WELL POSEDNESS FOR EVOLUTION INCLUSIONS

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(Received March, 1993; revised May, 1994)

ABSTRACT

We prove the existence of a continuous selection of the multivalued map $\varphi \rightarrow \Phi(\varphi)$ which is the set of all mild solutions of the evolution inclusion

$$\dot{x}(t) \in Ax(t) + F(t, x(t)) + \int_0^t h(t-s)g(x(s))ds$$

$$x(0) = \varphi.$$

Here F is a multivalued map, Lipschitzian with respect to x , and A is the infinitesimal generator of a C_0 -semigroup.

Key words: Well-posedness, Evolution Inclusion, Mild Solution.

AMS (MOS) subject classifications: 34A12, 34A60, 45J05, 54C65.

1. Introduction

According to a generally accepted terminology in the theory of differential equations, an initial value problem, whose solutions exist, are unique and depend continuously on the initial data, is called *well posed*. For problems lacking uniqueness, (i.e., those for which any solution through a point, can be embedded in a continuous, single valued family of solutions depending on the initial point) can be considered as the natural extension of the well posedness.

The existence of a continuous map $\varphi \rightarrow x_\varphi$, such that x_φ is a solution of the Cauchy problem

$$\dot{x} \in F(t, x), \quad x(0) = \varphi$$

where F is a nonempty set-valued function, Lipschitzian with respect to x , was proved first by Cellina in [4]. Then, the same problem for a differential inclusion with Lipschitzian right-hand side defined on an open set, was studied by several authors [3, 4, 6, 7]. Well posedness for a differential inclusion on closed sets was proved in [5]. A continuous function $f: X \rightarrow Y$ is said to be a *continuous selection* of a multivalued map $F: X \rightarrow 2^Y$ if $f(x) \in F(x)$ for all $x \in X$. Continuous selections exist due to continuous selection theorems. A detailed study of continuous selection theorems is given in [2]. The existence of a continuous selection of the set-valued

$$\dot{x}(t) \in Ax(t) + F(t, x(t)), \quad x(0) = \varphi,$$

function $\varphi \rightarrow \Phi(\varphi)$, where $\Phi(\varphi)$ is the set of all mild solutions of the Cauchy problem is established in [7, 9]. Here F is Lipschitzian with respect to x and A is the infinitesimal generator of a C_0 -semigroup.

In this paper, we consider the evolution inclusion of the form

$$\begin{aligned} \dot{x}(x) &\in Ax(t) + F(t, x(t)) + \int_0^t h(t-s)g(x(s))ds, \\ x(0) &= \varphi, \end{aligned} \tag{1}$$

where F is a set-valued function and $\varphi \in X$. We prove the existence of a continuous selection of the set-valued function $\varphi \rightarrow \Phi(\varphi)$, where $\Phi(\varphi)$ is the set of all mild solutions of the Cauchy problem (1), assuming that F is Lipschitzian with respect to x and A is the infinitesimal generator of a C_0 -semigroup. This work was motivated by the existence of unique mild solution of the evolution integrodifferential equation studied by Ahmed [1].

2. Preliminaries

Let $T > 0$, $I = [0, T]$ and denote $\mathcal{L}$ the σ -algebra of all Lebesgue measurable subsets of I . Let X be a real separable Banach space with the norm $\|\cdot\|$. Denote by $\mathfrak{B}(X)$ the family of all Borel subsets of X . For any subset $A \subset X$ and $x \in X$, we set $d(x, A) = \inf\{\|x - y\| : y \in A\}$. Furthermore, for two closed bounded nonempty subsets A and B of X , we denote by $h(A, B)$ the Hausdorff distance from A to B , that is, $h(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}$.

Denote by $C(I, X)$ the Banach space of all continuous functions $x: I \rightarrow X$ endowed with the norm $\|x\|_\infty = \sup\{\|x(t)\| : t \in I\}$ and by $\mathcal{L}^1(I, X)$, the Banach space of all Bochner integrable functions $x: I \rightarrow X$ with norm $\|x\|_1 = \int_0^T \|x(t)\| dt$. A subset K of $\mathcal{L}^1(I, X)$ is called *decomposable* if for every u and v in K and $A \in \mathcal{L}$ we have $u\chi_A + v\chi_{I-A} \in K$, where χ_A stands for the characteristic function of A . Denote by $\mathfrak{D}$ the family of all closed nonempty decomposable subsets of $\mathcal{L}^1(I, X)$.

Let X be a separable Banach space and $\{G(t): t \geq 0\} \subset \mathcal{L}(X, X)$ be a strongly continuous semigroup of bounded linear operators from X to X having infinitesimal generator A . Consider the Cauchy problem (1) where $\varphi \in X$ and $F: I \times X \rightarrow 2^X$ is a set-valued function satisfying the following hypotheses:

- (i) F is the $\mathcal{L} \otimes \mathfrak{B}(X)$ measurable,
- (ii) there exists $k \in \mathcal{L}^1(I, R)$ such that $h(F(t, x), F(t, y)) \leq k(t)\|x - y\|$, for all $x, y \in X$ a.e. in I ,
- (iii) there exists $\mu \in \mathcal{L}^1(I, R)$ such that $d(0, F(t, 0)) \leq \mu(t)$, $t \in I$ a.e.,
- (iv) $g: X \rightarrow X$ be a continuous function and there exists a constant $C > 0$ such that $\|g(x)\| \leq C(1 + \|x\|)$ and $\|g(x) - g(y)\| \leq C\|x - y\|$, for all $x, y \in X$,
- (v) $h \in \mathcal{L}^1(I, R)$ and there exists a constant $H \in \mathcal{L}^1(I, R)$ such that for each pair $s, t \in I$ with $s < t$, $\int_s^t |h(\tau)| d\tau \leq \int_0^T |h(\tau)| d\tau = H$.

Definition 1: A function $x(\cdot, \varphi): I \rightarrow X$ is called a *mild solution* of the Cauchy problem (1) if there exists $f(\cdot, \varphi) \in \mathcal{L}^1(I, X)$ such that

- (i) $f(t, \varphi) \in F(t, x(t, \varphi))$ for almost all $t \in I$; and

$$(ii) \quad x(t, \varphi) = G(t)\varphi + \int_0^t G(t-\tau)f(\tau, \varphi)d\tau + \int_0^t G(t-\tau)\left(\int_0^\tau h(\tau-s)g(x(s, \varphi))ds\right)d\tau$$

for each $t \in I$.

We denote by $\Phi(\varphi)$ the set of all mild solutions of (1).

Let S be a separable metric space. A set-valued function $G: S \rightarrow 2^X$ is called *lower semicontinuous* if the set $\{s \in S: G(s) \subset C\}$ is closed in S for any closed $C \subset X$. The following two lemmas are used in the sequel.

Lemma 1:[7] *Let $F: I \times S \rightarrow 2^X$ be $L \otimes \mathfrak{B}(S)$ measurable and lower semicontinuous in s . Then, the function $s \rightarrow G_F(s)$ given by*

$$G_F(s) = \{v \in \mathcal{L}^1(I, X): v(t) \in F(t, s) \text{ a.e. in } I\}$$

is lower semicontinuous from S into $\mathfrak{D}$ if and only if there exists a continuous function $\mu: S \rightarrow \mathcal{L}^1(I, R)$ such that for every $s \in S$, $\mu(s)(t) \leq d(0, F(t, s))$ a.e. in I .

Lemma 2: [7] *Consider a lower semicontinuous function $G: S \rightarrow \mathfrak{D}$ and assume that $p: S \rightarrow \mathcal{L}^1(I, X)$ and $q: S \rightarrow \mathcal{L}^1(I, R)$ are continuous functions and for every $s \in S$ the set*

$$H(s) = cl\{u \in G(s): \|u(t) - p(s)(t)\| \leq q(s)(t) \text{ a.e. in } I\}$$

is nonempty. Then the function $H: S \rightarrow \mathfrak{D}$ is lower semicontinuous, so it has a continuous selection.

3. Well Posedness

Theorem 1: *Let A be the infinitesimal generator of a C_0 -semigroup $\{G(t): t \geq 0\}$ and let the hypotheses (i)-(v) be satisfied. Then, there exists a function $x(\cdot, \cdot): I \times X \rightarrow X$ such that*

- (i) $x(\cdot, \varphi) \in \Phi(\varphi)$ for every X ; and
- (ii) $\varphi \rightarrow x(\cdot, \varphi)$ is continuous from X into $C(I, X)$.

Proof: Let $M = \sup\{\|G(t)\|: t \in I\}$, and for $\varphi \in X$ defined $x_0(\cdot, \varphi): I \rightarrow X$ by $x_0(t, \varphi) = G(t)\varphi$. Clearly $\varphi \rightarrow x_0(\cdot, \varphi)$ is continuous from X to $C(I, X)$. For each $\varphi \in X$, let $\alpha(\varphi): I \rightarrow R$ be given by

$$\alpha(\varphi)(t) = \mu(t) + k(t) \|x_0(t, \varphi)\|.$$

Clearly, $\alpha(\cdot)$ is continuous from X to $\mathcal{L}^1(I, R)$. Moreover, for each $\varphi \in X$,

$$d(0, F(t, x_0(t, \varphi))) \leq \beta(t) + k(t) \|x_0(t, \varphi)\| = \alpha(\varphi)(t).$$

Let $\epsilon > 0$ be fixed, and for $n \in N$, set $\epsilon_n = \epsilon/2^{n+1}$. Now define

$$G_0: X \rightarrow 2^{\mathcal{L}^1(I, X)} \text{ and } H_0: X \rightarrow 2^{\mathcal{L}^1(I, X)} \text{ by}$$

$$G_0(\varphi) = \{v \in \mathcal{L}^1(I, X): v(t) \in F(t, x_0(t, \varphi)) \text{ a.e. } t \in I\} \text{ and}$$

$$H_0(\varphi) = cl\{v \in G_0(\varphi): \|v(t)\| \leq \alpha(\varphi)(t) + \epsilon_0 \text{ a.e. } t \in I\}$$

Clearly by Lemma 1, $G_0(\cdot)$ is lower semicontinuous from X into $\mathfrak{D}$ and $H_0(\varphi) \neq \emptyset$ for each $\varphi \in X$, and, hence by Lemma 2, there exists a continuous function $h_0: X \rightarrow \mathcal{L}^1(I, X)$, which is a

continuous selection of $H_0(\cdot)$.

For each $\varphi \in X$, let $\beta(\varphi): I \rightarrow \mathbb{R}$ be given by $\beta(\varphi) = HC(1 + \|x_0(t, \varphi)\|)$. Clearly, $\beta(\cdot)$ is continuous from X to $\mathcal{L}^1(I, \mathbb{R})$. Let $K(t) = k(t) + CH$ and set $m(t) = \int_0^t K(\tau) d\tau$. For $n \geq 1$, define $\Gamma_n: X \rightarrow \mathcal{L}^1(I, \mathbb{R})$ by

$$\begin{aligned} \Gamma_n(\varphi)(t) &= M^n \int_0^t \alpha(\varphi(s)) \frac{[m(t) - m(s)]^{n-1}}{(n-1)!} ds \\ &\quad + M^n \int_0^t \beta(\varphi)(s) \frac{[m(t) - m(s)]^{n-1}}{(n-1)!} ds \\ &\quad + M^n t \left(\sum_{i=0}^n \epsilon_i \right) \frac{[m(t)]^{n-1}}{(n-1)!}, \quad t \in I. \end{aligned}$$

Set $f_0(t, \varphi) = h_0(\varphi)(t)$ and $g_0(x(t, \varphi)) = g(x_0(t, \varphi))$. Since $x_0(\cdot, \varphi)$ is continuous, $g_0(x(\cdot, \varphi))$ is also continuous. Define

$$x_1(t, \varphi) = G(t)\varphi + \int_0^t G(t-\tau) f_0(\tau, \varphi) d\tau + \int_0^t G(t-\tau) \left(\int_0^\tau h(\tau-s) g_0(x(s, \varphi)) ds \right) d\tau, \quad t \in I.$$

Then, $f_0(t, \varphi) \in F(t, x_0(t, \varphi))$, $\|f_0(t, s)\| \leq \alpha(s)(t) + \epsilon_0$, and $\|g_0(x(t, \varphi))\| = \|g(x_0(t, \varphi))\| \leq C(1 + \|x_0(t, \varphi)\|)$. For $t \in I \setminus \{0\}$ and by Fubini's theorem,

$$\begin{aligned} \|x_1(t, \varphi) - x_0(t, \varphi)\| &\leq \int_0^t \|G(t-\tau)\| \|f_0(\tau, \varphi)\| d\tau \\ &\quad + \int_0^t \left(\int_0^\tau \|G(t-\tau)\| \|h(\tau-s)\| g_0(x(s, \varphi)) ds \right) d\tau \\ &\leq M \int_0^t \|f_0(\tau, \varphi)\| d\tau + M \int_0^t H \|g(x_0(\tau, \varphi))\| d\tau \\ &\leq M \int_0^t (\alpha(\varphi)(\tau) + \epsilon_0) d\tau + M \int_0^t HC(1 + \|x_0(\tau, \varphi)\|) d\tau \\ &\leq M \int_0^t \alpha(\varphi)(\tau) d\tau + MT\epsilon_0 + M \int_0^t \beta(\varphi)(\tau) d\tau < \Gamma_1(\varphi)(t). \end{aligned}$$

We claim that there exist three sequence $\{f_n(\cdot, \varphi)\}_{n \in \mathbb{N}}$, $\{x_n(\cdot, \varphi)\}_{n \in \mathbb{N}}$ and $\{g_n(x(\cdot, \varphi))\}_{n \in \mathbb{N}}$ such that for each $n \geq 1$ the following conditions are satisfied.

- (a) $\varphi \mapsto f_n(\cdot, \varphi)$ is continuous from X into $\mathcal{L}^1(I, X)$,
- (b) $f_n(t, \varphi) \in F(t, x_n(t, \varphi))$ for each $\varphi \in X$ and a.e. $t \in I$,
- (c) $\|f_n(t, \varphi) - f_{n-1}(t, \varphi)\| \leq K(t)\Gamma_n(\varphi)(t)$,
- (d) $g_i(x(\cdot, \varphi)) = g(x_i(\cdot, \varphi))$ and $g_n(x(\cdot, \varphi))$ is continuous from X into $C(X, X)$ for $i = 1, 2, \dots$,
- (e) $\|g_n(x(t, \varphi)) - g_{n-1}(x(t, \varphi))\| \leq K(t)\Gamma_n(\varphi)(t)$,

$$(f) \quad x_{n+1}(t, \varphi) = G(t)\varphi + \int_0^t G(t-\tau)\delta f_n(\tau, \varphi)d\tau + \int_0^t G(t-\tau)\left(\int_0^\tau h(\tau-s) \cdot g_n(x(s, \varphi))ds\right)d\tau.$$

Suppose that we already constructed $f_1, f_2, \dots, f_n$, $x_1, x_2, \dots, x_n$ and $g_1, g_2, \dots, g_n$ satisfying (a)-(e). Define $x_{n+1}(\cdot, \varphi): I \rightarrow X$ by

$$x_{n+1}(t, \varphi) = G(t)\varphi + \int_0^t G(t-\tau)f_n(\tau, \varphi)d\tau + \int_0^t G(t-\tau)\left(\int_0^\tau h(\tau-s)g_n(x(s, \varphi))ds\right)d\tau.$$

Then, for $t \in I \setminus \{0\}$, we have that

$$\begin{aligned} \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\| &\leq \int_0^t \|G(t-u)\| \|f_n(u, \varphi) - f_{n-1}(u, \varphi)\| du \\ &+ \int_0^t \|G(t-u)\| \left(\int_0^u |h(u-s)| \|g_n(x(s, \varphi)) - g_{n-1}(x(s, \varphi))\| ds\right) du \\ &\leq M \int_0^t k(u) \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \\ &+ M \int_0^t CH \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \end{aligned}$$

(by Fubini's theorem and making use of (iv) and (v))

$$\begin{aligned} &\leq M \int_0^t (k(u) + CH) \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \\ &\leq M \int_0^t K(u)\Gamma_n(\varphi)(u)du. \end{aligned}$$

By making use of calculations provided in [2] we get

$$\begin{aligned} \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\| &\leq M^{n+1} \int_0^t \alpha(s)(\tau) \frac{[m(t) - m(\tau)]^n}{n!} d\tau \\ &+ M^{n+1} T \left(\sum_{i=0}^n \epsilon_i\right) \frac{[m(t)]^n}{n!} + M^{n+1} \int_0^t \beta(s)(\tau) \frac{[m(t) - m(\tau)]^n}{n!} d\tau \\ &\leq \Gamma_{n+1}(\varphi)(t) \end{aligned}$$

and $d(f_n(t, \varphi), F(t, x_{n+1}(t, \varphi))) \leq k(t) \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\|$

$$\begin{aligned} &\leq K(t) \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\| \\ &\leq K(t)\Gamma_{n+1}(\varphi)(t). \end{aligned}$$

Define $G_{n+1}: X \rightarrow 2^{\mathcal{L}(I, X)}$ and $H_{n+1}: X \rightarrow 2^{\mathcal{L}(I, X)}$ by

$$G_{n+1} = \{v \in \mathcal{L}^1(I, X): v(t) \in F(t, x_{n+1}(t, \varphi)) \text{ a.e. in } I\} \text{ and}$$

$$H_{n+1} = cl\{v \in G_{n+1}(\varphi): \|v(t) - f_n(t, \varphi)\| \leq K(t)\Gamma_{n+1}(\varphi)(t) \text{ a.e. in } I\}.$$

Again, by Lemma 1, $G_{n+1}(\cdot)$ is lower semicontinuous from S into $\mathfrak{D}$ and $H_{n+1}(\varphi)$ is not empty for each $\varphi \in X$. Hence, by Lemma 2 there exists $h_{n+1}: X \rightarrow \mathcal{L}^1(I, X)$ as a continuous selection of $H_{n+1}(\cdot)$. Then, $f_{n+1}(t, \varphi) = h_{n+1}(\varphi)(t)$ satisfies the conditions (a)-(c). Also

$$\begin{aligned} \|g_{n+1}(x(t, \varphi)) - g_n(x(t, \varphi))\| &= \|g(x_{n+1}(t, \varphi)) - g(x_n(t, \varphi))\| \\ &\leq C \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\| \\ &\leq K(t) \|x_{n+1}(t, \varphi) - x_n(t, \varphi)\| \\ &\leq K(t)\Gamma_{n+1}(\varphi)(t). \end{aligned}$$

Therefore, g_{n+1} satisfies (e) and (f) and since $x_n(\cdot, \varphi)$ is continuous, $g_{n+1}(t, \varphi)$ is continuous.

Now,

$$\begin{aligned} \|f_n(\cdot, \varphi) - f_{n-1}(\cdot, \varphi)\|_1 &= \int_0^T \|f_n(u, \varphi) - f_{n-1}(u, \varphi)\| du \\ &\leq \int_0^T k(u) \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \\ &\leq \int_0^T K(u)\Gamma_n(\varphi)(u) du \\ &\leq M^n \int_0^\tau \alpha(\varphi)(u) \frac{[m(T) - m(u)]^n}{n!} du \\ &\quad + M^n T \left(\sum_{i=0}^n \epsilon_i \right) \frac{[m(t)]^n}{n!} \\ &\quad + M^n \int_0^T \beta(\varphi)(u) \frac{[m(T) - m(u)]^n}{n!} du \\ &\leq \frac{[M \|K_1\|]^n}{n!} [\|\alpha(\varphi)\|_1 + \|\beta(\varphi)\|_1 + T\epsilon]. \end{aligned}$$

Since $\varphi \mapsto \|\alpha(\varphi)\|_1$ and $\varphi \mapsto \|\beta(\varphi)\|_1$ are continuous it is locally bounded. Therefore, $\{f_n(\cdot, \varphi)\}_{n \in N}$ is a Cauchy sequence in $\mathcal{L}^1(I, X)$. If $f(\cdot, \varphi) \in \mathcal{L}^1(I, X)$ is the limit of $\{f_n(\cdot, \varphi)\}_{n \in N}$, then $\varphi \mapsto f(\cdot, \varphi)$ is continuous from X into $\mathcal{L}^1(I, X)$.

Similarly,

$$\begin{aligned} \|g_n(x(\cdot, \varphi)) - g_{n-1}(x(\cdot, \varphi))\|_1 &\leq \int_0^T C \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \\ &\leq \int_0^T K(u)\Gamma_n(\varphi)(u) du \end{aligned}$$

$$\leq \frac{[M \|K_1\|]^n}{n!} (\|\alpha(\varphi)\|_1 + \|\beta(\varphi)\|_1 + T\epsilon)$$

and so, as previously, $\{g_n(x(\cdot, \varphi))\}_{n \in N}$ is a Cauchy sequence in $C(X, X)$. If $g(x(\cdot, \varphi)) \in C(X, X)$ is its limit, then $x(\cdot, \varphi) \rightarrow g(x(\cdot, \varphi))$ is continuous from X into $C(X, X)$.

On the other hand,

$$\begin{aligned} \|x_{n+1}(\cdot, \varphi) - x_n(\cdot, \varphi)\|_\infty &\leq \int_0^T M \|f_n(u, \varphi) - f_{n-1}(u, \varphi)\| du \\ &\quad + MH \int_0^T \|g_n(x(u, \varphi)) - g_{n-1}(x(u, \varphi))\| du \\ &\leq M \|f_n(u, \varphi) - f_{n-1}(u, \varphi)\|_1 + MH \|g_n(x(u, \varphi)) - g_{n-1}(x(u, \varphi))\|_1 \\ &\leq M \int_0^T [K(u) + CH] \|x_n(u, \varphi) - x_{n-1}(u, \varphi)\| du \\ &\leq \frac{[M \|K_1\|]^n}{n!} [M \|\alpha(\varphi)\|_1 + M \|\beta(\varphi)\|_1 + MT\epsilon]. \end{aligned}$$

Hence, $\{x_n(\cdot, \varphi)\}_{n \in N}$ is a Cauchy sequence in $C(I, X)$. If $x(\cdot, \varphi) \in C(I, X)$ is its limit then it follows that $\varphi \rightarrow x(\cdot, \varphi)$ is continuous from X into $C(I, X)$.

Since $x_n(\cdot, \varphi)$ converges to $x(\cdot, \varphi)$ uniformly, and $d(F_n(t, \varphi), F(t, x(t, \varphi))) \leq K(t) \|x_n(t, \varphi) - x(t, \varphi)\|$, the limit of a subsequence $\{f_{n_k}\}_{k \in N}$ of $\{f_n\}_{n \in N}$ converges pointwise to f , so we obtain $f(t, \varphi) \in F(t, x(t, \varphi))$ for $\varphi \in X$ and $t \in I$ a.e.

Furthermore, $g_n(x(\cdot, \varphi))$ converges to $g(x(\cdot, \varphi))$ uniformly, passing the limit in the condition (f) we obtain

$$\begin{aligned} x(t, \varphi) &= G(t)\varphi + \int_0^t G(t-\tau)f(\tau, \varphi)d\tau \\ &\quad + \int_0^t G(t-\tau) \left(\int_0^\tau h(\tau-s)g(x(s, \varphi))ds \right) d\tau \text{ for each } t \in I. \end{aligned}$$

Therefore, $x(\cdot, \varphi) \in \Phi(\varphi)$ for every $\varphi \in X$ and the proof is complete.

Acknowledgement

The authors are thankful to Professor Jewgeni H. Dshalalow for the improvement of the paper.

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