turn} decrease both in the turns A and B with the exception of some high rotation speed cases ($Ro_* = 2$) in which K_{turn} shows a slight increase. The value of K_{straight} in the second straight pass becomes negative, due to the positive pressure gradient in the streamwise direction which is caused by the coincidence of the main flow and buoyancy force directions; the main flow is driven by the buoyancy in the second straight pass (radially inward flow). The buoyancy increases and decreases K_{straight} in the first and second straight passes, respectively, and this opposing variation in K_{straight} cancels each other. As a total, the pressure loss behavior of the channel is controlled by K_{turn} , and the f factor decreases as $Gr_{m,q}/Re_m^2$ increases.

The variation of K_{straight} due to the buoyancy is smaller for the higher Reynolds number case. The large f factor for the high rotation speed ($Ro_* = 2$) is due to the increased K_{turn} .

Figure 12 shows the Colburn's j factor normalized by using j_{∞} calculated from Nu_{∞} in (13). The precise values of j/j_{∞} are shown in Table 1. In order to further examine the heat transfer efficiency taking the pressure loss into account, the heat transfer efficiency index, η_{eff} , was calculated by using the following equation [41]:

$$\eta_{\text{eff}} = \frac{St/St_{\text{smooth,stationary}}}{(f/f_{\text{smooth,stationary}})^{1/3}}. \quad (17)$$

As explained in [41], η_{eff} is the index of the heat conductance for equal pumping power and heat transfer surface area. In this study, the smooth and stationary result of $Re_* = 1000$ [22] was used for calculating (17) for all the cases. As shown in Figure 12, the j factor is insensitive to the buoyancy and stays almost constant. As a result, η_{eff} slightly increases with the increase of the buoyancy due to the decreased f factor seen in Figure 11. η_{eff} is lowered as the Reynolds and rotation numbers increase, and it becomes almost constant with respect to the buoyancy parameter increase for the higher rotation number case ($Ro_* = 2$). This is because the friction factor is less sensitive to the buoyancy for the higher rotation number as seen in Figure 11; in other words, the friction factor is dominated by the pressure loss in the turn section which is increased by the high rotation speed and is insensitive to the buoyancy.

4. CONCLUSIONS

The large eddy simulation of the two-pass angled-rib-roughened square channel with the 180° sharp turns was performed changing the Reynolds, Rayleigh, and rotation numbers for the rib arrangement of the 60° rib NP. From the numerical results, the following conclusions were drawn.

When the Reynolds number was increased, the flow structure became finer which resulted in the finer instantaneous distribution of the local Nusselt number. When the centrifugal buoyancy was introduced, the aiding and opposing contributions of the centrifugal buoyancy worked in the different way depending on the radial flow directions (radially outward and inward flows). The pressure surfaces of the radially outward and inward flows came to the aiding and opposing buoyancy contribution sides, respectively, and the heat transfer was larger in the aiding buoyancy contribution, though the detailed profile differs depending on the Reynolds number, the rotation number, and the rib existence. On the other hand, the suction surfaces of the radially outward and inward flows came to the opposing and aiding buoyancy contribution sides, respectively, and both showed the slightly increased heat transfer.

The friction factor was decreased as the buoyancy increased which was dominated by the sharp-turn induced pressure loss because the increased and decreased pressure losses in the first and second straight passes, respectively, cancelled each other. Especially for the high rotation number case, the sharp-turn induced pressure loss became large and

it made the friction factor very high. On the other hand, the channel averaged Nusselt number was insensitive to the buoyancy, and it was almost constant. The heat transfer efficiency index taking the pressure loss into account gave the lower values for the higher Reynolds and rotation number cases with a slight increase due to the buoyancy.

Here, it is emphasized that the novel findings of this study are in disclosing the flow field, the two-dimensional profile of the local heat transfer coefficient and the local shear stress on the wall, and the friction factor in the heating and rotating conditions. These findings cannot be supplied by the presently available experimental methods.

NOMENCLATURE

$A_{\text{total,ndim}}$	Dimensionless total heat transfer area of channel
c_p	Specific heat, J/(kg K)
C_S	Smagorinsky constant
C_θ	Coefficient of linear component in energy equation ($= A_{\text{total,ndim}}/32$)
D	Hydraulic diameter (=side length of straight-pass cross-section), m
e	Height of rib, m
f	Friction factor ($= \Delta p_{\text{total}} D / (2\rho U_m^2 L)$)
F_i	External force term
Gr_*	Grashof number based on friction temperature ($= \beta T_r \omega^2 R_{m,\text{dim}} \ell^3 / \nu^2$)
$Gr_{m,q}$	Grashof number based on wall heat flux ($= \beta \dot{q} \omega^2 R_{m,\text{dim}} D^4 / (\nu^2 \lambda)$)
$Gr_{m,\Delta T}$	Grashof number based on temperature difference ($= \beta \Delta T \omega^2 R_{m,\text{dim}} D^3 / \nu^2$)
h	Heat transfer coefficient, W/(m ² K)
H_0	Dimensionless distance between rotation axis and x axis ($= H_{0,\text{dim}}/\ell$)
$H_{0,\text{dim}}$	Distance between rotation axis and x axis ($z = 0$), m
j	Colburn's j factor ($= Nu_m / (Re_m Pr^{1/3})$)
K_{straight}	Pressure loss coefficient of straight pass ($= \Delta p_{\text{straight}} / ((1/2)\rho U_m^2)$)
K_{turn}	Pressure loss coefficient of sharp turn ($= \Delta p_{\text{turn}} / ((1/2)\rho U_m^2)$)
ℓ	Length scale ($= 0.5D$), m
L	Total duct-axial length of two-pass channel ($= 22D$), m
Nu	Nusselt number ($= hD/\lambda$)
P	Rib pitch ($= D$), m
$\Delta p_{\text{straight}}$	Pressure loss at straight pass, Pa
Δp_{total}	Pressure loss between channel inlet and outlet, Pa
Δp_{turn}	Pressure loss between turn inlet and outlet, Pa
Pr	Prandtl number ($= 0.71$)
Pr_{SGS}	Prandtl number of subgrid-scale model ($= 0.5$)
$\dot{q}$	Wall heat flux, W/m ²
Q	Second invariant of deformation tensor
R_m	Dimensionless mean rotation radius ($= R_{m,\text{dim}}/\ell$)
$R_{m,\text{dim}}$	Mean rotation radius, m
Ra_*	Rayleigh number ($= Gr_* Pr$)

Re_m	Reynolds number based on bulk mean velocity ($= U_m D / \nu$)
Ro_m	Rotation number based on bulk mean velocity ($= \omega D / U_m$)
Ro_*	Rotation number based on friction velocity ($= \omega \ell / u_*$)
S_{ij}	Rate-of-strain tensor
St	Stanton number ($= Nu_m / (Re_m Pr)$)
t	Dimensionless time
ΔT	Mean temperature difference between wall and fluid ($= (T_w - T_b)_m$), K
T	Temperature, K
T_{linear}	Linearly increasing component of temperature, K
T_r	Friction temperature ($= \dot{q} / (\rho c_p u_*)$), K
u, v, w	Dimensionless velocities in x, y, z directions
u_*	Mean friction velocity calculated from mean pressure gradient in x direction, m/s
$u_{*\text{mod}}$	Mean friction velocity estimated by using bulk mean velocity, m/s
$\bar{u}_m$	Dimensionless bulk mean velocity in x direction calculated at the entrance ($= (1/4) \int_{-1}^1 \bar{u}_x dz$)
U_m	Bulk mean velocity, m/s
x, y, z	Dimensionless Cartesian coordinates
$a_{\text{SGS},j}$	Subgrid-scale energy flux
β	Expansion coefficient, 1/K
$\Delta_1, \Delta_2, \Delta_3$	Grid spacing in ξ, η, ζ directions expressed in (x, y, z) coordinates' scale
η_{eff}	Heat transfer efficiency index ($= (St / St_{\text{smooth,stationary}}) / (f / f_{\text{smooth,stationary}})^{1/3}$)
λ	Thermal conductivity, W/(mK)
ν	Kinematic viscosity, m ² /s
ν_{SGS}	Dimensionless subgrid-scale eddy viscosity
ω	Angular velocity, rad/s
ρ	Density, kg/m ³
θ	Dimensionless temperature ($= (T - T_{\text{linear}}) / T_r$)
$\tau_{\text{SGS},ij}$	Subgrid-scale stress tensor
$\tau_{w,s}$	Streamwise component of wall shear stress, Pa
ξ, η, ζ	Curvilinear coordinates.

SUBSCRIPTS AND SUPERSSCRIPTS

b	Bulk value
B	Blasius
L	Local value
m	Duct average or based on bulk mean velocity
w	Wall
∞	Fully developed
$*$	Friction velocity or defined by using u_*
$+$	Dimensionless value based on inner scales
$-$	Grid resolvable component.

REFERENCES

- [1] J.-C. Han, "Recent studies in turbine blade cooling," in *Proceedings of the 9th International Symposium on Transport Phenomena and Dynamics of Rotating Machinery (ISRO-MAC '02)*, pp. 1–6, Honolulu, Hawaii, USA, February 2002, Invited Lecture #5.

- [2] A. Murata, S. Mochizuki, and M. Fukunaga, "Detailed measurement of local heat transfer in a square-cross-section duct with a sharp 180-deg turn," in *Proceedings of the 10th International Heat Transfer Conference*, vol. 4, pp. 291–296, Brighton, UK, August 1994.
- [3] S. Mochizuki, A. Murata, and M. Fukunaga, "Effects of rib arrangements on pressure drop and heat transfer in a rib-roughened channel with a sharp 180 deg turn," *Journal of Turbomachinery*, vol. 119, no. 3, pp. 610–616, 1997.
- [4] P. R. Chandra, J.-C. Han, and S. C. Lau, "Effect of rib angle on local heat/mass transfer distribution in a two-pass rib-roughened channel," *Journal of Turbomachinery*, vol. 110, no. 2, pp. 233–241, 1988.
- [5] M. Hirota, H. Fujita, A. Syuhada, S. Araki, T. Yoshida, and T. Tanaka, "Heat/mass transfer characteristics in two-pass smooth channels with a sharp 180-deg turn," *International Journal of Heat and Mass Transfer*, vol. 42, no. 20, pp. 3757–3770, 1999.
- [6] S. V. Ekkad and J.-C. Han, "Detailed heat transfer distributions in two-pass square channels with rib turbulators," *International Journal of Heat and Mass Transfer*, vol. 40, no. 11, pp. 2525–2537, 1997.
- [7] T. M. Liou, C. C. Chen, and T. W. Tsai, "Liquid crystal measurements of heat transfer in a 180° sharp turning duct with different divider thicknesses," in *Proceedings of the 8th International Symposium on Flow Visualization (ISFV '98)*, pp. 1–9, Sorrento, Italy, September 1998.
- [8] T. Astarita and G. Cardone, "Thermofluidynamic analysis of the flow in a sharp 180° turn channel," *Experimental Thermal and Fluid Science*, vol. 20, no. 3–4, pp. 188–200, 2000.
- [9] S. Y. Son, K. D. Kihm, and J.-C. Han, "PIV flow measurements for heat transfer characterization in two-pass square channels with smooth and 90° ribbed walls," *International Journal of Heat and Mass Transfer*, vol. 45, no. 24, pp. 4809–4822, 2002.
- [10] J.-C. Han, Y.-M. Zhang, and K. Kalkuehler, "Uneven wall temperature effect on local heat transfer in a rotating two-pass square channel with smooth walls," *Journal of Heat Transfer*, vol. 115, no. 4, pp. 912–920, 1993.
- [11] J. H. Wagner, B. V. Johnson, and F. C. Kopper, "Heat transfer in rotating serpentine passages with smooth walls," *Journal of Turbomachinery*, vol. 113, no. 3, pp. 321–330, 1991.
- [12] S. Mochizuki, J. Takamura, S. Yamawaki, and W.-J. Yang, "Heat transfer in serpentine flow passages with rotation," *Journal of Turbomachinery*, vol. 116, no. 1, pp. 133–140, 1994.
- [13] J. A. Parsons, J.-C. Han, and Y.-M. Zhang, "Effect of model orientation and wall heating condition on local heat transfer in a rotating two-pass square channel with rib turbulators," *International Journal of Heat and Mass Transfer*, vol. 38, no. 7, pp. 1151–1159, 1995.
- [14] B. V. Johnson, J. H. Wagner, G. D. Steuber, and F. C. Yeh, "Heat transfer in rotating serpentine passages with trips skewed to the flow," *Journal of Turbomachinery*, vol. 116, no. 1, pp. 113–123, 1994.
- [15] S. Mochizuki, M. Beier, A. Murata, T. Okamura, and Y. Hashidate, "Detailed measurement of convective heat transfer in rotating two-pass rib-roughened coolant channels," ASME paper no. 96-TA-6, 1996.
- [16] T.-M. Liou, C.-C. Chen, and M.-Y. Chen, "TLCT and LDV measurements of heat transfer and fluid flow in a rotating sharp turning duct," *International Journal of Heat and Mass Transfer*, vol. 44, no. 9, pp. 1777–1787, 2001.
- [17] B. Bonhoff, U. Tömm, B. V. Johnson, and I. Jennions, "Heat transfer predictions for rotating U-shaped coolant channels with skewed ribs and with smooth walls," in *Proceedings of ASME International Gas Turbine and Aeroengine Congress and Exhibition*, Orlando, Fla, USA, June 1997, ASME paper no. 97-GT-162.
- [18] Y.-L. Lin, T. I.-P. Shih, M. A. Stephens, and M. K. Chyu, "A numerical study of flow and heat transfer in a smooth and ribbed U-duct with and without rotation," *Journal of Heat Transfer*, vol. 123, no. 2, pp. 219–232, 2001.
- [19] S. Gavrilakis, "Numerical simulation of low-Reynolds-number turbulent flow through a straight square duct," *Journal of Fluid Mechanics*, vol. 244, pp. 101–129, 1992.
- [20] A. Murata and S. Mochizuki, "Large eddy simulation of turbulent heat transfer in a rotating two-pass smooth square channel with sharp 180° turns," *International Journal of Heat and Mass Transfer*, vol. 47, no. 4, pp. 683–698, 2004.
- [21] A. Murata and S. Mochizuki, "Effect of rib orientation and channel rotation on turbulent heat transfer in a two-pass square channel with sharp 180° turns investigated by using large eddy simulation," *International Journal of Heat and Mass Transfer*, vol. 47, no. 12–13, pp. 2599–2618, 2004.
- [22] A. Murata and S. Mochizuki, "Centrifugal buoyancy effect on turbulent heat transfer in a rotating two-pass smooth square channel with sharp 180-deg turns," *International Journal of Heat and Mass Transfer*, vol. 47, no. 14–16, pp. 3215–3231, 2004.
- [23] A. Murata and S. Mochizuki, "Aiding and opposing contributions of centrifugal buoyancy on turbulent heat transfer in a two-pass transverse- or angled-rib-roughened channel with sharp 180° turns," *International Journal of Heat and Mass Transfer*, vol. 47, no. 17–18, pp. 3721–3743, 2004.
- [24] M. E. Taslim and S. D. Spring, "Effects of turbulator profile and spacing on heat transfer and friction in a channel," *Journal of Thermophysics and Heat Transfer*, vol. 8, no. 3, pp. 555–562, 1994.
- [25] G. J. Korotky and M. E. Taslim, "Rib heat transfer coefficient measurements in a rib-roughened square passage," ASME paper no. 96-GT-356, 1996.
- [26] M. E. Taslim, T. Li, and D. M. Kercher, "Experimental heat transfer and friction in channels roughened with angled, V-shaped, and discrete ribs on two opposite walls," *Journal of Turbomachinery*, vol. 118, no. 1, pp. 20–28, 1996.
- [27] J. C. Han, J. S. Park, and C. K. Lei, "Heat transfer enhancement in channels with turbulence promoters," *Journal of Engineering for Gas Turbines and Power*, vol. 107, no. 3, pp. 628–635, 1985.
- [28] S. V. Patankar, C. H. Liu, and E. M. Sparrow, "Fully developed flow and heat transfer in ducts having streamwise-periodic variations of cross-sectional area," *Journal of Heat Transfer*, vol. 99, no. 2, pp. 180–186, 1977.
- [29] T. Kajishima, T. Ohta, K. Okazaki, and Y. Miyake, "High-order finite-difference method for incompressible flows using collocated grid system," *JSME International Journal, Series B*, vol. 41, no. 4, pp. 830–839, 1998.
- [30] C. Meneveau, T. S. Lund, and W. H. Cabot, "A Lagrangian dynamic subgrid-scale model of turbulence," *Journal of Fluid Mechanics*, vol. 319, pp. 353–385, 1996.
- [31] P. Moin, K. Squires, W. Cabot, and S. Lee, "A dynamic subgrid-scale model for compressible turbulence and scalar transport," *Physics of Fluids A*, vol. 3, no. 11, pp. 2746–2757, 1991.
- [32] F. H. Harlow and J. E. Welch, "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface," *Physics of Fluids*, vol. 8, no. 12, pp. 2182–2189, 1965.

- [33] J. Kim and P. Moin, "Application of a fractional-step method to incompressible Navier-Stokes equations," *Journal of Computational Physics*, vol. 59, no. 2, pp. 308–323, 1985.
- [34] Y. Zang, R. L. Street, and J. R. Koseff, "A non-staggered grid, fractional step method for time-dependent incompressible Navier-Stokes equations in curvilinear coordinates," *Journal of Computational Physics*, vol. 114, no. 1, pp. 18–33, 1994.
- [35] D. K. Tafti, "Evaluating the role of subgrid stress modeling in a ribbed duct for the internal cooling of turbine blades," *International Journal of Heat and Fluid Flow*, vol. 26, no. 1, pp. 92–104, 2005.
- [36] W. M. Kays and M. E. Crawford, *Convective Heat and Mass Transfer*, McGraw-Hill, New York, NY, USA, 3rd edition, 1993.
- [37] A. Murata and S. Mochizuki, "Effect of cross-sectional aspect ratio on turbulent heat transfer in an orthogonally rotating rectangular smooth duct," *International Journal of Heat and Mass Transfer*, vol. 42, no. 20, pp. 3803–3814, 1999.
- [38] A. Murata and S. Mochizuki, "Effect of cross-sectional aspect ratio on turbulent heat transfer in an orthogonally rotating rectangular duct with angled rib turbulators," *International Journal of Heat and Mass Transfer*, vol. 46, no. 16, pp. 3119–3133, 2003.
- [39] J. C. R. Hunt, A. A. Wray, and P. Moin, "Eddies, stream, and convergence zones in turbulent flows," Research Report CTR-S88, Center for Turbulence Research, Stanford, Calif, USA, 1988.
- [40] D. E. Metzger, C. W. Plevich, and C. S. Fan, "Pressure loss through sharp 180 deg turns in smooth rectangular channels," *Journal of Engineering for Gas Turbines and Power*, vol. 106, no. 3, pp. 677–681, 1984.
- [41] D. L. Gee and R. L. Webb, "Forced convection heat transfer in helically rib-roughened tubes," *International Journal of Heat and Mass Transfer*, vol. 23, no. 8, pp. 1127–1136, 1980.