

Research Article

Certain Transformation Formulae for Polybasic Hypergeometric Series

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Making use of Bailey's transformation and certain known summations of truncated series, an attempt has been made to establish transformation formulae involving polybasic hypergeometric series.

1. Introduction

The remarkable contribution in the field of hypergeometric and basic hypergeometric series mainly due to Bailey [1] has appeared in Proceeding of London Mathematical society in 1947. The key result of the paper later on recognized as Bailey's transformation is as follows:

$$\begin{aligned} \text{if } \beta_n &= \sum_{r=0}^n \alpha_r u_{n-r} v_{n+r}, \\ \gamma_n &= \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{n+r}, \\ \text{then } \sum_{n=0}^{\infty} \alpha_n \gamma_n &= \sum_{n=0}^{\infty} \beta_n \delta_n, \end{aligned} \tag{1.1}$$

where $\alpha_r, \delta_r, u_r, v_r$ are functions of r only, such that the series for γ_n exists. Bailey's paper [2] published in the London Mathematical society in 1949, that strengthened the importance of Bailey's transformation. The main result of the paper [2] was recognized as Bailey's lemma during the 20th century. Making use of celebrated transformation, Bailey [1, 2] developed a number of transformations for both ordinary and basic hypergeometric series, and later on

he successfully used these transformations to obtain a number of identities of the Rogers-Ramanujan type. The extensive use of Bailey transformation appeared in the papers of Slater [3, 4] and these papers were published in 1951 and 1952, respectively. Slater established 130 identities of the Rogers-Ramanujan type in [3, 4]. The platform provided by Bailey and Slater motivated a number of mathematicians namely Agarwal [5, 6], Andrews [7–9], Andrews and Warner [10], Bressoud et al. [11, 12], Denis et al. [13], Joshi and Vyas [14], Schilling and Warnaar [15], Singh [16], Srivastava [17], Verma and Jain [18, 19] and due to the contribution of these mathematicians, literatures of ordinary and basic hypergeometric series were enriched. In the present paper, making use of certain known summations of truncated series, an attempt has been made to establish transformation formulae involving poly-basic hypergeometric series.

2. Definitions and Notations

For real or complex q ($|q| < 1$), put

$$(\lambda; q)_\infty = \prod_{n=0}^{\infty} (1 - \lambda q^n). \quad (2.1)$$

Let $(\lambda; q)_\mu$ be defined by

$$(\lambda; q)_\mu = \frac{(\lambda; q)_\infty}{(\lambda q^\mu; q)_\infty}. \quad (2.2)$$

For arbitrary parameters λ and μ , so that

$$(\lambda; q)_n = \begin{cases} 1, & n = 0, \\ (1 - \lambda)(1 - \lambda q) \cdots (1 - \lambda q^{n-1}), & n \in \{1, 2, 3, \dots\}, \end{cases} \quad (2.3)$$

the generalized basic hypergeometric series is defined by:

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q; z \\ b_1, b_2, \dots, b_s \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_s; q)_n}, \quad (2.4)$$

where $(a_1, a_2, \dots, a_r; q)_n = (a_1; q)_n (a_2; q)_n \cdots (a_r; q)_n$ and $\max(|q|, |z| < 1)$ for convergence.

The truncated basic hypergeometric series is defined by

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q; z \\ b_1, b_2, \dots, b_s \end{matrix} \right]_N = \sum_{n=0}^N \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_s; q)_n}. \quad (2.5)$$

The polybasic hypergeometric series is defined by (cf. Gasper and Rahman [20, (3.9.1) page 85]):

$$\begin{aligned} & \Phi \left[\begin{matrix} a_1, a_2, \dots, a_r : c_{1,1}, \dots, c_{1,r_1}; \dots; c_{m,1}, \dots, c_{m,r_m}; q, q_1, \dots, q_m; z \\ b_1, b_2, \dots, b_{r-1} : d_{1,1}, \dots, d_{1,r_1}; \dots; d_{m,1}, \dots, d_{m,r_m} \end{matrix} \right] \\ &= \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_{r-1}; q)_n} \prod_{j=1}^m \frac{(c_{j,1}, \dots, c_{j,r_j}; q_j)_n}{(d_{j,1}, \dots, d_{j,r_j}; q_j)_n}, \end{aligned} \quad (2.6)$$

where $\max(|z|, |q|, |q_1|, \dots, |q_m|) < 1$ for convergence.

The other notations appearing in this paper have their usual meaning. We will use the following summation formulae in our analysis:

$${}_2\phi_1 \left[\begin{matrix} a, y; & q, q \\ ayq; & \end{matrix} \right]_n = \frac{(aq, yq; q)_n}{(q, ayq; q)_n}, \quad (2.7)$$

see [5, App.II(8)]

$${}_4\phi_3 \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e; & q, \frac{1}{e} \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{e}; & \end{matrix} \right]_n = \frac{(\alpha q, eq; q)_n}{(q, \alpha q/e; q)_n e^n}, \quad (2.8)$$

see [5, App.II(8)]

$${}_6\phi_5 \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; & q, q \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta}; & \end{matrix} \right]_n = \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_n}{(q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_n}, \quad (2.9)$$

see [5, App.II(25)] provided $\alpha = \beta\gamma\delta$,

$$\sum_{r=0}^n \frac{(1-ap^r q^r)(a;p)_r (c;q)_r c^{-r}}{(1-a)(q;q)_r (ap/c;p)_r} = \frac{(ap;p)_n (cq;q)_n}{(q;q)_n (ap/c;p)_n c^n}, \quad (2.10)$$

see [20, App.II(II.34)]

$$\sum_{r=0}^n \frac{(1-ap^r q^r)(1-bp^r q^{-r})(a,b;p)_r (c, a/bc; q)_r q^r}{(1-a)(1-b)(q, aq/b; q)_r (ap/c, bcp;p)_r} = \frac{(ap, bp;p)_n (cq, aq/bc; q)_n}{(q, aq/b; q)_n (ap/c, bcp;p)_n}, \quad (2.11)$$

see [20, App.II(II.35)]

$$\begin{aligned}
& \sum_{r=0}^n \frac{(1-adp^r q^r)(1-bp^r/dq^r)(a,b;p)_r(c,ad^2/bc;q)_r q^r}{(1-ad)(1-b/d)(dq,adq/b;q)_r(adp/c,bcp/d;p)_r} \\
& = \frac{(1-a)(1-b)(1-c)(1-ad^2/bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \\
& \quad \times \left\{ \frac{(ap,bp;p)_n(cq,ad^2q/bc;q)_n}{(dq,adq/b;q)_n(adp/c,bcp/d;p)_n} - \frac{(c/ad,d/bc;p)_1(1/d,b/ad;q)_1}{(1/c,bc/ad^2;q)_1(1/a,1/b;p)_1} \right\}, \tag{2.12}
\end{aligned}$$

which is $m = 0$, case of [20, App. II (II. 36)].

3. Main Results

In this section we have established the following main results.

$$\begin{aligned}
\Phi \left[\begin{matrix} \alpha q, \beta q : a, y; & \\ \alpha \beta q : p, ayp; & q, p; p \end{matrix} \right] &= \frac{[ap, yp; p]_\infty}{[p, ayp; p]_\infty} \frac{[\alpha q, \beta q; q]_\infty}{[q, \alpha \beta q; q]_\infty} \\
&\quad - \frac{q(1-\alpha)(1-\beta)}{(1-q)(1-\alpha \beta q)} \Phi \left[\begin{matrix} ap, yp : \alpha q, \beta q; & \\ ayp : q^2, \alpha \beta q^2; & p, q; q \end{matrix} \right], \tag{3.1}
\end{aligned}$$

$$\begin{aligned}
\Phi \left[\begin{matrix} \alpha q, eq : a, y; & \\ \frac{\alpha q}{e} : p, ayp; & q, p; \frac{p}{e} \end{matrix} \right] &= -\frac{(1-\alpha q^2)(1-e)}{e(1-q)(1-\alpha q/e)} \\
&\quad \times \Phi \left[\begin{matrix} ap, yp : \alpha q, q^2\sqrt{\alpha}, -q^2\sqrt{\alpha}, eq; & \\ ayp : q^2, q\sqrt{\alpha}, -q\sqrt{\alpha}, \frac{\alpha q^2}{e}; & p, q; \frac{1}{e} \end{matrix} \right], \tag{3.2}
\end{aligned}$$

$$\begin{aligned}
\Phi \left[\begin{matrix} \alpha q, \beta q, \gamma q, \delta q : a, y; & \\ \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} : p, ayp; & q, p; p \end{matrix} \right] \\
&= \frac{[ap, yp; p]_\infty [\alpha q, \beta q, \gamma q, \delta q; q]_\infty}{[p, ayp; p]_\infty [q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q]_\infty} \\
&\quad - \frac{(1-q^2\alpha)(1-\beta)(1-\gamma)(1-\delta)q}{(1-q)(1-\alpha q/\beta)(1-\alpha q/\gamma)(1-\alpha q/\delta)} \\
&\quad \times \Phi \left[\begin{matrix} ap, yp : \alpha q, q^2\sqrt{\alpha}, -q^2\sqrt{\alpha}, \beta q, \gamma q, \delta q; & \\ ayp : q^2, q\sqrt{\alpha}, -q\sqrt{\alpha}, \frac{\alpha q^2}{\beta}, \frac{\alpha q^2}{\gamma}, \frac{\alpha q^2}{\delta}; & p, q; q \end{matrix} \right], \tag{3.3}
\end{aligned}$$

$$\begin{aligned} & \Phi \left[\begin{matrix} x, y : ap : cp; \\ xyP : \frac{ap}{c} : q; \end{matrix} \quad P, p, q; \frac{P}{c} \right] \\ &= \frac{(1 - apq)(1 - c)}{(1 - q)(1 - ap/c)c} \end{aligned} \tag{3.4}$$

$$\begin{aligned} & \times \Phi \left[\begin{matrix} xP, yP : ap : cq : ap^2q^2; \\ xyP : \frac{ap^2}{c} : q^2 : apq; \end{matrix} \quad P, p, q, pq; \frac{1}{c} \right], \\ & \Phi \left[\begin{matrix} x, y : ap, bp : cq, \frac{aq}{bc}; \\ xyP : \frac{ap}{c}, bcp : q, \frac{aq}{b}; \end{matrix} \quad P, p, q; P \right] \\ &= \frac{[xP, yP; P]_\infty [ap, bp; p]_\infty [cq, aq/bc; q]_\infty}{[P, xyP; P]_\infty [q, aq/b; q]_\infty [ap/c, bcp; p]_\infty} \\ & - \frac{(1 - apq)(1 - bp/q)(1 - c)(1 - a/bc)q}{(1 - q)(1 - aq/b)(1 - ap/c)(1 - bcp)} \end{aligned} \tag{3.5}$$

$$\begin{aligned} & \times \Phi \left[\begin{matrix} xP, yP : ap^2q^2 : \frac{bp^2}{q^2} : ap, bp : cq, \frac{aq}{bc}; \\ xyP : apq : \frac{bp}{q} : \frac{ap^2}{c}, bcp^2 : q^2, \frac{aq^2}{b}; \end{matrix} \quad P, pq, \frac{p}{q}, p, q; q \right], \\ & \Phi \left[\begin{matrix} x, y : ap, bp : cq, \frac{ad^2q}{bc}; \\ xyP : \frac{adp}{c}, \frac{bcp}{d} : dq, \frac{adq}{b}; \end{matrix} \quad P, p, q; P \right] \\ &= \frac{[xP, yP; P]_\infty [ap, bp; p]_\infty [cq, ad^2q/bc; q]_\infty}{[P, xyP; P]_\infty [dq, adq/b; q]_\infty [adp/c, bcp/d; p]_\infty} \\ & - \frac{dq(1 - adpq)(1 - bp/dq)(1 - c/d)(1 - ad/bc)}{(1 - dq)(1 - adq/b)(1 - adp/c)(1 - bcp/d)} \end{aligned} \tag{3.6}$$

$$\times \Phi \left[\begin{matrix} xP, yP : adp^2q^2 : \frac{bp^2}{dq^2} : ap, bp : cq, \frac{ad^2q}{bc}; \\ xyP : adpq : \frac{bp}{dq} : \frac{adp^2}{c}, \frac{bcp^2}{d} : dq^2, \frac{adq^2}{b}; \end{matrix} \quad P, pq, \frac{p}{q}, p, q; q \right].$$

4. Proof of Main Results

Taking $u_r = v_r = 1$ in (1.1), Bailey's transformation takes the following form:

$$\text{If } \beta_n = \sum_{r=0}^n \alpha_r, \quad (4.1)$$

$$\gamma_n = \sum_{r=0}^{\infty} \delta_r, \quad (4.2)$$

$$\text{then } \sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n. \quad (4.3)$$

Proof of Result (3.1). Taking $\alpha_r = (\alpha, \beta; q)_r q^r / (q, \alpha\beta q; q)_r$ and $\delta_r = (a, y; p)_r p^r / (p, ayp; p)_r$ in (4.1) and (4.2), respectively, and making use of (2.7), we get

$$\beta_n = \frac{(\alpha q, \beta q; q)_n}{(q, \alpha\beta q; q)_n}, \quad \gamma_n = \frac{(ap, yp; p)_\infty}{(p, ayp; p)_\infty} - \frac{(1 - ay)(1 - p^n)(a, y; p)_n}{(1 - a)(1 - y)(p, ay; p)_n}. \quad (4.4)$$

Putting these values in (4.3), we get the following transformation:

$$\begin{aligned} & \Phi \left[\begin{matrix} \alpha q, \beta q : a, y; \\ \alpha\beta q : p, ayp; \end{matrix} \middle| q, p; p \right] + \frac{(1 - ay)}{(1 - a)(1 - y)} \Phi \left[\begin{matrix} \alpha, \beta : a, y; \\ \alpha\beta q : p, ay; \end{matrix} \middle| q, p; q \right] \\ &= \frac{(ap, yp; p)_\infty}{(p, ayp; p)_\infty} \frac{(\alpha q, \beta q; q)_\infty}{(q, \alpha\beta q; q)_\infty} + \frac{(1 - ay)}{(1 - a)(1 - y)} \Phi \left[\begin{matrix} \alpha, \beta : a, y; \\ \alpha\beta q : p, ay; \end{matrix} \middle| q, p; pq \right], \end{aligned} \quad (4.5)$$

which on simplification gives the result (3.1). \square

Proof of Result (3.2). Taking $\alpha_r = (\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e; q)_r / (q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/e; q)_r e^r$ and $\delta_r = (a, y; p)_r p^r / (p, ayp; p)_r$ in (4.1) and (4.2), respectively, and making use of (2.8) and (2.7), we get

$$\beta_n = \frac{(\alpha q, eq; q)_n}{(q, \alpha q/e; q)_n e^n}, \quad \gamma_n = \frac{(ap, yp; p)_\infty}{(p, ayp; p)_\infty} - \frac{(1 - ay)(1 - p^n)(a, y; p)_n}{(1 - a)(1 - y)(p, ay; p)_n}. \quad (4.6)$$

Substituting these values in (4.3), we get the following transformation for $|e| > 1$:

$$\begin{aligned} & \Phi \left[\begin{matrix} \alpha q, eq : a, y; \\ \frac{\alpha q}{e} : p, ayp; \end{matrix} \middle| q, p; \frac{p}{e} \right] = \frac{(1 - ay)}{(1 - a)(1 - e)} \times \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e : a, y; \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{e} : p, ay; \end{matrix} \middle| q, p; \frac{p}{e} \right] \\ & \quad - \frac{(1 - ay)}{(1 - a)(1 - y)} \Phi \left[\begin{matrix} \alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, e : a, y; \\ \sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{e} : p, ay; \end{matrix} \middle| q, p; \frac{1}{e} \right], \end{aligned} \quad (4.7)$$

which on simplification gives result (3.2). \square

Proof of Result (3.3). Taking $\alpha_r = (\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta; q)_r q^r / (q, \sqrt{\alpha}, -\sqrt{\alpha}, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_r$, where $\alpha = \beta\gamma\delta$ and $\delta_r = (a, y; p)_r p^r / (p, ayp; p)_r$ in (4.1) and (4.2), respectively, and making use of (2.9) and (2.7), we get

$$\begin{aligned}\beta_n &= \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_n}{(q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_n}, \\ \gamma_n &= \frac{(ap, yp; p)_\infty}{(p, ayp; p)_\infty} - \frac{(1 - ay)(1 - p^n)(a, y; p)_n}{(1 - a)(1 - y)(p, ay; p)_n}.\end{aligned}\quad (4.8)$$

Substituting these values in (4.3), we get the following transformation for $\alpha = \beta\gamma\delta$:

$$\begin{aligned}&\Phi \left[\frac{\alpha q, \beta q, \gamma q, \delta q : a, y}{\beta, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} : p, ayp; \quad q, p; p} \right] + \frac{(1 - ay)}{(1 - a)(1 - y)} \times \Phi \left[\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta : a, y}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} : p, ay; \quad q, p; q} \right] \\ &= \frac{(ap, yp; p)_\infty}{(p, ayp; p)_\infty} \times \frac{(\alpha q, \beta q, \gamma q, \delta q; q)_\infty}{(q, \alpha q/\beta, \alpha q/\gamma, \alpha q/\delta; q)_\infty} + \frac{(1 - ay)}{(1 - a)(1 - y)} \\ &\quad \times \Phi \left[\frac{\alpha, q\sqrt{\alpha}, -q\sqrt{\alpha}, \beta, \gamma, \delta : a, y}{\sqrt{\alpha}, -\sqrt{\alpha}, \frac{\alpha q}{\beta}, \frac{\alpha q}{\gamma}, \frac{\alpha q}{\delta} : p, ay; \quad q, p; pq} \right],\end{aligned}\quad (4.9)$$

which on simplification gives result (3.3). \square

Proof of Result (3.4). Taking $\alpha_r = (apq; pq)_r (a; p)_r (c; q)_r c^{-r} / ((a; pq)_r (q; q)_r (ap/c; p)_r)$ and $\delta_r = (x, y; P)_r P^r / (P, xyP; P)_r$ in (4.1) and (4.2), respectively and making use of (2.10) and (2.7), we get

$$\beta_n = \frac{(ap; p)_n (cq; q)_n c^{-n}}{(q; q)_n (ap/c; p)_n}, \quad \gamma_n = \frac{(xP, yP; P)_\infty}{(P, xyP; P)_\infty} - \frac{(1 - xy)(1 - P^n)(x, y; P)_n}{(1 - x)(1 - y)(P, xy; P)_n}. \quad (4.10)$$

Putting these values in (4.3), we get the following transformation for $|c| > 1$:

$$\begin{aligned}&\Phi \left[\frac{x, y : ap : cq; \quad P, p, q; \frac{P}{c}}{xyP : \frac{ap}{c} : q; \quad P, p, q; \frac{P}{c}} \right] = \frac{(1 - xy)}{(1 - x)(1 - y)} \times \Phi \left[\frac{x, y : apq : a : c; \quad P, pq, p, q; \frac{P}{c}}{xy : a : \frac{ap}{c} : q; \quad P, pq, p, q; \frac{P}{c}} \right] \\ &\quad - \frac{(1 - xy)}{(1 - x)(1 - y)} \times \Phi \left[\frac{x, y : apq : a : c; \quad P, pq, p, q; \frac{1}{c}}{xy : a : \frac{ap}{c} : q; \quad P, pq, p, q; \frac{1}{c}} \right],\end{aligned}\quad (4.11)$$

which on simplification gives result (3.4). \square

Proof of Result (3.5). Taking $\alpha_r = (apq; pq)_r (bp/q; p/q)_r (a, b; p)_r (c, a/bc; q)_r q^r / ((a; pq)_r (b; p/q)_r (q, aq/b; q)_r (ap/c, bcp; p)_r)$ and $\delta_r = (x, y; P)_r P^r / (P, xyP; P)_r$ in (4.1) and (4.2), respectively, and making use of (2.11) and (2.7), we get

$$\beta_n = \frac{(ap, bp; p)_n (cq, aq/bc; q)_n}{(q, aq/b; q)_n (ap/c, bcp; p)_n}, \quad \gamma_n = \frac{(xP, yP; P)_\infty}{(P, xyP; P)_\infty} - \frac{(1 - xy)(1 - P^n)(x, y; P)_n}{(1 - x)(1 - y)(P, xy; P)_n}. \quad (4.12)$$

Putting these values in (4.3), we get the following transformation:

$$\begin{aligned} & \Phi \left[\begin{matrix} x, y : ap, bp : cq, \frac{aq}{bc}; \\ xyP : \frac{ap}{c}, bcp : q, \frac{aq}{b}; \end{matrix} \quad P, p, q; P \right] + \frac{(1 - xy)}{(1 - x)(1 - y)} \\ & \times \Phi \left[\begin{matrix} x, y : apq : \frac{bp}{q} : a, b : c, \frac{a}{bc}; \\ xy : a : b : \frac{ap}{c}, bcp : q, \frac{aq}{b}; \end{matrix} \quad P, pq, \frac{p}{q}, p, q; q \right] \\ & = \frac{(xP, yP; P)_\infty}{(P, xyP; P)_\infty} \frac{(ap, bp; p)_\infty}{(q, aq/b; q)_\infty} \frac{(cq, aq/bc; q)_\infty}{(ap/c, bcp; p)_\infty} + \frac{(1 - xy)}{(1 - x)(1 - y)} \\ & \times \Phi \left[\begin{matrix} x, y : apq : \frac{bp}{q} : a, b : c, \frac{a}{bc}; \\ xy : a : b : \frac{ap}{c}, bcp : q, \frac{aq}{b}; \end{matrix} \quad P, pq, \frac{p}{q}, p, q; Pq \right], \end{aligned} \quad (4.13)$$

which on simplification gives result (3.5). \square

Proof of Result (3.6). Taking $\alpha_r = (adpq; pq)_r (bp/dq; p/q)_r (a, b; p)_r (c, ad^2/bc; q)_r q^r / ((ad; pq)_r (b/d; p/q)_r (dq, adq/b; q)_r (adp/c, bcp/d; p)_r)$ and $\delta_r = (x, y; P)_r P^r / (P, xyP; P)_r$ in (4.1) and (4.2), respectively, and making use of (2.12) and (2.7), we get

$$\begin{aligned} \beta_n &= \frac{(1 - a)(1 - b)(1 - c)(1 - ad^2/bc)}{d(1 - ad)(1 - b/d)(1 - c/d)(1 - ad/bc)} \\ &\times \left\{ \frac{(ap, bp; p)_n (cq, ad^2q/bc; q)_n}{(dq, adq/b; q)_n (adp/c, bcp/d; p)_n} - \frac{(b - ad)(c - ad)(d - bc)(1 - d)}{d(1 - a)(1 - b)(1 - c)(bc - ad^2)} \right\}, \quad (4.14) \\ \gamma_n &= \frac{(xP, yP; P)_\infty}{(P, xyP; P)_\infty} - \frac{(1 - xy)(1 - P^n)(x, y; P)_n}{(1 - x)(1 - y)(P, xy; P)_n}. \end{aligned}$$

Putting these values in (4.3), we get the following transformation:

$$\begin{aligned}
& \frac{(1-a)(1-b)(1-c)(1-ad^2/bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \times \Phi \left[\begin{array}{l} x, y : ap, bp : cq, \frac{ad^2q}{bc}; \\ xyP : \frac{adp}{c}, \frac{bcp}{d} : dq, \frac{adq}{b}; \end{array} P, p, q; P \right] \\
& + \frac{(1-xy)}{(1-x)(1-y)} \times \Phi \left[\begin{array}{l} x, y : adpq : \frac{bp}{dq} : a, b : c, \frac{ad^2}{bc}; \\ xy : ad : \frac{b}{d} : \frac{adp}{c}, \frac{bcp}{d} : dq, \frac{adq}{b}; \end{array} P, pq, \frac{p}{q}, p, q; q \right] \\
& = \frac{(1-a)(1-b)(1-c)(1-ad^2/bc)}{d(1-ad)(1-b/d)(1-c/d)(1-ad/bc)} \\
& \times \frac{(xP, yP; P)_\infty (ap, bp; p)_\infty (cq, ad^2q/bc; q)_\infty}{(P, xyP; P)_\infty (dq, adq/b; q)_\infty (adp/c, bcp/d; p)_\infty} \\
& + \frac{(1-xy)}{(1-x)(1-y)} \times \Phi \left[\begin{array}{l} x, y : adpq : \frac{bp}{dq} : a, b : c, \frac{ad^2}{bc}; \\ xy : ad : \frac{b}{d} : \frac{adp}{c}, \frac{bcp}{d} : dq, \frac{adq}{b}; \end{array} P, pq, \frac{p}{q}, p, q; Pq \right], \tag{4.15}
\end{aligned}$$

which on simplification gives result (3.6). \square

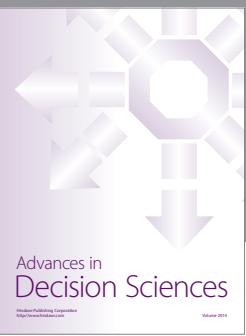
References

- [1] W. N. Bailey, "Some identities in combinatory analysis," *Proceedings of the London Mathematical Society*, vol. 49, no. 2, pp. 421–425, 1947.
- [2] W. N. Bailey, "Identities of the Rogers-Ramanujan type," *Proceedings of the London Mathematical Society*, vol. 50, pp. 1–10, 1948.
- [3] L. J. Slater, "A new proof of Rogers's transformations of infinite series," *Proceedings of the London Mathematical Society*, vol. 53, pp. 460–475, 1951.
- [4] L. J. Slater, "Further identities of the Rogers-Ramanujan type," *Proceedings of the London Mathematical Society*, vol. 54, pp. 147–167, 1952.
- [5] R. P. Agarwal, "Generalized hypergeometric series and its applications to the theory of combinatorial analysis and partition theory," unpublished monograph.
- [6] R. P. Agarwal, *Resonance Of Ramanujan's Mathematics, Vol .I*, New Age International, New Delhi, India, 1996.
- [7] G. E. Andrews, "A general theory of identities of the Rogers-Ramanujan type," *Bulletin of the American Mathematical Society*, vol. 80, pp. 1033–1052, 1974.
- [8] G. E. Andrews, "An analytic generalization of the Rogers-Ramanujan identities for odd moduli," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 71, pp. 4082–4085, 1974.
- [9] G. E. Andrews, "Bailey's transform, lemma, chains and tree," in *Special Functions 2000: Current Perspective and Future Directions*, Tempe, AZ, vol. 30 of *NATO Science Series. Series II, Mathematics, Physics and Chemistry*, pp. 1–22, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2001.
- [10] G. E. Andrews and S. O. Warnaar, "The Bailey transform and false theta functions," *The Ramanujan Journal*, vol. 14, no. 1, pp. 173–188, 2007.

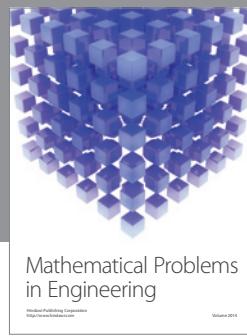
- [11] D. M. Bressoud, "Some identities for terminating q -series," *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 89, no. 2, pp. 211–223, 1981.
- [12] D. M. Bressoud, M. E. H. Ismail, and D. Stanton, "Change of base in Bailey pairs," *The Ramanujan Journal*, vol. 4, no. 4, pp. 435–453, 2000.
- [13] R. Y. Denis, S. N. Singh, and S. P. Singh, "On certain transformation formulae for abnormal q -series," *South East Asian Journal of Mathematics and Mathematical Sciences*, vol. 1, no. 3, pp. 7–19, 2003.
- [14] C. M. Joshi and Y. Vyas, "Extensions of Bailey's transform and applications," *International Journal of Mathematics and Mathematical Sciences*, no. 12, pp. 1909–1923, 2005.
- [15] A. Schilling and S. O. Warnaar, "A higher-level Bailey lemma," *International Journal of Modern Physics B*, vol. 11, no. 1-2, pp. 189–195, 1997.
- [16] U. B. Singh, "A note on a transformation of Bailey," *The Quarterly Journal of Mathematics*, vol. 45, no. 177, pp. 111–116, 1994.
- [17] P. Srivastava, "A note on Bailey's transform," *South East Asian Journal of Mathematics and Mathematical Sciences*, vol. 2, no. 2, pp. 9–14, 2004.
- [18] A. Verma and V. K. Jain, "Transformations between basic hypergeometric series on different bases and identities of Rogers-Ramanujan type," *Journal of Mathematical Analysis and Applications*, vol. 76, no. 1, pp. 230–269, 1980.
- [19] A. Verma and V. K. Jain, "Transformations of nonterminating basic hypergeometric series, their contour integrals and applications to Rogers-Ramanujan identities," *Journal of Mathematical Analysis and Applications*, vol. 87, no. 1, pp. 9–44, 1982.
- [20] G. Gasper and M. Rahman, *Basic Hypergeometric Series*, Encyclopedia of Mathematics and Its Applications, Cambridge University Press, New York, NY, USA, 1991.



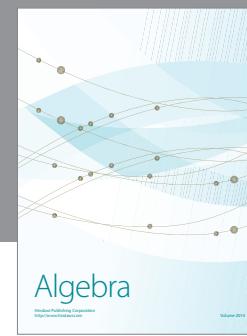
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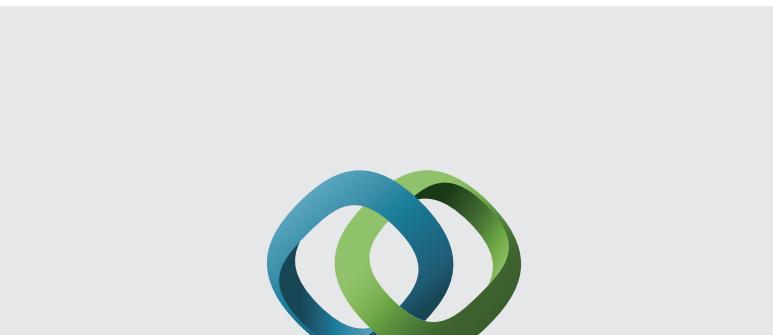
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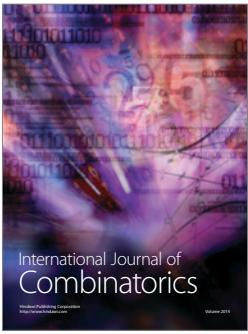


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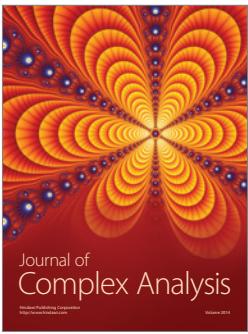
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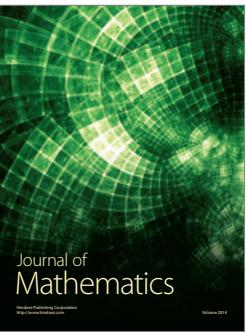
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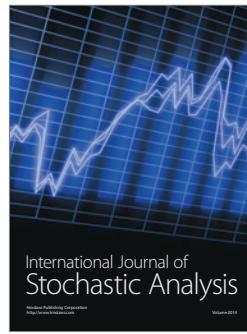
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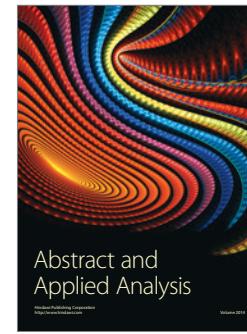
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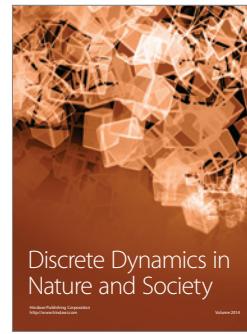
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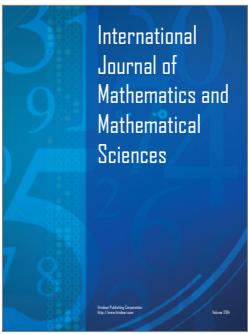
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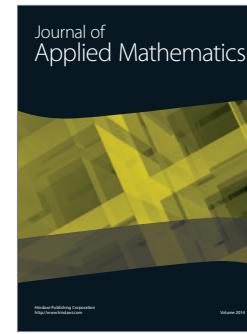
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